APPLETONS' MATHEMATICAL SERIES.

A

PRACTICAL ARITHMETIC.

BY

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AUTHOR OF

"A PRIMARY ARITHMETIC;" "AN ELEMENTARY ARITHMETIC;" "AN ENGLISH GRAMMAR;" "FIRST LESSONS IN COMPOSITION;" "ADVANCED COURSE OF COMPOSITION AND RHETORIC;" "A NATURAL PHILOSOPHY;" "ILLUSTRATED SCHOOL HISTORY OF THE UNITED STATES;" "PRIMARY HISTORY OF THE UNITED STATES;" ETC.

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PREFACE.

The Third of our Series of Arithmetics, designed for all ordinary classes in our Public and Private Schools, is now presented to the public. The aim has been to make it comprehensive, clear, free from verbiage in its definitions and explanations, inductive in its development of the subject, and well adapted to the school-room.

It is believed that the study of Arithmetic, apart from its necessity as a practical branch, may be rendered invaluable as a mental discipline. Every device has been resorted to in this work to make it useful as a means of intellectual training, of teaching the young learner to reflect and reason, at the same time without requiring anything that is not fairly within his reach. Acting on this principle, the author has not laid down rules arbitrarily, but shown the reasons for them by means of preliminary analyses. He has also placed occasional questions or suggestions after examples, in the belief that such hints, starting the learner in the right direction, would encourage him to attempt the solution for himself, rather than apply for aid to his teacher,—a practice as destructive of self-reliance in the one as it is annoying to the other.

To impress principles on the mind, as well as to impart facility in operating, much practice is necessary; and, to secure this, numerous examples are presented, applying the rules in a great variety of ways. The answers in most cases are given, but, to test the learner, a few under almost every rule are omitted. Answers are apt to suggest the processes used; and, if they are invariably given, even the most faithful will unconsciously fall

into the habit of depending upon them. A Key for the teacher's use will prevent any inconvenience at recitation.

A "*Practical*" Arithmetic should deserve its name, and we have kept this in view throughout. We have asked, What applications of Arithmetic is the pupil likely to need in life? What are the shortest methods, and those actually used by business men? The branches of Mercantile Arithmetic have received special attention,—the making out of bills, the casting of interest, partial payments, operations in profit and loss, averaging accounts, equation of payments, &c. Much collateral information on business subjects has been embodied. In a word, the author has weighed every line, with the view of giving what would be most useful and best prepare the learner for the duties of the counting-room.

The great distinguishing feature of this book is that it is adapted to the present state of things. The last five years have been five years of financial changes; specie payments have been suspended, prices have doubled, the tariff has been altered, a national tax levied, &c. No Arithmetic that ignores these changes should be placed in the hands of our youth. Time is too precious to be wasted in learning things wrong, only to unlearn them on entering into active life. Our examples are adapted to the present: the prices given are those of to-day; the difference between gold and currency is recognized and taught; the rates of duties agree with the present tariff; the mode of computing the national income tax is explained; a full description is given of the different classes of United States securities, with examples to show the comparative results of investments in them. These are matters that children, as well as adults, ought to know and understand.

It is hoped that these, with other features that will be obvious on examination but need not be mentioned here, may commend the work to teachers generally.

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PRACTICAL ARITHMETIC.

CHAPTER I.

NUMBERS.

1. One, a single thing, is called a **Unit.**

2. If we join another unit to ONE, we have TWO; if another, THREE; and so, adding a unit each time, we get FOUR, FIVE, SIX, SEVEN, EIGHT, NINE.

3. One, two, three, &c., are called Numbers. A **Number** is, therefore, one unit or more.

4. **Arithmetic** treats of Numbers.

5. Numbers are either **Abstract** or **Concrete**. They are Abstract, when not applied to any particular thing; as, *one*, *eight*. They are Concrete, when applied to particular things; as, *one* pound, *eight* dollars.

6. That to which a concrete number is applied, is called its **Denomination.** In the last example, *dollars* is the denomination of the number *eight.*

7. **Counting** is naming the numbers in order; as, *one, two, three, four, five,* &c.

8. We may express numbers by writing out their names, as *one, two, three;* or by characters, as 1, 2, 3.

QUESTIONS.—1. What is a single thing called?—2. What do we get by successive additions of a unit to *one*?—3. What are *one, two, three,* &c., called? What is a Number?—4. Of what does Arithmetic treat?—5. How are numbers distinguished? When are they called Abstract? When, Concrete?—6. What is meant by the Denomination of a concrete number?—7. What is Counting?—8. How may we express numbers?

CHAPTER II.

NOTATION.

9. Notation is the art of expressing numbers by characters.

10. Two systems of notation are used, the Ar'abic and the Roman.

The Arabic Notation.

11. The Arabic Notation is so called because it was introduced into Europe by the Arabs, who obtained it from India. It uses ten characters, called **Figures**:—

0	1	2	3	4	5	6	7	8	9
NAUGHT	ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE

12. The first of these figures, **0**, is called Naught, Cipher, or Zero. It implies the absence of number.

The other nine are called Significant Figures, or Digits,—each signifying a certain number.

13. The greatest number that can be expressed with one figure is *nine*, 9. For numbers above nine, we combine two or more figures.

First, 1 is placed at the left of each of the ten figures, forming 10, ten; 11, eleven; 12, twelve; 13, thirteen; 14, fourteen; 15, fifteen; 16, sixteen; 17, seventeen; 18, eighteen; 19, nineteen.

Then 2, forming 20, twenty; 21, twenty-one; 22, twenty-two; 23, twenty-three; 24, twenty-four; 25, twenty-five; 26, twenty-six; 27, twenty-seven; 28, twenty-eight; 29, twenty-nine.

Then 3, forming 30 (thirty), 31, 32, 33, 34, 35, 36, 37, 38, 39.

Then 4, forming 40 (forty), 41, &c. Then 5: 50 (fifty), 51, &c. Then 6: 60 (sixty), 61, &c. Then 7: 70 (seventy), 71, &c. Then 8: 80 (eighty), 81, &c. Then 9: 90 (ninety), 91, &c.

---

9. What is Notation?—10. How many systems of notation are used? What are they called?—11. Why is the Arabic Notation so called? How many characters does it use? What are they?—12. What is the first of these figures called? What does it imply? What are the other nine called? Why are they called *Significant*?—13. What is the greatest number that can be expressed with one figure? How do we express numbers above nine? Show how 1, 2, 3, &c., are combined in turn with each of the ten figures, and what numbers are thus formed.

14. UNITS, TENS, HUNDREDS.—The first or right-hand place is called the **units' place**; the second, the **tens' place.**

1 in the units' place (1) is 1 unit.
1 in the tens' place (10) is 1 ten, or ten units.
2 in the tens' place (20) is 2 tens, or twenty units.
3 in the tens' place (30) is 3 tens, or thirty units.

A figure, therefore, in the second place denotes so many *tens*, and its value is ten times as great as if it stood in the first place.

15. The value of a figure standing alone or in the first place is called its Simple Value. Its value in any other place is called its Local Value.

16. The greatest number that can be expressed with two figures is *ninety-nine*, 99. Next comes *one hundred*—100—expressed by putting 1 in the third place, which is called the **hundreds' place.**

To express hundreds, set the several figures in the third place with naughts after them:—

One hundred,	100.	Three hundred,	300.
Two hundred,	200.	Four hundred,	400, &c.

17. Observe how the numbers between the hundreds are expressed:—

One hundred and one,	101	— 1 hundred, 0 tens, 1 unit.
One hundred and two,	102	— 1 hundred, 0 tens, 2 units, &c.
One hundred and ten,	110	— 1 hundred, 1 ten, 0 units.
One hundred and eleven,	111	— 1 hundred, 1 ten, 1 unit, &c.
Two hundred and one,	201	— 2 hundreds, 0 tens, 1 unit.
Three hundred and one,	301	— 3 hundreds, 0 tens, 1 unit, &c.

18. Figures are grouped in Periods of three each. These three places, *units*, *tens*, *hundreds*, form the first period, or **Period of Units.**

---

14. What is the first or right-hand place called? The second? What is the value of 1 in the tens' place? 2 in the tens' place? 3 in the tens' place? What does a figure in the second place denote?—15. What is meant by the Simple Value of a figure, and what by its Local Value? Which of these always remains the same?—16. What is the greatest number that can be expressed with two figures? What comes after 99? How is one hundred expressed? How are the hundreds expressed?—17. Show by examples how the numbers between the hundreds are expressed.—18. How are figures grouped? What is the first period called? Of what three places does it consist?

EXERCISE IN NOTATION.

Express in figures, remembering that vacant places *on the right* must be filled with naughts:—

1. Five units. Four tens.
2. Three hundreds.
3. Eight tens, nine units.
4. Six hundreds, four tens.
5. Two hundreds, two units.
6. Seven hundreds, five tens.
7. 1 hundred, 1 ten, 5 units.
8. 1 hundred, 6 tens, 1 unit.
9. Five hundred and sixty.
10. Eighty-three. Thirty-nine.
11. One hundred and thirteen.
12. Nine hundred and nine.
13. Seven hundred and fifty.
14. Two hundred and twelve.
15. Four hundred and eighty-seven.

19. THOUSANDS:—The second period is that of **Thousands**. It consists of three places,—**thousands, ten-thousands, hundred-thousands.**

Observe how thousands are expressed:—

One thousand,	1,000.	Ten thousand,	10,000.
Two thousand,	2,000.	Fifty thousand,	50,000.
Three thousand,	3,000, &c.	Sixty thousand,	60,000, &c.

One hundred thousand,	100,000.
Seven hundred thousand,	700,000.
Eight hundred thousand,	800,000, &c.

20. *To express a given number of thousands, write the number in the second period. If there are numbers corresponding to the places of the first period, set them there; if not, supply naughts.*

EXAMPLE 1.—Write seven hundred and nine thousand.

To do this, write seven hundred and nine (709) in the second period. Supply three naughts for the units' period—709,000.

EXAMPLE 2.—Write seven hundred and nine thousand, and forty.

To do this, write 709, as before, in the second period, and 40 in the first, supplying a naught for the vacant hundreds' place—709,040.

---

19. What is the second period? Of what three places does it consist? Show how thousands are expressed.—20. Recite the rule for expressing a given number of thousands.—How do you write seven hundred and nine thousand? Seven hundred and nine thousand, and forty?

So, five hundred and fifty-one thousand,	551,000.
Ten thousand, six hundred and eighteen,	10,618.
Four hundred and sixty thousand, nine hundred,	460,900.

EXERCISE IN NOTATION.

Write the following numbers in figures :—

1. Fifty thousand. Four hundred thousand.
2. Two thousand, two hundred and twelve.
3. Two hundred thousand, six hundred and sixty-one.
4. Eight hundred and twenty thousand, and thirty.
5. Nine thousand, three hundred and seventy-one.
6. Forty-seven thousand, one hundred and nineteen.
7. Eighty-one thousand, and seven.
8. Sixty thousand, four hundred and eighty-two.
9. Seven hundred and twenty-eight thousand, eight hundred and fifty-seven.

21. MILLIONS, BILLIONS, TRILLIONS, &c.—The third period is that of **Millions.** It consists of three places,—**millions, ten-millions, hundred-millions.**

EXAMPLES.—	One million,	1,000,000.
	Ten million,	10,000,000.
	One hundred million,	100,000,000.

22. The fourth period is that of **Billions.** It consists of three places,—**billions, ten-billions, hundred-billions.**

EXAMPLES.—	One billion,	1,000,000,000.
	Ten billion,	10,000,000,000.
	One hundred billion,	100,000,000,000.

23. The Periods above billions are seldom used. They are called Trillions, Quadrillions, Quintillions, Sextillions, Septillions, Octillions, Nonillions, Decillions, &c.

Beginning at the right, name first the periods in order, then the places, as shown in the following Table:—

---

21. What is the third period? Of what three places does it consist?—22. What is the fourth period? Of what three places does it consist? Give examples of the mode of expressing millions and billions.—23. Name the periods above billions. How many places must we fill, to express a million? To express a billion? To express ten thousand?

Hundred-quadrillions.	Ten-quadrillions.	Quadrillions.	Hundred-trillions.	Ten-trillions.	Trillions.	Hundred-billions.	Ten-billions.	Billions.	Hundred-millions.	Ten-millions.	Millions.	Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.
6th Per.			5th Per.			4th Per.			3d Per.			2d Per.			1st Per.		
QUADRILLIONS			TRILLIONS			BILLIONS			MILLIONS			THOUSANDS			UNITS		

24. One ten is equivalent to ten units; one hundred, to ten tens. Hence, removing a figure one place to the right, diminishes its value ten times; removing it one place to the left, increases it ten times.

25. RULE FOR NOTATION.—*Write in each period, beginning with the highest mentioned, the number belonging to it, filling vacant places on the right with naughts.*

The left-hand period need not contain three places, but every other must.—Naughts before a number do not affect its value, and should not be written. Every naught placed after a number throws its figures one place farther to the left, and therefore increases its value ten times.

A person counting 100 every minute since the birth of Christ would not yet have reached a trillion.

EXERCISE IN NOTATION.

Write the following numbers in figures, placing units under units, tens under tens, &c.:—

1. Nine hundred and eighty-one million, seven hundred.
2. Ninety-six billion, one hundred million, and twelve.
3. Three hundred and twenty quadrillion, five thousand.
4. Fifteen quintillion, four quadrillion, ten thousand.
5. Eight trillion, twelve billion, seven hundred.
6. Two hundred and fifty-seven million, one hundred and ninety-one thousand, seven hundred and sixty-three.

---

24. What is the effect of removing a figure one place to the right? Of removing it one place to the left?—25 Give the rule for Notation. Which of the periods must contain three places, and which need not? What is the effect of prefixing a cipher to a number? Of annexing a cipher? What remark is made, to show how great a trillion is?

7. Ninety-eight sextillion, three hundred million, eleven thousand, four hundred and thirteen.

8. Seven hundred and seven trillion, forty-one million, seven hundred and twenty thousand, and one.

9. Four hundred and fifty trillion, five hundred and forty billion, forty-five million, fifty-four thousand, and eleven.

10. 475 decillion, 200 nonillion, 84 octillion, 7 septillion, 63 sextillion, 450 quintillion, 2 trillion, three hundred and sixty.

11. Eight hundred and ninety-one quadrillion, one trillion, fifty billion, six hundred and nine million, and seventy.

12. Two nonillion, fourteen septillion, two hundred and eleven quadrillion, thirteen trillion, five hundred and forty-six billion, twenty-seven thousand, and ninety-five.

13. Twenty quintillion, two hundred and seven billion, six hundred million, six thousand, and fifty-nine.

14. Two hundred sextillion, and sixty-nine.

15. One trillion, one hundred billion, and eleven.

The Roman Notation.

26. The Roman Notation, so called because it was used by the ancient Romans, employs seven letters. I. denotes one; V., five; X., ten; L., fifty; C., one hundred; D., five hundred; M., one thousand.

V resembles the outline of the hand with the five fingers spread. X is two V's, or fives, joined at their points. C begins the Latin word *centum*, one hundred. It was sometimes written in this form **C**, and the lower half, afterwards written as L, denoted fifty. M begins the Latin word *mille*, a thousand. A thousand was sometimes written CIƆ; hence IƆ, afterwards changed to D, denoted 500.—Some think that V was used to denote five, because U, which was anciently written V, was the fifth vowel.

27. These letters are combined to express numbers, according to the following principles:—

1. If a letter is repeated, its value is repeated. XX. is twenty; III. is three.

26. Why is the Roman Notation so called? What does it use to represent numbers? Why is it supposed that V was used for 5, and X for 10? How did C come to denote 100, and L 50? Why was M used for 1000, and D for 500?—27. In combining these characters to express numbers, what is the effect of repeating a letter?

2. A letter of less value, placed after one of greater, unites its value to that of the latter. VI. is six.

3. A letter of less value, placed before one of greater, takes its value from that of the latter. IV. is four.

4. A letter of less value, placed between two of greater, takes its value from that of the other two united. LIV. is fifty-four.

5. A bar over a letter increases its value a thousand times. $\overline{\text{V}}$. is five thousand.

TABLE.

I.	One.	L.	Fifty.
II.	Two.	LX.	Sixty.
III.	Three.	LXX.	Seventy.
IV.	Four.	LXXX.	Eighty.
V.	Five.	XC.	Ninety.
VI.	Six.	C.	One hundred.
VII.	Seven.	CI.	One hundred and one.
VIII.	Eight.	CC.	Two hundred.
IX.	Nine.	CCC.	Three hundred.
X.	Ten.	CCCC.	Four hundred.
XI.	Eleven.	D.	Five hundred.
XII.	Twelve.	DC.	Six hundred.
XIII.	Thirteen.	DCC.	Seven hundred.
XIV.	Fourteen.	DCCC.	Eight hundred.
XV.	Fifteen.	DCCCC.	Nine hundred.
XVI.	Sixteen.	M.	One thousand.
XVII.	Seventeen.	MM.	Two thousand.
XVIII.	Eighteen.	MMM.	Three thousand.
XIX.	Nineteen.	MMMM.	Four thousand.
XX.	Twenty.	$\overline{\text{V}}$.	Five thousand.
XXI.	Twenty-one.	$\overline{\text{X}}$.	Ten thousand.
XX.	Thirty.	$\overline{\text{L}}$.	Fifty thousand.
XL.	Forty.	$\overline{\text{M}}$.	One million.

What is the effect of placing a letter of less value after one of greater? Of placing a letter of less value before one of greater? Of placing one of less value between two of greater? Of placing a bar over a letter? Learn the Table.

The Roman Notation is now used chiefly in expressing dates, marking the hours on clock and watch faces, paging prefaces, and numbering volumes, chapters, or lessons of books. It was ill adapted for use in calculating or keeping accounts, and was for most purposes superseded with great advantage, in the 16th century, by the Arabic Notation, which had been introduced among the learned of Europe two hundred years before, and had been gradually made known by means of almanacs.

EXERCISE IN NOTATION.

Write the following numbers, first by the Arabic, and then by the Roman, Notation:—

1. Eighteen.
2. Forty-five.
3. Six hundred.
4. Three thousand.
5. Seventy-nine.
6. Eight hundred.
7. Ten thousand.
8. Twenty-nine.
9. Fifteen hundred (or, one thousand, five hundred).
10. Nineteen hundred and five.
11. Twelve hundred and thirty-eight.
12. One million, one thousand, and one.
13. Five thousand, seven hundred and ninety.
14. One hundred thousand, and eleven.
15. Fifty thousand, four hundred and fifty-four.
16. Sixteen hundred and ninety-nine.
17. One thousand, six hundred and sixty.
18. Two thousand, two hundred and eighty-seven.

Express the following numbers according to the Roman Notation: 325; 13; 10,500; 81; 119; 50,909; 1,000,000; 48; 5,555; 76; 1,864; 4,200; 14; 15,000; 849; 1,111; 52; 660; 101,000.

Express the following numbers according to the Arabic Notation: $\overline{\mathrm{M}}$. XXV. MDCCC. LXVIII. $\overline{\mathrm{VV}}$. $\overline{\mathrm{L}}\mathrm{CCC}$. XVII. LI. XII. DC. CCCCIV. $\overline{\mathrm{CX}}$. VI. XI. IX. MCXIII. MDCCCCLXXXIX. CXIX.

iv. lii. cx. xciii. xxxviii. lxx. xiv. cxliv. lxxxi.

For what is the Roman Notation now chiefly used? When was it superseded by the Arabic? When was the Arabic Notation first introduced, and how was it made known?

CHAPTER III.

NUMERATION.

28. Numeration is the art of reading numbers expressed by characters.

29. Rule for Numeration.—1. ***Beginning at the right, divide the number into periods of three figures each.***

2. ***Beginning at the left, read the figures in each period as if they stood alone, adding the name of the period in every case except the last.***

The figures of the right hand period are never named as units, the word *units* being understood. We read 500 *five hundred*, not *five hundred units.*

Places containing 0 must be passed over in reading. We read 2043 *two thousand and forty-three*, not *two thousand, no hundred and forty-three.*

Examples.

1,100	One thousand, one hundred; or, eleven hundred.
140,000	One hundred and forty thousand.
20,009,000	Twenty million, nine thousand.
11,000,000,017	Eleven billion, and seventeen.
9,000,070,212,005	Nine trillion, seventy million, two hundred and twelve thousand, and five.
6,020,100,009,000,400	Six quadrillion, twenty trillion, one hundred billion, nine million, four hundred.

EXERCISE IN NUMERATION.

Name the places in order—*units, tens, hundreds,* &c.; then read the number:—

1.	840000	7.	3712300000041586З
2.	75819	8.	714629300000927000
3.	30451000	9.	46327723207003
4.	1673549000227	10.	15615000000111101
5.	84001100206	11.	827716420018
6.	290902029092209	12.	7423065056506650128

28. What is Numeration?—29. Give the rule for Numeration. Which period has its name understood in reading? How do we read places containing 0? Give an example.

13.	4987043790080465	21.	LXVI.
14.	224000000600317010	22.	MCXCII.
15.	563745119	23.	$\overline{\text{MX}}$DCCCL.
16.	1612875962	24.	$\overline{\text{D}}$CCCXIX.
17.	18459223	25.	CXVIII.
18.	DXLIX.	26.	MCCLXXIV.
19.	CCCCXCVII.	27.	$\overline{\text{L}}$MCXXII.
20.	DCCCLXXXII.	28.	$\overline{\text{V}}$CCCXXXIII.

29. Fill nine periods with ones. Read this number.

30. Fill seven periods with fours. Read this number.

31. Set down 7 ten-thousands, 1 thousand, 5 hundreds, 6 tens, 2 units. Read the number thus formed.

32. Set down 2 trillions, 3 ten-billions, 8 ten-millions, 6 millions, 1 thousand, 9 tens, 5 units, filling vacant places with naughts. Read this number.

30. English Numeration Table.—The division of numbers into periods of *three* figures each, as shown on page 12, is that followed in the United States, France, and the continent of Europe generally. The English divide into periods of *six* figures each, naming them and the places they contain as follows:—

4th Per. TRILLIONS	3d Per. BILLIONS	2d Per. MILLIONS	1st Per. UNITS
Hundred-thousands of trillions.	Hundred-thousands of billions.	Hundred-thousands of millions.	Hundred-thousands.
Ten-thousands of trillions.	Ten-thousands of billions.	Ten-thousands of millions.	Ten-thousands.
Thousands of trillions.	Thousands of billions.	Thousands of millions.	Thousands.
Hundred-trillions.	Hundred-billions.	Hundred-millions.	Hundreds.
Ten-trillions.	Ten-billions.	Ten-millions.	Tens.
Trillions.	Billions.	Millions.	Units.

30. How does the English Numeration Table differ from ours? Name the first eighteen places according to our table; according to the English table. When we speak of a billion in this country, how many million do we mean?

CHAPTER IV.

ADDITION.

31. Four girls and three boys went a riding; how many went in all?

Here we are required to find one number containing as many units as 4 and 3 together. This process is called Addition.

32. Addition is the process of uniting two or more numbers in one, called their **Sum.** Adding 4 and 3, we have 7 for their sum.

Addition Table.

0 and 1 are 1; 0 and 2 are 2, 0 and any number make that number.
1 and 0 are 1; 2 and 0 are 2; any number and 0 make that number.

1 and	2 and	3 and	4 and	5 and
1 are 2	1 are 3	1 are 4	1 are 5	1 are 6
2 are 3	2 are 4	2 are 5	2 are 6	2 are 7
3 are 4	3 are 5	3 are 6	3 are 7	3 are 8
4 are 5	4 are 6	4 are 7	4 are 8	4 are 9
5 are 6	5 are 7	5 are 8	5 are 9	5 are 10
6 are 7	6 are 8	6 are 9	6 are 10	6 are 11
7 are 8	7 are 9	7 are 10	7 are 11	7 are 12
8 are 9	8 are 10	8 are 11	8 are 12	8 are 13
9 are 10	9 are 11	9 are 12	9 are 13	9 are 14
10 are 11	10 are 12	10 are 13	10 are 14	10 are 15
6 and	**7 and**	**8 and**	**9 and**	**10 and**
1 are 7	1 are 8	1 are 9	1 are 10	1 are 11
2 are 8	2 are 9	2 are 10	2 are 11	2 are 12
3 are 9	3 are 10	3 are 11	3 are 12	3 are 13
4 are 10	4 are 11	4 are 12	4 are 13	4 are 14
5 are 11	5 are 12	5 are 13	5 are 14	5 are 15
6 are 12	6 are 13	6 are 14	6 are 15	6 are 16
7 are 13	7 are 14	7 are 15	7 are 16	7 are 17
8 are 14	8 are 15	8 are 16	8 are 17	8 are 18
9 are 15	9 are 16	9 are 17	9 are 18	9 are 19
10 are 16	10 are 17	10 are 18	10 are 19	10 are 20

31. In the given example, what are we required to do? What is this process called?—32. What is Addition? What is the result of addition called? How much do 0 and any number make? How much do any number and 0 make? Recite the Table.

33. Addition is denoted by an erect cross +, called **Plus**, placed between the numbers to be added. $6+5$ is read *six plus five*, and means that *six and five are to be added.*

34. Two short horizontal lines =, placed between two quantities or sets of quantities, denote that they are equal. $6+4=10$ is read *six plus four equals ten*, and means that *the sum of six and four is ten.*

35. Observe the following:—

$3+2=5$	$4+5=9$	$3+7=10$	$5+8=13$
then	then	then	then
$13+2=15$	$4+35=39$	$53+7=60$	$5+28=33$
$23+2=25$	$4+45=49$	$63+7=70$	$45+8=53$
$33+2=35$, &c.	$4+55=59$, &c.	$73+7=80$, &c.	$5+88=93$, &c.

36. Observe that $4+5=9$, and $5+4=9$.

Hence, when numbers are to be added, it makes no difference which is taken first.

EXERCISE ON THE ADDITION TABLE.

How many are 7 and 6? 6 and 7? 17 and 6? 16 and 7? 6 and 27? 6 and 47? 57 and 6? 8 and 5? 5 and 8?

How many are 5 and 7? 85 and 7? 87 and 5? 7 and 85? 5 and 87? 4 and 8? 4 and 38? 38+1+3? 4+2? 104+2? 164+2? 174+2? 274+2? 92+4? 402+4?

How many are 9 and 3? 29 and 3? 8 and 2? 48 and 2? 108 and 2? 102 and 8? 8 and 3? 38 and 3? 3 and 78? 178 and 3? 278 and 3? 11 and 1? 15 and 2? 10 and 2? 20 and 3? 5 and 30? 6 and 3? 23 and 6?

Add 9 and 7. 9 and 9. 9 and 8. 9 and 57. 9 and 59. 9 and 58. 9 and 47. 9 and 5. 4, 5, and 9. 5, 4, and 19.

Add 9 and 2. 6, 3, and 2. 89 and 2. 2 and 29. 7, 2, and 1. 49 and 1. 51, 1, and 8. 9 and 161. 1 and 179.

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33. How is addition denoted?—34. What do two short horizontal lines denote? How is 6+4=10 read?—35. How much is 3+2? 13+2? 33+2? 4+5? 4+35? 4+45? 3+7? 53+7? 63+7? 3+77?—36. How much is 4+5? How much is 5+4? What principle is deduced from this?

What does 6+6 equal? 6+106? 126+6? 226+6? 6+36? 46+3+3? 56+4+2? 5+1+46?

What does 4+6 equal? 104+6? 24+6? 6+14? 5+6? 85+6? 5+76? 9+4? 9+44?

What is the sum of 10 and 10? 10 and 20? 10 and 30? 4, 6, and 70? 7, 3, and 80? 10 and 5? 4, 5, and 8? 3, 6, and 19? 4, 2, and 7? 7, 3, and 2? 8, 4, and 8? 21, 5, and 3? 93, 4, and 3?

Count by twos, commencing 2, 4, 6, 8, &c., up to 100.

Count by twos, commencing 1, 3, 5, 7, &c., up to 99.

Count by threes, commencing 3, 6, 9, 12, &c., up to 99.

Count by threes, commencing 2, 5, 8, 11, &c., up to 98.

Count by threes, commencing 1, 4, 7, 10, &c., up to 100.

Count by fours, commencing 4, 8, 12, 16, &c., up to 100.

Count by fives, commencing 5, 10, 15, 20, &c., up to 100.

37. Applications of Addition.—To find a whole, when its parts are given, add the parts.

38. To find the selling price, when the cost and gain are given, add the cost and gain.

39. To find the cost, when the selling price and loss are given, add the selling price and loss.

40. To find a later date, when an earlier date (A. D., or after Christ) and the difference of years are given, add the earlier date and the difference of years.

MENTAL EXERCISES.

1. A boy has 20 cents in one pocket, 9 in another, and 3 in a third; how many cents has he in all?

Ans. 20+9+3 cents, or 32 cents.

2. A farmer buys a cow for $32*, and sells her so as to gain $7; what does she bring? (See § 38.)

37. How do you find a whole, when its parts are given?—38. How do you find the selling price, when the cost and gain are given?—39. How do you find the cost, when the selling price and loss are given?—40. How do you find a later date, when an earlier date and the difference of years are given?

* This mark ($) denotes *dollars*. It is always placed before the number. $32 is read *thirty-two dollars*.

3. Sold a picture for $38, at a loss of $8; what did the picture cost? (See § 39.)

4. In what year will Charles be eight years old, if he was born in 1861? (See § 40.)

5. How many strokes will a clock strike in twelve hours, commencing at 1 o'clock?

6. I spent $10 for a coat, $4 for a vest, and $6 for a hat; what did the whole cost?

7. Five cows in one field, three in another, and seventeen in a third, make how many cows altogether?

8. A grocer has 10 barrels of flour, 5 of sugar, and 6 of potatoes; how many barrels has he in all?

9. Clay was born in 1777. Webster was born 5 years later; what was the date of Webster's birth?

10. There are 14 bones in the face, 6 in the ears, and 8 in the back of the skull; how many bones in the whole head?

11. Washington became president in 1789. He held the office eight years; when did he leave it?

12. There are 31 days in July, and 31 in August; how many days in both months?

MODEL.—Thirty-one is 3 tens and 1 unit. 3 tens and 1 unit, added to 3 tens and 1 unit, make 6 tens and 2 units, or 62. *Ans.* 62 days.

13. A person travelled 40 miles by railroad, and 35 miles by stage; how far did he go in all?

14. Genesis contains fifty chapters, and Exodus forty; how many chapters in both?

15. I have three lots of land; the first contains 30 acres, the second 10, and the third as many as the other two together. How many acres in all three?

16. A certain orchard contains 16 apple-trees that bear, and 4 that do not; 7 pear-trees that bear, and 3 that do not; and 10 cherry-trees. How many apple-trees does it contain? How many pear-trees? How many trees altogether?

17. If a person spends $10 on Monday, $5 on Tuesday, and as much on Wednesday as on both the previous days, how many dollars does he spend altogether?

41. Adding by Columns.—When the numbers are large, we set them down and add the columns separately.

Example.—A merchant gives \$5261 for one lot of goods, \$432 for another, and \$303 for a third. How much do they all cost him?

That we may unite things of the same kind, in setting the numbers down, place units under units, tens under tens, &c.

Begin to add at the bottom of the right-hand column.

3 and 2 units are 5 units, and 1 makes 6. Set down 6 under the units.

0 and 3 tens are 3 tens, and 6 make 9. Set down 9 under the tens.

3 and 4 hundreds are 7 hundreds, and 2 make 9. Set down 9 under the hundreds.

Bring down the 5 thousands.—*Ans.* \$5996.

	\$5261
	432
	303
Ans.	\$5996

42. Proof of Addition.—**Proving** a sum is finding whether the work is correct.

Addition is proved by adding the columns from the top downward. If the sum is the same as when they are added from the bottom upward, we infer that the work is right. If an error has been made in the first addition, it is not likely to be repeated in the second, when the numbers are taken in a different order.

Example.—Prove the example in § 41. Add each column from the top downward. 1 and 2 are 3, and 3 is 6. 6 and 3 are 9. 2 and 4 are 6, and 3 is 9. Bring down 5. *Ans.* \$5996,—the same as before. Hence the work is right.

	\$5261
	432
	303
Ans.	\$5996

EXAMPLES FOR PRACTICE.

Read and add the following numbers; prove each example:—

(1)	(2)	(3)
62317	1100264	29761005483314732
4330	53105	47211574162
722321	58234510	22851040001004

41. How do we deal with the numbers, when they are large? In the given example, how must we set down the numbers to be added? Why so? Proceed with the addition.—42. What is meant by *proving* a sum? How is addition proved? Why should the result be the same when you add in the opposite direction? (§ 36) Why is not an error in the first addition likely to be repeated in the second? Prove the example in § 41.

4. Add 90153, 321, 405301, and 3214. *Ans.* 498989.

5. Add 84600325, 70, 55402, and 123201. *Ans.* 84778998.

6. Add 41, 725, 12, 200, 4001, 20, and 3000. *Ans.* 7999.

7. Add 11, 232, 2543, 14201, 870012. *Ans.* 886999.

8. What is the sum of 1640263, 1501, 214123, and 23011?

9. What is the value of 26+231041+711+55101+2110?

10. Add thirteen hundred; nineteen million, two hundred thousand, five hundred; forty-two; five hundred and twenty-four thousand, and thirteen; and twenty million, fourteen thousand, and thirty-two. *Ans.* 39739887.

11. Add eleven million, two hundred and twenty-three thousand, four hundred and fifty-one; five hundred and ten thousand, two hundred and fifteen; five million, one hundred and forty-one thousand, one hundred and twenty-two; and twelve thousand, two hundred. *Ans.* 16886988.

12. What is the sum of 14 billion, three hundred and twenty-one; 2 billion, 15 million, 111 thousand, three hundred and five; 420 million, 12 thousand, and fifty-three; and 131 million, 600 thousand? *Ans.* 16566723679.

13. Find the sum of twenty trillion, two hundred billion, two million, and seventeen; thirty-one billion, three hundred and seventy-one million, six hundred and thirty-four thousand; thirteen thousand, three hundred and twenty-one; and five billion, eleven million, twenty-one thousand, four hundred and forty.
Ans. 20236384668778.

14. Add eleven million and eleven; five million, two hundred and ninety-two thousand, one hundred and twenty-three; and six thousand, five hundred and fifty. *Ans.* 16298684.

15. One town contains 16735 inhabitants, another 22242; what is the population of both?

16. How many acres in three farms, if the first contains 427 acres, the second 250 acres, the third 211 acres?

17. A person who is worth $145250, makes $10000 more, and has $220700 left to him. What is he then worth?

18. If an army of 23452 men is reënforced with 15316 men, how many will it then contain?

19. A boat starts with 1652 bushels of wheat aboard; 43 miles down the river, it receives 27 barrels of flour and 3335 bushels of wheat. How many bushels of wheat are then aboard?

Ans. 4987 bushels.

43. CARRYING.—The sum of a column may consist of more than one figure. In this case, *set down the right-hand figure, and carry the left-hand figure or figures to the next column.*

If the sum of a column is 64, set down 4 and carry 6; if it is 93, set down 3 and carry 9; if it is 127, set down 7 and carry 12, &c.

EXAMPLE.—Add 3658, 4903, 7006, and 734.

Set down the numbers, units under units, tens under tens, &c. Begin to add at the right. The sum of the units is 21,—that is, 2 tens and 1 unit. Set down the 1 unit in the units' place, and carry the 2 tens to the tens' column.

2 and 3 are 5, and 5 is 10. 10 tens are 1 hundred and 0 tens. Set down 0 in the tens' place, and carry the 1 hundred to the hundreds' column.

1 and 7 are 8, and 9 is 17, and 6 is 23. 23 hundreds are 2 thousands and 3 hundreds. Set down 3 in the hundreds' place, and carry the 2 thousands to the column of thousands.

2 and 7 are 9, and 4 is 13, and 3 is 16. 16 thousands are 1 ten-thousand and 6 thousands. This being the last column, set them down. *Ans.* 16301.

	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.
		3	6	5	8
		4	9	0	3
		7	0	0	6
			7	3	4
Ans.	1	6	3	0	1

EXAMPLES FOR PRACTICE.

Read and add the following numbers; prove each example:—

(20)	(21)	(22)
49778	857215	24313755596
112	29524	24464485
352243	8461489	5325273374
5314	828	306482265
5154432	58327317	3694817601153
65423216	874516526	12365498705618

43. When the sum of a column consists of more than one figure, what must we do? If the sum of a column is 64, what do we do? If it is 93? If it is 127? If the sum of a column is expressed by three figures, how many do we set down, and how many do we carry? Add the numbers in the given example, and show how we carry.

44. Rule for Addition.

1. *Write units under units, tens under tens, &c.*

2. *Beginning at the right, find the sum of each column.*

3. *If the sum is expressed by one figure, write it under the column added; if not, set down the right-hand figure, and carry the left-hand figure or figures to the next column.*

4. *Prove by adding in the opposite direction.*

EXAMPLES FOR PRACTICE.

1. Add 123405, 2354210, 354, 794327, and 36547.

2. Add 27562, 345607, 2461, 4567801, and 365.

3. Add 1034001, 78954, 379205, 367001, and 45637.

4. Add 11, 4562, 773, 15266, 8958, and 66666.

5. Add 100375, 406780, 4673005, 4112, 18365791, 2478, and 164357. *Ans.* 23716898.

6. What is the sum of three hundred thousand, six hundred and fifty; seven thousand, eight hundred and thirty-two; eleven thousand, five hundred and sixty-seven; ten thousand and fifty-six; four hundred and seventy-two? *Ans.* 330577.

7. A man drew five loads of bricks. In the first load, he had 1209; in the second, 1453; in the third, 1101; in the fourth, 1212; in the fifth, 1303. How many bricks were there in all?

8. If there are shipped from the United States, 15624 barrels of flour to Sweden, 250 barrels to Holland, 205154 to England, 6401 to Cuba, and 19602 to Mexico, how many barrels are shipped altogether?

9. Find the sum of eighty-eight million, and nineteen; forty-seven thousand, and sixty-eight; nine million, seven hundred and eighty-five thousand; nine hundred and eighty-six; eight billion, seven million. *Ans.* 8104833073.

10. How many square miles in British America, there being 2,436,000 square miles in the Hudson's Bay Territories, 357822 in Canada, 27704 in New Brunswick, 18746 in Nova Scotia, 2134 in

Prince Edward's Island, 35913 in Newfoundland, and 19 in the Bermuda Islands? *Ans.* 2878338 square miles.

11. It is computed that there are two million pagans in North America, two million in South America, one million in Europe, five hundred and ten million in Asia, sixty-five million in Africa, and twenty-four million in Oceania. How many pagans are there in the world? *Ans.* 604,000,000.

12. Maine, the largest of the New England States, contains 31766 square miles. New York, the largest of the Middle States, contains 15234 square miles more than Maine. How many square miles in New York?

13. A person has $1557 in one bank, $2343 in another, and in a third as much as in both the other two. How much has he in the third bank? How much in all three?

14. A lady gave $3445 for a house and $1055 for furniture. She then bought some adjoining land for as much as both house and furniture cost. What did she give for the whole?

15. Wellington's army at Waterloo consisted of 49608 infantry, 12402 cavalry, and 5645 artillery-men. How many men did it contain in all?

16. Napoleon's army at Waterloo consisted of 48950 infantry, 15765 cavalry, and 7232 artillery-men. How many men did it contain in all?

17. How many men did both Wellington's and Napoleon's army at Waterloo contain? *Ans.* 139602 men.

18. How many bushels of wheat are there on four boats, each of which contains 5250 bushels of wheat and 45 barrels of flour? *Ans.* 21000 bushels.

19. President Madison was born in 1751, and attained the age of eighty-five; in what year did he die?

20. How many strokes does a clock strike in 24 hours?

21. A lady gave each of her three daughters $9250, and her son $8345. How much did she distribute among them?

22. The earth is 95298260 miles from the sun. The planet Neptune is 2767105740 miles farther. What is Neptune's distance from the sun?

23. A has $4250; B has $375 more than A; C has as much as A and B together. What are all three worth? *Ans.* $17750.

24. How far is it from New York to Buffalo, the distance from New York to Albany being 150 miles, from Albany to Rochester 251 miles, and from Rochester to Buffalo 75 miles?

25. A man worth $12500 makes as much more, and has $5490 left to him. What is he then worth? *Ans.* $30490.

26. Required the whole population of the world, that of N. America being estimated at 46000000; S. America, 20000000; Europe, 280000000; Asia, 680000000; Africa, 80000000; and Oceania, 28000000.

27. How many men in an army consisting of four regiments, two of nine hundred and eighty men each, and two of twelve hundred and forty? *Ans.* 4440 men.

28. A merchant bought $1786 worth of books, and $875 worth of stationery. On the books he gained $549, and on the stationery $228. What did he sell the books for? What did he sell the stationery for? What was his whole gain?

Ans. Books, $2335. Stationery, $1103. Gain, $777.

29. In 1862, the postal revenue of the U. S. amounted to $8299820; in 1863, it was $2863969 more. What did it then amount to?

30. If I invest $2356 in pork, and $1977 in beef, and sell them so as to gain $395, how much do I receive for the whole?

31. A man left his wife $95000; each of his three sons, $15000; his daughter, $34000; and the rest of his property, which amounted to $47250, to charitable societies. What was the whole value of his estate? *Ans.* $221250.

32. A's orchard contains 146 apple-trees; B's, 22 pear-trees, 9 plum-trees, and 27 apple-trees; C's, 18 plum-trees, 139 apple-trees, and 38 pear-trees. How many apple-trees in all three orchards? How many pear-trees? How many plum-trees? How many trees altogether?

33. The salary of the Vice-President of the United States is $8000 a year; that of the President is $17000 more. What do the yearly salaries of both amount to?

84. A certain school opens with 78 boys and 129 girls. Within 30 days there is an addition of 42 boys and 39 girls. How many does the school then contain?

85. Add LXVI., MDXIX., CCIV., XVIII. *Ans.* 1807.

86. Add $\overline{\text{MD}}$., $\overline{\text{V}}$CXXX., XLIV., CXV., $\overline{\text{X}}$. *Ans.* 1015789.

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45. Practise the following examples till they can be added at sight up and down, naming the results only. Thus, in Example 1:—*three, eight, fourteen, sixteen, seventeen, twenty-three, thirty, thirty-four, thirty-six*—set down 6, and carry 3. *Three, five, nine, sixteen,* &c.

(1)	(2)	(3)	(4)
1179582	28204681	76456789	5234567
2295344	17130579	76789123	6346789
3381437	96792468	13123456	2678912
4574296	85246835	63456789	7891235
5275011	74683579	55789123	1124567
6443322	63357924	21123456	4456789
7109876	52642753	44456789	4678913
8123345	31297531	32789123	8892345
9345123	97468864	76123456	8123456

(5)	(6)	(7)	(8)
7389234	66694375	34751212	7634725
6188345	53693025	23586259	8583614
5267345	35215354	11310344	9472583
1856234	86424443	97311581	8361472
2945245	14644636	80678363	7258361
3134123	53301445	72846256	6747258
4723345	49321435	62562172	5136147
2312234	21648673	57166249	4825836
5689345	36623535	47691554	3614725
6490133	55615525	34426235	2583614
3567345	41623573	20934389	9472583
1289234	24635521	19281213	8361472
7631405	36159247	42816354	7258361

46. When the sum of a column consists of three figures, the two left-hand figures must be carried. Thus, in Example 9, the sum of the first column is 108—set down 8, and carry 10.

(9)	(10)	(11)	(12)
846750	683621	635643	802950
846750	562961	428744	395135
846750	963421	935613	801915
846759	137653	453824	717486
846759	412623	254872	496395
846759	525636	662816	585289
846759	547213	654883	674198
846759	325443	544865	717487
846759	202652	111834	496396
846759	517650	653851	585285
846759	531021	452840	674199
846759	446133	232890	717488
846759	107553	623886	496397
846759	547605	651893	585286
353759	838714	734882	674195
12208358	7349899	8033336	9420101

(13)	(14)	(15)
1399736957	3826486095	7587897897
7228226934	6667483794	4182596897
6179552483	988253592	5469675797
5598504587	8759236294	4829827957
4499246912	486755	2789494849
3398715975	7631486745	4589296895
4297751987	4782884793	9286795997
5181736661	3292692	1743394797
6548424326	5884276195	6949417875
7464352987	85486785	7489386822
6394756980	8926386411	3589095897
5298656957	6867141394	9791297897
4164157256	3788445795	2278096895
3547399992	4949951678	7639155256
71201220994	63161299018	78215431728

CHAPTER V.

SUBTRACTION.

47. Five hens are on a roost. Three fly down; how many remain?

Here we are required to take 3 from 5, or to find the difference between 3 and 5. This process is called Subtraction.

48. Subtraction is the process of taking one number from another.

SUBTRACTION TABLE.

0 from 1 leaves 1; 0 from 2, 2; 0 from any number leaves that number.

1 from	2 from	3 from	4 from	5 from
1 leaves 0	2 leaves 0	3 leaves 0	4 leaves 0	5 leaves 0
2 leaves 1	3 leaves 1	4 leaves 1	5 leaves 1	6 leaves 1
3 leaves 2	4 leaves 2	5 leaves 2	6 leaves 2	7 leaves 2
4 leaves 3	5 leaves 3	6 leaves 3	7 leaves 3	8 leaves 3
5 leaves 4	6 leaves 4	7 leaves 4	8 leaves 4	9 leaves 4
6 leaves 5	7 leaves 5	8 leaves 5	9 leaves 5	10 leaves 5
7 leaves 6	8 leaves 6	9 leaves 6	10 leaves 6	11 leaves 6
8 leaves 7	9 leaves 7	10 leaves 7	11 leaves 7	12 leaves 7
9 leaves 8	10 leaves 8	11 leaves 8	12 leaves 8	13 leaves 8
10 leaves 9	11 leaves 9	12 leaves 9	13 leaves 9	14 leaves 9
6 from	**7 from**	**8 from**	**9 from**	**10 from**
6 leaves 0	7 leaves 0	8 leaves 0	9 leaves 0	10 leaves 0
7 leaves 1	8 leaves 1	9 leaves 1	10 leaves 1	11 leaves 1
8 leaves 2	9 leaves 2	10 leaves 2	11 leaves 2	12 leaves 2
9 leaves 3	10 leaves 3	11 leaves 3	12 leaves 3	13 leaves 3
10 leaves 4	11 leaves 4	12 leaves 4	13 leaves 4	14 leaves 4
11 leaves 5	12 leaves 5	13 leaves 5	14 leaves 5	15 leaves 5
12 leaves 6	13 leaves 6	14 leaves 6	15 leaves 6	16 leaves 6
13 leaves 7	14 leaves 7	15 leaves 7	16 leaves 7	17 leaves 7
14 leaves 8	15 leaves 8	16 leaves 8	17 leaves 8	18 leaves 8
15 leaves 9	16 leaves 9	17 leaves 9	18 leaves 9	19 leaves 9

47. Repeat the example. What are we here required to do?—48. What is Subtraction? What does 0 from any number leave? Recite the Table.

49. The number to be subtracted, is called the **Subtrahend**; that from which it is to be taken, the **Minuend.** The result is called the **Remainder,** or **Difference.**

3 from 5 leaves 2; 3 is the subtrahend, 5 the minuend, 2 the remainder or difference.—If the minuend is less than the subtrahend, the subtraction can not be performed; we can not take 3 from 2.

50. Subtraction is denoted by a short horizontal line —, called **Minus,** placed before the subtrahend. 5—3 is read *five minus three,* and means that 3 *is to be subtracted from* 5.

51. Observe the following:—

3—2=1	7—3=4	7—3=4	11—10=1
then	then	then	then
13—2=11	47—3=44	27—23=4	31—10=21
23—2=21	57—3=54	77—73=4	51—10=41
33—2=31, &c.	67—3=64, &c.	87—83=4, &c.	91—10=81, &c.

EXERCISE ON THE SUBTRACTION TABLE.

Subtract 4 from 5. 4 from 15. 14 from 15. 4 from 24. 4 from 44. 54 from 55. 2 from 6. 2 from 66. 62 from 66.

Take 3 from 5. 3 from 75. 3 from 85. 83 from 85. 1 from 9. 1 from 19. 11 from 19. 3 from 8. 3 from 38.

How much is 1—1? 3—3? 23—23? 33—23? 43—33? 43—10? 53—10? 54—10? 6—3? 86—3? 86—83? 86—10? 8—4? 28—4? 28—24? 9—5? 49—5? 9—7? 69—7?

Take 3 from 9. 3 from 59. 3 from 99. 2 from 10. 2 from 20. 2 from 60. 6 from 10. 6 from 50. 6 from 80.

How much is 12—7? 22—7? 42—7? 92—7? 15—8? 65—8? 75—8? 14—9? 34—9? 44—9? 64—9? 15—10? 25—10? 16—9? 26—9? 76—9? 86—9? 10—4? 10—3?

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49. What is the number to be subtracted called? What is the number from which the subtrahend is to be taken called? What is the result called? 3 from 5 leaves 2; select the minuend, subtrahend, and remainder. In what case can the subtraction not be performed?—50. How is subtraction denoted? How is 5—3 read? What does it mean?—51. How much is 3—2? What follows? How much is 7—3? 27—23? 77—73? How much is 11—10? 31—10? 51—10?

Subtract 8 from 14. 8 from 64. 8 from 84. 6 from 13. 6 from 83. 7 from 14. 7 from 74. 8 from 17. 8 from 87.

Take 7 from 10. 9 from 18. 6 from 15. 2 from 89. 3 from 47. 4 from 56. 25 from 29. 36 from 38. 7 from 11.

Count backward by twos from 100. Thus: 100, 98, 96, &c.

Count backward by twos from 99. Thus: 99, 97, 95, &c.

Count backward by threes from 99. Thus: 99, 96, 93, &c.

Count backward by fours from 100. Thus: 100, 96, 92, &c.

Count backward by fives from 100. Thus: 100, 95, 90, &c.

52. APPLICATIONS OF SUBTRACTION.—When a whole and one of its parts are given, to find the other part, subtract the given part from the whole.

53. When a whole and all its parts but one are given, to find that one, subtract the sum of the given parts from the whole.

54. When the cost and selling price are given, to find the gain, subtract the cost from the selling price.

55. When the cost and loss are given, to find the selling price, subtract the loss from the cost.

56. When the selling price and gain are given, to find the cost, subtract the gain from the selling price.

57. When a later date and the difference of years are given, to find an earlier date (A. D., or after Christ), subtract the difference of years from the later date.

MENTAL EXERCISES.

1. A grocer who has 19 barrels of flour, sells 10 of them. How many has he left? *Ans.* 19—10, or 9, barrels.

2. Leaving home with $17, I spend $5 and give $4 away. How much have I left? (See § 53.)

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52. When a whole and one of its parts are given, how do we find the other part?—53. When a whole and all its parts but one are given, how do we find that one?—54. When the cost and selling price are given, how do we find the gain?—55. When the cost and loss are given, how do we find the selling price?—56. When the selling price and gain are given, how do we find the cost?—57. When a later date and the difference of years are given, how do we find an earlier date?

3. A colt was bought for $81, and sold for $88. What was the gain? (See § 54.)

4. A butcher lost $7 on a cow that cost $49. What did he sell her for? (See § 55.)

5. A jeweller sold a ring for $29, and thereby gained $3. What did the ring cost him? (See § 56.)

6. La Fayette was born in 1757. President Madison was born six years earlier. What year was that? (See § 57.)

7. If ten gallons of wine are drawn out of a hogshead containing 63 gallons, how many are left?

8. I sold a watch for $57, and by so doing gained $5. How much did it cost?

9. Napoleon died in 1821. When was the battle of Waterloo fought, which took place six years before his death?

10. A farmer who has 89 sheep, sells 52 of them. How many does he retain?

MODEL.—Eighty-nine is 8 tens and 9 units; fifty-two is 5 tens and 2 units. 5 tens and 2 units from 8 tens and 9 units leave 3 tens and 7 units, or 37. *Ans.* 37 sheep.

11. If I buy some cloth for $95 and sell it at a loss of $32, what do I get for it?

12. A person lays out $4 for books, $2 for paper, and $1 for pens. How much change must he receive for a $20 dollar bill?

13. A boy who has 58 cents, gives 32 cents to the poor. How many cents has he left?

14. If a man buys a cow for $45 and a calf for $6, and sells both for $62, how much does he make by the operation?

58. When the numbers are too large to perform the operation mentally, write the smaller number under the greater and subtract each figure from the one above it.

EXAMPLE.—A person who has $87945, gives away $6035. How much has he left?

He has the difference between $6035 and $87945, which is to be found by subtraction. Set the smaller number under the greater,—units under

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58. When the numbers are too large to perform the operation mentally, how do we deal with them? Go through the given example.

units, tens under tens, &c., because things can be taken only from others of the same kind.

Begin to subtract at the right. 5 units from 5 units leave 0 units; set down 0 in the units' column. 3 tens from 4 tens leave 1 ten; set it down. 0 hundreds from 9 hundreds leave 9 hundreds. 6 thousands from 7 thousands leave 1 thousand. Bring down 8. *Ans.* $81910.

Minuend	$87945
Subtrahend	6035
Remainder	$81910

59. Proof of Subtraction.—*Add the remainder and subtrahend. If their sum equals the minuend, the work is right.*—This follows, because *a whole is equal to the sum of its parts.* The minuend is the whole; the remainder and subtrahend are its parts.

Example.—Prove the above example. Add the remainder and subtrahend. Their sum is $87945, which equals the minuend. Hence the work is right.

Rem.	$81910
Sub.	6035
Sum	$87945

EXAMPLES FOR PRACTICE.

Read minuend, subtrahend, and remainder. Prove each example:—

	(1)	(2)	(3)
From	8267054	93847765138	87945568746598
Take	145031	624324122	52345154133273

4. Subtract 61425626377889 from 573929699387989.

5. From $\overline{\text{V}}$DCCCLXXXIV. take MCCCCXLIII.

6. How much more is cxcviii. than xxxvi.?

7. From five hundred and sixty-three billion fifty-nine thousand and seven, take two hundred and twenty billion thirty-five thousand and four. *Ans.* 343000024003.

8. Subtrahend, four billion five million and three; minuend, eighteen trillion seven billion nineteen million and six; required the remainder. *Ans.* 18003014000003.

9. From sixty-eight million nine hundred thousand and sixteen, take seven million two hundred thousand and two.

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59. How is subtraction proved? Why must the sum of the remainder and subtrahend equal the minuend? Prove the example in § 58.

60. Borrowing and Carrying.—The lower figure may be greater than the one above it.

Example.—From 964 take 839.

We can not take 9 units from 4 units. Hence from the 6 tens we borrow 1, leaving 5 tens. 1 ten is equal to 10 units, which we add to the 4 units, making 14. Now subtracting 9 units from 14 units, we have 5 units left; set down 5. 3 tens from 5 (*not* 6) tens leave 2 tens. 8 hundreds from 9 hundreds leave 1 hundred. *Ans.* 125.	5 9~~6~~4 839 —— *Ans.* 125
In stead of taking 1 from the upper figure, as was done above, it is usual to add 1 to the figure below it, which is more convenient, while it gives the same result. Thus: 9 from 14, 5. Carry 1; 1 and 3 are 4; 4 from 6, 2. 8 from 9, 1. *Ans.* 125.	964 839 —— *Ans.* 125

61. This adding of 10 to the upper figure is called **Borrowing**; adding 1 to the next lower figure is called **Carrying**.—We may have to borrow and carry several times in succession.

Example.—From 980000 take 969893.

3 from 10, 7. Carry 1; 1 and 9 are 10; 10 from 10, 0. Carry 1; 1 and 8 are 9; 9 from 10, 1. Carry 1; 1 and 9 are 10; 10 from 10, 0. Carry 1; 1 and 6 are 7; 7 from 8, 1. 9 from 9, 0; naughts at the left are not written. *Ans.* 10107.	980000 969893 —— *Ans.* 10107

62. Rule for Subtraction.

1. *Set the smaller number under the greater, units under units, tens under tens, &c.*

2. *Beginning at the right, take each figure of the subtrahend from the one above it, and set the remainder under the figure subtracted.*

3. *If any lower figure is greater than the one above it, add* 10 *to the upper figure, subtract, and carry* 1 *to the next lower figure.*

4. *Prove by adding remainder and subtrahend.*

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60. From 964 take 839, explaining the several steps.—61. What is this adding of 10 to the upper figure called? What is adding 1 to the next lower figure called? Show, with the given example, how we may have to borrow and carry several times in succession.—62. Recite the rule for subtraction.

EXAMPLES FOR PRACTICE.

Read the numbers. Subtract. Prove each example:—

(1)	(2)	(3)
801732641	156241098755	743812634378021
516238264	64980099668	56424152883322

(4)	(5)	(6)
89900321098	5601312499324	4385768506870
11127717749	999746446289	4039299991989

7. From 243008 take 14652.
8. From 814630 take 79999.
9. Take 90643 from 300652.
10. Take 89989 from 89990.
11. Take 42329 from 52330.
12. From 8264531 take 7642.
13. Take 15623 from 824618.
14. From 900061 take 10378.
15. 8630145416—9218682.
16. 245610035—81740305.
17. 51849035—22683518.
18. 426834260—97958473.
19. 98765432—23456789.
20. 10008674—10007987.
21. 576301498—85600534.
22. 7000338251—531258.

23. A merchant sells a lot of flour for $12085, and thereby gains $996; what did it cost him? (See § 56.)

24. Victoria became queen in 1837, 771 years after the Norman Conquest. What was the date of the Conquest? (See § 57.)

25. From thirteen billion subtract 8621356, and from their difference take the same subtrahend. *Ans.* 12982757288.

26. The population of the United States in 1863 was estimated at 34,344,926; in 1862, at 33,344,589. What was the increase?

27. A warehouse containing goods valued at $295125 took fire. Only $27250 worth of goods was saved; what was the value of those consumed?

28. Georgia was first settled by Oglethorpe in 1733. How long was that after the settlement of Virginia at Jamestown, which took place in 1607?

29. A man gave $21460 for a farm, and $1635 for stock. If he sold the whole for $25000, did he gain or lose, and how much?

30. If a person who has 36043 bushels of wheat sells one lot of 1845 bushels, and another of 12067 bushels, how much has he left? *Ans.* 22131 bushels.

31. C sells D 58 barrels of apples for $116, 225 watermelons for $30, and 100 chickens for $28. D pays $95 cash; how much does he still owe C? *Ans.* $79.

32. A lady divided $10000 among her three children. She gave the eldest $3585, and the second $3196. How much did the youngest receive? (See § 53.)

33. At an election, 12847 votes were cast; the successful candidate received 8968; how many were for his opponent? What was the majority of the former? *Ans.* Maj. 5089.

34. A broker, at the end of a day's business, had on hand $54253. How much of this was in bills, $14160 of it being in gold, and $1789 in silver?

35. A library contained 1429 volumes in English, 376 in French, and 198 in German. 642 of these books were burned up, and 183 sold; how many were left? *Ans.* 1178 vols.

36. P and Q begin business with $4500 each. P gains $868; Q loses $419. Which is worth the most, and how much?

37. The Imperial Library of Paris contains 1000000 printed volumes and 84000 manuscripts. The Royal Library of Munich contains 800000 volumes and 18600 manuscripts. How many volumes in both libraries? How many manuscripts? How many more volumes than manuscripts?

38. The native population of New York state in 1860 was 2882095; that of Pennsylvania, 2475710. The foreign-born population of N. Y. was 998640; that of Pennsylvania, 430505. What was the population of both states? How many more native inhabitants in N. Y. than in Penn.? How many more inhabitants of foreign birth? How many more inhabitants altogether?

39. Add the difference between 86458 and 64987 to the difference between 7000 and 5999. *Ans.* 22467.

40. From the sum of 26348 and 14275, take their difference.

41. What is the value of 18643 + 270967 − 4689?

42. From forty-one billion subtract 863246 + 579863.

CHAPTER VI.

MULTIPLICATION.

63. What will 5 lemons cost, at 3 cents each?

If 1 lemon costs 3 cents, 5 lemons will cost 5 times 3 cents, or 15 cents. Here we are required to take 3 five times. This process is called Multiplication.

64. Multiplication is the process of taking one of two numbers as many times as there are units in the other.

Multiplication Table.

Once 0 is 0; twice 0 is 0; 0 taken any number of times is 0.
0 times 1 is 0; 0 times 2 is 0; 0 times any number is 0.

Once	Twice	3 times	4 times	5 times	6 times
1 is 1	1 is 2	1 is 3	1 is 4	1 is 5	1 is 6
2 is 2	2 is 4	2 is 6	2 is 8	2 is 10	2 is 12
3 is 3	3 is 6	3 is 9	3 is 12	3 is 15	3 is 18
4 is 4	4 is 8	4 is 12	4 is 16	4 is 20	4 is 24
5 is 5	5 is 10	5 is 15	5 is 20	5 is 25	5 is 30
6 is 6	6 is 12	6 is 18	6 is 24	6 is 30	6 is 36
7 is 7	7 is 14	7 is 21	7 is 28	7 is 35	7 is 42
8 is 8	8 is 16	8 is 24	8 is 32	8 is 40	8 is 48
9 is 9	9 is 18	9 is 27	9 is 36	9 is 45	9 is 54
10 is 10	10 is 20	10 is 30	10 is 40	10 is 50	10 is 60
11 is 11	11 is 22	11 is 33	11 is 44	11 is 55	11 is 66
12 is 12	12 is 24	12 is 36	12 is 48	12 is 60	12 is 72
7 times	**8 times**	**9 times**	**10 times**	**11 times**	**12 times**
1 is 7	1 is 8	1 is 9	1 is 10	1 is 11	1 is 12
2 is 14	2 is 16	2 is 18	2 is 20	2 is 22	2 is 24
3 is 21	3 is 24	3 is 27	3 is 30	3 is 33	3 is 36
4 is 28	4 is 32	4 is 36	4 is 40	4 is 44	4 is 48
5 is 35	5 is 40	5 is 45	5 is 50	5 is 55	5 is 60
6 is 42	6 is 48	6 is 54	6 is 60	6 is 66	6 is 72
7 is 49	7 is 56	7 is 63	7 is 70	7 is 77	7 is 84
8 is 56	8 is 64	8 is 72	8 is 80	8 is 88	8 is 96
9 is 63	9 is 72	9 is 81	9 is 90	9 is 99	9 is 108
10 is 70	10 is 80	10 is 90	10 is 100	10 is 110	10 is 120
11 is 77	11 is 88	11 is 99	11 is 110	11 is 121	11 is 132
12 is 84	12 is 96	12 is 108	12 is 120	12 is 132	12 is 144

65. The number to be taken, or multiplied, is called the **Multiplicand.** The number by which we multiply, or which shows how often the multiplicand is to be taken, is called the **Multiplier.** The result, or number obtained by multiplication, is called the **Product.**

3 times 2 is 6. 2 is the multiplicand, 3 the multiplier, 6 the product.

66. The multiplicand and multiplier are called **Factors** of the product. 2 and 3 are factors of 6.

67. Multiplication is denoted by a slanting cross $\times$, placed between the factors. 2×3 is read, and denotes, *two multiplied by three.*

68. The multiplier shows how many times the multiplicand is to be taken. Multiplying 2 by 3 is taking 2 three times: $2 + 2 + 2 = 6$. $2 \times 3 = 6$. Multiplying is, therefore, a short way of adding a number to itself.

69. When two numbers are to be multiplied together, it makes no difference in the result which is taken as the multiplicand, and which as the multiplier. $4 \times 3 = 12$. $3 \times 4 = 12$.

Here we have 12 stars, whether we take them crosswise as forming 3 rows of 4 each, or lengthwise as forming 4 rows of 3 each.

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70. When the multiplier is an abstract number, the product is of the same denomination as the multiplicand. 3 times 2 *men* is 6 *men*; 3 times 2 apples is 6 apples, &c.

71. Application of Multiplication.—When the number of articles and the cost of one are given, multiply them together, to find the whole cost.

63. In the given example, what are we required to do? What is this process called?—64. What is Multiplication? How much is 0 taken any number of times? How much is 0 times any number? Recite the Table.—65. What is the number to be multiplied called? What is the number by which we multiply called? What is the result called? Give an example of these definitions.—66. What are the multiplicand and multiplier called? Name the factors of 6; of 10.—67. How is multiplication denoted? How is 2×3 read?—68. What does the multiplier show? Give an example. Multiplying is, therefore, a short way of doing what?—69. In multiplying two numbers, what is found to make no difference? Prove this.—70. When the multiplier is an abstract number, of what denomination is the product?—71. How do you find the cost of a number of articles, when the cost of one is given?

MENTAL EXERCISES.

How much is 8 times 6? 6 times 8? 9 times 5? 5 times 9? 7 times 10? 10 times 7? 11 times 5? 5 times 11?

How much is 3×12? 6×6? 2×9? 4×7? 12×5? 1×6? 0×10? 1×10? 11×2? 10×3? 5×0? 9×6? 8×7? 9×9?

What is the product of 10 and 11? 6 and 3? 12 and 10? 9 and 8? 11 and 11? 5 and 5? 12 and 12? 8 and 12?

How much is $4\times3\times7$? $2\times5\times8$? $1\times3\times8$? $2\times6\times1$? $6\times5\times0$? $3\times3\times3$? $4\times3\times6$? $2\times3\times2\times7$?

1. If 6 marbles are bought for 1 cent, how many can be bought for 4 cents? *Ans.* 4 times 6, or 24, marbles.

2. 7 days make a week. How many days in 3 weeks? In 5 weeks? In 7 weeks? In 11 weeks?

3. A certain boy earns $3 a week. How much will he earn in 4 weeks? In 6 weeks? In 9 weeks?

4. At 10 cents apiece, what will 2 writing-books cost? 8 writing-books? 12 writing-books?

5. Two boys have four pair of ducks each. How many ducks have they in all?

6. How many bushels of pears in four gardens, each containing three trees, if each tree yields two bushels?

7. A person having six broods of eleven chickens each, sells two of the broods. How many chickens has he left?

8. If a boat goes 12 miles an hour, how far will it go in 2 hours? In 5 hours? In 8 hours?

9. If a boy reads 4 pages every morning and 5 every afternoon, how many pages will he read in 7 days?

10. If a man earns $12 a week, and spends $7, how much will he lay up in 1 week? In 4 weeks?

72. MULTIPLYING BY 12 OR LESS.—The Multiplication Table carries us as far as 12 times. We can, therefore, multiply by 12 or any less number in one line.

RULE.—*Set the multiplier under the multiplicand, units under units, tens under tens. Beginning at the*

right, multiply each figure of the multiplicand in turn, setting each product in the same column with the figure multiplied.

EXAMPLE.—Multiply 53201 by 3.

Multiplicand	53201
Multiplier	3
Product	159603

Setting 3 under the units' figure of the multiplicand, begin to multiply at the right. 3 times 1 is 3; 3 times 0 is 0; 3 times 2 is 6; 3 times 3 is 9; 3 times 5 is 15. The last product consists of two figures; set it down with its right-hand figure under the figure multiplied. *Ans.* 159603.

73. CARRYING.—In the above example, each product except the last consists of but one figure. *When any product consists of two or three figures, set the right-hand one under the figure multiplied, and carry the rest to the next product.*

EXAMPLE.—Multiply 5309 by 12.

	5309
	12
Ans.	63708

Set the multiplier under the multiplicand, units under units, tens under tens. Begin at the right.

12 times 9 units are 108 units,—or, 10 tens and 8 units. Set the 8 units in the units' place, and carry the 10 tens to the next product.

12 times 0 tens are 0 tens, and the 10 carried make 10 tens,—or, 1 hundred, 0 tens. Set 0 in the tens' place; carry 1.

12 times 3 hundreds are 36 hundreds, and 1 makes 37 hundreds,—or, 3 thousands, 7 hundreds. Set 7 in the hundreds' place, and carry 3 thousands.

12 times 5 thousands are 60 thousands, and 3 makes 63. This being the last product, set down both figures. *Ans.* 63708.

EXAMPLES FOR PRACTICE.

	(1)	(2)	(3)	(4)
Multiply	73214	832614	901432	20613
By	2	3	4	5

(5)	(6)	(7)	(8)	(9)
802716	291503	237016	654321	61423
6	7	8	9	10

10. How much is 11 times 75992638401? *Ans.* 835919022411.

72. With how great a multiplier can we multiply in one line? Recite the rule. Illustrate the process with an example.—73. When any product consists of more than one figure, how do we proceed? Multiply 5309 by 12, and show how we carry.

11. How much is 9 times 3608498751? *Ans.* 32476488759.

12. What is the value of 847352619 × 12? *Ans.* 10168231428.

13. 372918635 × 2.	19. 7294380756 × 9.
14. 896140753 × 1.	20. 960527834 × 10.
15. 579063245 × 3.	21. 243781659 × 11.
16. 472938619 × 5.	22. 581629473 × 12.
17. 639845728 × 7.	23. 1759268423 × 4.
18. 576838492 × 8.	24. 867341395 × 11.

25. Multiply 31486 by 2; 3; 4; 5; 6; 7; 8; 9; and add the products. *Ans.* 1385384.

26. Multiply 8976201 by 9; 8; 7; 6; 5; 4; 3; 2; and add the products. *Ans.* 394952844.

27. Multiply 652081 first by 8, then by 3; and find the difference between the products. *Ans.* 3260405.

28. How much is twelve times six hundred and forty-nine thousand and thirty-seven? *Ans.* 7788444.

29. Six is one factor, ninety-six thousand and seventy-three is the other. What is the product? *Ans.* 576438.

30. What cost 1785 coats, at $11 each?

31. What is the product of CXCVIII. and XI.? *Ans.* 2178.

32. Multiply $\overline{\text{X}}$DCCCXXXIV. by VIII. *Ans.* 86672.

74. Multiplying by numbers above 12.—When the multiplier exceeds 12, multiply by its figures separately.

Example.—Multiply 287 by 156.

We can not multiply by 156 at once. Hence we first multiply by the 6 units; then by the 5 tens, or 50; then by the 1 hundred. Thus we get three **Partial Products**, as they are called; adding these, we get the whole product.

Multiplicand	287	
Multiplier	156	
Partial Products	1722	= 287 × 6
	1435	= 287 × 50
	287	= 287 × 100
Product	44772	= 287 × 156

Here, when we come to multiply by 5, we set the first figure of the partial product under the 5. This is because the 5 is 5 tens, or 50, 0 being omitted on the right. So, when we multiply by 1 hundred, we set the first figure of the partial product under the 1, two naughts being omitted on the right. *Take care, then, always to set the first figure of each partial product under the figure used in multiplying.*

75. Proof of Multiplication.—*Multiply the multiplier by the multiplicand. If this product agrees with the former one, the work is right.*

Example.—Prove the last example.

Multiply the multiplier 156, by the multiplicand 287. The product is 44772, which agrees with the former one. Hence the work is right.

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  156
  287
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 1092
1248
312
-----
44772
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76. When two numbers are to be multiplied together, it is usual to take the one with the fewer figures for the multiplier.

EXAMPLES FOR PRACTICE.

Find the product. Prove each example:—

1. Multiply 263 by 157.
2. Multiply 418 by 234.
3. Multiply 537 by 856.
4. Multiply 916 by 729.
5. Multiply 846 by 4823.
6. 2463 $\times$ 1857.
7. 1974 $\times$ 9436.
8. 2684 $\times$ 2631.
9. 47685 $\times$ 8249.
10. 16853 $\times$ 62583.

77. Rule for Multiplication.

1. *Set the multiplier under the multiplicand, units under units, tens under tens, &c.*

2. *If the multiplier is* 12 *or less, multiply by it each figure of the multiplicand in turn, beginning at the right; set down the right-hand figure of each product, and carry the remaining figure or figures, if any, to the next product.*

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74. When the multiplier exceeds 12, what are we to do? Multiply 287 by 156, explaining the process. What are the products obtained by multiplying by the different figures of the multiplier called? Where must we take care to set the first figure of each partial product? Why so?—75. How is multiplication proved? Prove the last example.—76. When two numbers are to be multiplied together, which do we take for the multiplier?—77. Recite the rule for Multiplication.

3. *If the multiplier exceeds* 12, *multiply by each of its figures in turn, setting the first figure of each partial product under the figure used in multiplying. Then add the partial products.*

4. *Prove by multiplying multiplier by multiplicand.*

78. Naught in the Multiplier.—When 0 occurs in the multiplier, bring it down, and go on multiplying by the next figure, all in the same line.

Example.—Multiply 7967 by 4005.

First multiply by 5. Bring down the two naughts, each in its own column. Then multiply by 4, setting the product in the same line; its first figure is thus brought under the 4. Finally, add the partial products.

$$\begin{array}{r} 7967 \\ 4005 \\ \hline 39835 \\ 3186800 \\ \hline \textit{Ans.}\ 31907835 \end{array}$$

EXAMPLES FOR PRACTICE.

1. Multiply four thousand two hundred and ninety-three, by eight thousand and seventy-six. *Ans.* 34670268.

2. Multiply fifty-seven thousand and three, by seventy-five thousand and four. *Ans.* 4275453012.

3. The factors of a certain product are 51, 4, 6, and 108. Required the product. *Ans.* 132192.

4. How much money must be divided among 2065 persons, that each may have $908?

5. A drover who had 967 head of cattle, bought 92 more, and then sold the whole for $63 apiece. How much did he receive?

6. How many books in three rooms, furnished with four book-cases apiece, each case containing 108 books?

7. What cost 825 horses at $145 apiece?

8. What is the product of 9263 and 7603?

9. What is the value of 68753×10408?

10. Multiply MDLXV. by $\overline{\text{V}}$IX. *Ans.* 7839085.

11. Multiply $\overline{\text{L}}$CCXLI. by $\overline{\text{X}}$VII. *Ans.* 502761687.

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78. When 0 occurs in the multiplier, how must we proceed? Illustrate this with the given example.

79. Naughts at the right.—We learned in § 25 that every naught placed after a number increases its value ten times. Hence, *to multiply by* 10, 100, 1000, *&c.*, *annex as many naughts as are in the multiplier.*

$76 \times 10 = 760$ $76 \times 100 = 7600$ $76 \times 10000 = 760000$

80. Example.—Multiply 4200 by 40.

$$\begin{array}{r|l} 42 & 00 \\ 4 & 0 \\ \hline 168 & 000 \end{array}$$

Multiplying in the usual way, we find the product to be 168000. The result is the same as if we had multiplied the significant figures together, and annexed the naughts at the right of both factors. Hence,

$$\begin{array}{r} 4200 \\ 40 \\ \hline \textit{Ans.}\ 168000 \end{array}$$

When there are naughts at the right of either factor or both, multiply the other figures, and annex to their product as many naughts as are at the right of both factors.

EXAMPLES FOR PRACTICE.

Find the value of the following:—

1. 80632 × 10.
2. 42635 × 100.
3. 62 × 100000.
4. 8541 × 1000.
5. 6000 × 1000.
6. 14000 × 10000.
7. 28000 × 146.
8. 976 × 25000.
9. 15000 × 1500.
10. 64700 × 39000.
11. 839100 × 60000.
12. 6287000 × 7800.

13. One gold eagle is worth $20; how many dollars are 6500 eagles worth?

14. If 100 pounds make one hundred-weight, how many pounds in seventy-eight hundred-weight?

15. Light travels 192000 miles in a second; how many miles will it travel in 60 seconds?

16. What will 140 miles of railroad cost, at an average of $42900 a mile?

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79. What is the effect of annexing a naught to a number? How, then, may we multiply by 10, 100, 1000, &c.?—80. Multiply 4200 by 40 in the usual way. What other way of obtaining the result is there? When there are naughts at the right of either factor or both, what is the shortest mode of proceeding?

17. If an army consume 840 barrels of provisions in one day, how many will it consume in 1 year, or 365 days?

18. If sound travels 1120 feet in a second, how far will it travel in 20 seconds?

19. A farmer has 6 orchards, each containing 10 piles of apples. In each pile are 1200 apples; how many apples in all?

20. Multiply 640 by 10; by 50; by 940: add the products.

21. How much more is 6800×140 than 970×850?

81. Multiplying by a Composite Number.—A **Composite Number** is the product of two or more factors greater than 1. 16 is a composite number, being equal to 8×2.

When the multiplier is a composite number, we may either multiply by the whole at once, or by its factors in turn. The result will be the same. Multiplication by a composite number may, therefore, be proved by multiplying by its factors.

Example.—Multiply 93 by 24.

$24 = 6 \times 4$ or, 8×3 or, 12×2

93	93	93	93
24	6	8	12
372	558	744	1116
186	4	3	2
2232	2232	2232	2232

What are the factors of 33—that is, what two numbers multiplied together produce 33? What are the factors of 108? 72? 44? 21? 132? 35? 99? 42? 54? 49? 121?

EXAMPLES FOR PRACTICE.

In examples 1—6, first multiply by the whole multiplier; then prove the result by multiplying by its factors.

1. What cost 63 firkins of butter, at $16 apiece?

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81. What is a Composite Number? How may we multiply by a composite number? How may multiplication by a composite number be proved?

2. What is the weight of 84 barrels of flour, averaging 196 pounds each?

3. How many hours in 365 days, there being twenty-four hours in one day?

4. If a person travels 96 miles a day, for 108 days, how far does he travel in all?

5. What will 27 miles of plank road cost, at $4200 a mile?

6. How many bushels of apples does an orchard of 107 trees yield, if each tree produces 12 bushels?

7. What will 13 square miles of land cost, at $17 an acre, there being 640 acres in one square mile? *Ans.* $141440.

8. How many miles will a locomotive go in 7 days of 24 hours each, if it moves 29 miles an hour? *Ans.* 4872 miles.

9. The earth moves in its orbit 68000 miles an hour. How far will it move in 365 days of 24 hours each?

10. In an orchard of 219 apple-trees, the average yield of each tree was 3 barrels of fruit, worth $3 a barrel. What was the whole yield worth? *Ans.* $1971.

11. A man owing $8213, gives in payment 5 horses valued at $175 each, 29 cows at $38 each, and $765 in cash. How much remains unpaid? *Ans.* $5471.

12. A ship, after sailing 23 hours east at the rate of 8 miles an hour, is driven west by a storm 10 miles an hour for 5 hours. At the end of the 28 hours, how far is she from where she started? *Ans.* 134 miles.

13. In one year there are 31556929 seconds. How much does one trillion exceed the seconds in 1864 years?

14. A man bought two farms; one of 87 acres, at $54 an acre; the other of 101 acres, at $37 an acre. He paid $8140; how much was still due? *Ans.* $295.

15. If I give 3 horses, each worth $150, and 13 cows, each worth $36, for 50 acres of land, valued at $16 an acre, do I gain or lose, and how much? *Ans.* Lose $118.

16. If a man travels 47 miles a day for 5 days, and then goes 54 miles a day for four days, how many more miles will he have to go, to complete a journey of 600 miles?

CHAPTER VII.

DIVISION.

82. Two pints make a quart; how many quarts in 8 pints?

If 2 pints make 1 quart, in 8 pints there are as many quarts as 2 is contained times in 8. Here we are required to find how many times 2 is contained in 8. This process is called Division.

83. Division is the process of finding how many times one number is contained in another.

Division Table.

Any number is contained in 0, 0 times.

1 in 1, once; 1 in 2, twice; 1 in 3, 3 times; 1 in 4, 4 times, &c.

2 in	3 in	4 in	5 in	6 in	7 in
2, once.	3, once.	4, once.	5, once.	6, once.	7, once.
4, twice.	6, twice.	8, twice.	10, twice.	12, twice.	14, twice.
6, 3 times.	9, 3 times.	12, 3 times.	15, 3 times.	18, 3 times.	21, 3 times.
8, 4 times.	12, 4 times.	16, 4 times.	20, 4 times.	24, 4 times.	28, 4 times.
10, 5 times.	15, 5 times.	20, 5 times.	25, 5 times.	30, 5 times.	35, 5 times.
12, 6 times.	18, 6 times.	24, 6 times.	30, 6 times.	36, 6 times.	42, 6 times.
14, 7 times.	21, 7 times.	28, 7 times.	35, 7 times.	42, 7 times.	49, 7 times.
16, 8 times.	24, 8 times.	32, 8 times.	40, 8 times.	48, 8 times.	56, 8 times.
18, 9 times.	27, 9 times.	36, 9 times.	45, 9 times.	54, 9 times.	63, 9 times.
20, 10 times.	30, 10 times.	40, 10 times.	50, 10 times.	60, 10 times.	70, 10 times.
22, 11 times.	33, 11 times.	44, 11 times.	55, 11 times.	66, 11 times.	77, 11 times.
24, 12 times.	36, 12 times.	48, 12 times.	60, 12 times.	72, 12 times.	84, 12 times.

8 in	9 in	10 in	11 in	12 in
8, once.	9, once.	10, once.	11, once.	12, once.
16, twice.	18, twice.	20, twice.	22, twice.	24, twice.
24, 3 times.	27, 3 times.	30, 3 times.	33, 3 times.	36, 3 times.
32, 4 times.	36, 4 times.	40, 4 times.	44, 4 times.	48, 4 times.
40, 5 times.	45, 5 times.	50, 5 times.	55, 5 times.	60, 5 times.
48, 6 times.	54, 6 times.	60, 6 times.	66, 6 times.	72, 6 times.
56, 7 times.	63, 7 times.	70, 7 times.	77, 7 times.	84, 7 times.
64, 8 times.	72, 8 times.	80, 8 times.	88, 8 times.	96, 8 times.
72, 9 times.	81, 9 times.	90, 9 times.	99, 9 times.	108, 9 times.
80, 10 times.	90, 10 times.	100, 10 times.	110, 10 times.	120, 10 times.
88, 11 times.	99, 11 times.	110, 11 times.	121, 11 times.	132, 11 times.
96, 12 times.	108, 12 times.	120, 12 times.	132, 12 times.	144, 12 times.

82. Solve the given example. What are we here required to do? What is this process called?—83. What is Division? How many times is any number contained in 0? How many times is 1 contained in any number? Recite the Table.

84. The number to be divided, is called the **Dividend**; that by which we divide, the **Divisor.** The result, or number obtained by dividing, is called the **Quotient.** It shows how many times the divisor is contained in the dividend.

85. When the divisor is not contained an exact number of times in the dividend, what is left over is called the **Remainder.** 5 in 17, 3 times and 2 over; 17 is the dividend, 5 the divisor, 3 the quotient, and 2 the remainder.

86. The divisor is contained in the dividend as many times as it can be subtracted in succession from the dividend. Dividing is, therefore, a short way of performing successive subtractions of the divisor from the dividend.

$$\text{4 in twelve, 8 times.}\left\{\begin{array}{r} 12-4=8 \\ 8-4=4 \\ 4-4=0 \end{array}\right.$$

87. Division is generally denoted by a short horizontal line between two dots $\div$; the dividend is placed before it, and the divisor after it. $12 \div 4$ is read, and denotes, *twelve divided by four.*

Division is also denoted by a line, with the dividend above it and the divisor below it; as, $\frac{12}{4}$. When there is a remainder, it is often placed over the divisor with this line between, and thus written as part of the quotient. Thus: $17 \div 5 = 3\frac{2}{5}$.

Division is also denoted by a curved line, placed between the dividend on the right and the divisor on the left; as, 4)12.

88. When the divisor is an abstract number, the quotient is of the same denomination as the dividend. 12 men $\div 4 = 3$ men; 12 apples $\div 4 = 3$ apples.

89. Applications of Division.—Division is the converse of multiplication. The dividend corresponds with

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84. What is the number to be divided called? What is the number we divide by called? What is the result obtained by dividing called? What does the quotient show?—85. What is meant by the Remainder? Give an example of these definitions. —86. How many times is the divisor contained in the dividend? Dividing is a short way of doing what?—87. How is division generally denoted? How is 12÷4 read? In what other way is division denoted? How is a remainder often written? What is the third way in which division may be denoted?—88. What denomination is the quotient, when the divisor is an abstract number?—89. Of what is division the converse? With what does the dividend correspond? The divisor and quotient?

the product, the divisor and quotient with the factors. That is,

	dividend = divisor × quotient	$32 = 8 \times 4$
Hence,	dividend ÷ divisor = quotient	$32 \div 8 = 4$
	dividend ÷ quotient = divisor	$32 \div 4 = 8$

When, then, a product and one factor are given, to find the other factor, divide the product by the given factor.

90. We found in § 71 that the whole cost of any number of articles equals the cost of one multiplied by the number of articles. Hence,

Divide the whole cost by the number of articles, to find the cost of one article.

Divide the whole cost by the cost of one article, to find the number of articles.

MENTAL EXERCISES.

How many times is 5 contained in 25? In 40? In 30? 8 in 48? 7 in 49? 3 in 12? 6 in 54? 7 in 21? 9 in 36? 12 in 144? 10 in 70? 11 in 110? 12 in 48? 9 in 54? 11 in 121?

8 in 57, how many times? (*Ans.* 7 times, and 1 over.) 2 in 17? 3 in 14? 4 in 39? 5 in 33? 6 in 45? 7 in 18? 8 in 69? 9 in 87? 12 in 65? 11 in 58?

Find the quotient and remainder: 87÷10. 9÷2. 66÷6. 66÷11. 119÷12. 100÷11. 39÷10. 26÷12. 62÷9. 88÷7. 90÷8. 70÷6. 47÷11. 59÷6. 38÷12.

26÷3. 57÷12. 10÷1. 36÷10. 130÷12. 24÷5. 53÷7. 33÷4. 33÷8. 72÷9. 101÷12. 102÷11. 97÷9. 19÷8. 40÷7. 31÷6. 90÷12. 140÷12. 42÷9.

1. 99 is a product; 11 is one of its factors; what is the other? (See § 89.)

2. How many times is 7 contained in 7×6? 9 in 8×9?

When a product and one factor are given, how can we find the other factor? —90. When the whole cost and the number of articles are given, how can we find the cost of one article? When the whole cost and the cost of one article are given, how can we find the number of articles?

3. The dividend is 121, the divisor 11; what is the quotient?

4. If 8 pencils cost 48 cents, how much is that apiece? (See § 90.)

5. How often is 4 times 2 contained in 8 times 5?

6. How many two-quart pitchers will 24 quarts of water fill?

7. How many weeks in 56 days, there being 7 days in one week?

8. How many albums at $3 each can be bought for $30? (See § 90.)

9. If 96 cents are distributed equally among twelve beggars, how much will each receive?

10. How many dresses of ten yards each can be cut from a piece of calico containing forty yards?

11. A merchant lays out $72 for dresses, at $12 apiece; how many dresses does he buy?

12. How many twelve-cent loaves can be bought for 132 cents?

91. Short Division.—When the divisor is 12 or less, the process is called **Short Division.**

Rule.—*Set the divisor at the left of the dividend, with a curved line between. Begin to divide at the left. See how many times the divisor is contained in each figure of the dividend, and set the quotient under the figure divided.*

Example.—If 4 houses cost $12048, how much do they cost apiece?

They cost as many dollars apiece as 4 (the number of articles) is contained times in $12048 (the whole cost). Set the divisor at the left of the dividend; begin to divide at the left.

4 is not contained in 1; see, therefore, how many times it is contained in the first two figures. 4 in 12, 3 times. Set down 3 under 2, the right-hand figure of the two divided.

$$\begin{array}{r} 4)\underline{12048} \\ 3012 \end{array}$$

4 in 0, 0 times; set it down. 0 must never be omitted in the quotient, unless it is the first figure.

4 in 4, once. 4 in 8, twice. *Ans.* $3012.

Ans. $3012.

91. When is the process called Short Division? Give the rule. Illustrate the rule with the given example.

EXAMPLES FOR PRACTICE.

1. Divide eighteen thousand and six, by six. *Ans.* 3001.
2. Divide 100208406 by two. *Ans.* 50104203.
3. Divide ninety thousand and sixty-three, by three.
4. Divide forty-five thousand and five, by five.
5. Divide 9876543201 by one.
6. Divide seventy-two billion by 6; by 8; by 9; by 12.
7. Divide 1800402068 by 2.
8. Divide twenty thousand eight hundred and four, by 4.

92. CARRYING.—When all the figures of the dividend have been divided, if there is a remainder, set it down as such. If before this a remainder occurs, *prefix it (in the mind) to the next figure of the dividend, and continue the division.* This prefixing is called **Carrying.**

EXAMPLE.—How many stoves, at $12 apiece, can be bought with $25009?

As many stoves as $12 (the price of one stove) is contained times in $25009 (the whole number of dollars).

```
      1 10 4
12) 2 5 0 0 9
Ans. 2 0 8 4, 1
```

12 is not contained in 2. 12 in 25, twice and 1 over; set down 2 under the 5, and carry 1. 12 in 10, 0 times and 10 over; set down 0, and carry 10. 12 in 100, 8 times and 4 over; set down 8, and carry 4. 12 in 49, 4 times and 1 over; set down 4 in the quotient, and 1 as remainder. *Ans.* 2084 stoves, and $1 over.

93. PROOF OF DIVISION.—Dividend = divisor × quotient (§ 89). Hence, *Multiply the divisor and quotient together; add in the remainder, if there is any. If this result equals the dividend, the work is right.*

EXAMPLE.—Prove the last example.

Multiply the quotient 2084 by the divisor 12. Add in the remainder 1. The result equals the dividend; hence the work is right.

```
 2084
   12
25008
    1
25009
```

92. If there is a remainder after dividing all the figures of the dividend, what must be done with it? If before this a remainder occurs, what must be done with it? What is this prefixing called? Solve the given example, showing how the remainders are carried.—93. How is division proved? Prove the example in § 92.

EXAMPLES FOR PRACTICE.

Find the quotient:—

1. 184766÷2. *Ans.* 92383.
2. 312015÷5. *Ans.* 62403.
3. 817643330552÷4.
4. 902436421248÷6.
5. 325276814796÷3.
6. 864547999637÷7.
7. 190034012867÷6.
8. 715784206÷8. *Rem.* 6.
9. 487465834÷9. *Rem.* 4.
10. 367324169÷12. *Rem.* 5.
11. 269694625÷11. *Rem.* 2.
12. 903528658÷9. *Rem.* 1.
13. 117850860÷12. *Rem.* 0.
14. 552819043÷10. *Rem.* 3.

94. LONG DIVISION.—When the divisor exceeds 12, the process is called **Long Division.**

95. In Short Division, we subtract and prefix the remainder to the next figure, *in the mind.* In Long Division, we take the same steps, but *write down* all the figures used.

EXAMPLE.—Divide 361296 by 72.

Divr.	Dividend.	Quo.
	72) 361296 (5018	
72 × 5 =	360	
	129	
72 × 1 =	72	
	576	
72 × 8 =	576	

In long division, the quotient is set *at the right* of the dividend. Beginning at the left of the dividend, take as many figures as are required to contain the divisor. 72 is not contained in 3, or in 36; it is contained in 361, 5 times. Set 5 in the quotient as the first figure.

Multiply the divisor by 5; set the product under 361, and subtract. The remainder is 1, which (as in short division) we prefix, by bringing down 2, the next figure of the dividend.

Now repeat the same steps. 72 in 12, 0 times. Set 0 in the quotient, and bring down 9, the next figure of the dividend. 72 in 129, once. Set 1 in the quotient, multiply the divisor by it, and subtract the product from 129. The remainder is 57, to which bring down the next figure 6.

Repeat again the same steps. 72 in 576, 8 times. Set 8 in the quotient, multiply the divisor by it, and subtract. There is no remainder, and, as all the figures of the dividend have been brought down, the work is finished. *Ans.* 5018.

360, 12, 129, and 576, are called **Partial Dividends.**

---

94. When is the process called Long Division?—95. As regards the mode of operating, what is the difference between Short and Long Division? Divide 361296 by 72, explaining the several steps in full, and pointing out the Partial Dividends.

96. We may not always, on the first trial, get the right quotient figure.

If, on multiplying the divisor by any quotient figure, the product comes greater than the partial dividend, the quotient figure is too great, and must be diminished.

If, on the other hand, on subtracting, we have a remainder greater than the divisor, the quotient figure is too small, and must be increased.

```
72) 361296 (6
    432
    ———
```

Thus, in the last example, if we say 72 is contained 6 times in 361, we get a product greater than the partial dividend, and must therefore diminish the quotient figure.

If we say it is contained 4 times, on multiplying and subtracting, we get a remainder greater than the divisor, and must therefore increase the quotient figure.

```
72) 361296 (4
    288
    ———
     73
```

97. If the divisor will not go into the partial dividend, set 0 in the quotient, and bring down the next figure of the dividend. If several figures are brought down before the divisor will go into the partial dividend, *set a naught in the quotient for each.*

EXAMPLES FOR PRACTICE.

Find the quotient. Prove each sum (§ 95) :—

1. 772326 ÷ 321.	*Ans.* 2406.	10. 427854262 ÷ 95.	*Rem.* 7.
2. 705083 ÷ 547.	*Ans.* 1289.	11. 23981539 ÷ 349.	*Rem.* 4.
3. 713513 ÷ 89.	*Ans.* 8017.	12. 17235969 ÷ 4208.	*Rem.* 1.
4. 938986 ÷ 74.	*Ans.* 12689.	13. 9281746 ÷ 76.	
5. 922623 ÷ 39.	*Ans.* 23657.	14. 6955070 ÷ 1682.	
6. 961919 ÷ 106879.	*Rem.* 8.	15. 3368955 ÷ 49763.	
7. 16360358 ÷ 6307.	*Rem.* 0.	16. 52580797048 ÷ 8762.	
8. 829765304 ÷ 486.	*Rem.* 8.	17. 91426833296 ÷ 45761.	
9. 97329468 ÷ 265.	*Rem.* 3.	18. 110028314741 ÷ 89123.	

---

96. In the course of the division, what indicates that the quotient figure must be diminished? What shows that the quotient figure must be increased? Give examples.—97. If the divisor will not go into the partial dividend, what must be done? If several figures are brought down before the divisor will go into the partial dividend, what must be done?

19. 171051930356÷300089.	22. 42563008104÷82654.
20. 5763447÷678509.	23. 81000633357÷461305.
21. 73411659875÷46398.	24. 24639875555÷5362.

25. Divide 246515999541 by 28653. *Ans.* 8603497.
26. Divide 11963109376 by 109376. *Ans.* 109376.
27. Divide 166168212890625 by 12890625. *Ans.* 12890625.
28. Divide 1521808704 by 6503456. *Ans.* 234.
29. Divide 3278031150 by 46825. *Rem.* 200.
30. Divide 4000102955925 by 800095. *Rem.* 100.
31. Divide 8976014236 by 1280819. *Rem.* 978046.
32. Divide 243166625648 by 3471032. *Rem.* 7856.
33. Divide 9281746 by 27; by 44; by 98; by 294.
34. Divide 7200651897 by 2498; by 76389; by 32174.
35. Divide 8976014236 by 298701; by 4853684.

98. Rule for Division.

1. *Set the divisor at the left of the dividend. Take as many figures at the left of the dividend as will contain the divisor, and find how many times it will go into them.*

2. *If the divisor is* 12 *or less, set this first quotient figure under the figure divided, or under the right-hand figure of those divided, if more than one are taken. Divide each figure of the dividend in turn, carrying what is over, and setting each quotient figure under the figure divided.*

3. *If the divisor is over* 12, *set the first quotient figure at the right of the dividend. Multiply the divisor by it, and subtract the product from the partial dividend. Bring down the next figure of the dividend. Find the next quotient figure, multiply, and subtract, as before. Go on thus, till all the figures of the dividend are brought down.*

4. *Prove by multiplying quotient and divisor together, and adding in the remainder if there is one.*

---

98. Recite the rule for Division.

EXAMPLES FOR PRACTICE.

1. The product of two factors is 67048164. One of the factors is 9876; what is the other? (See § 89.) *Ans.* 6789.

2. If 1264 acres of land cost $21488, how much is that an acre? (See § 90.)

3. How many barrels of pork, costing $24 a barrel, can be bought for $95160?

4. If a merchant sells 221988 bushels of corn in 12 months, what is the average sale per month?

5. The earth's circumference is 25000 miles; how long would it take to traverse it, at the rate of 200 miles a day?

6. The cost of a certain railroad is $8490018. How long is the road, if the average cost is $52086 a mile?

7. A man worth $278195 in real estate, and $49990 in stocks, divides the whole equally among his wife, six sons, and four daughters. What is the share of each? *Ans.* $29835.

8. How many days will 128200 pounds of flour last a garrison of 641 men, allowing each man 4 pounds a day?

9. What number multiplied by 66 will produce 5148?

10. Divide nineteen million into 9 equal parts. *Ans.* $2111111\frac{1}{9}$.

11. If 46 persons consume 158 pounds of flour every day, how long will 12482 pounds last them? *Ans.* 79 days.

12. How many firkins holding 56 pounds each will be required for putting down 49000 pounds of butter?

13. There are 5280 feet in a mile. How many miles in 971520 feet? How many in 1943040 feet?

14. How many bales will 270630 pounds of cotton make, allowing 465 pounds to the bale?

15. If a tax of $44013645 is collected from six thousand and forty-five towns, what is the average amount paid by each town? *Ans.* $7281.

16. A forest containing 1995 trees was thinned by cutting down one tree in seven. How many trees were left?

17. There are 6 rows of cannon-balls, each containing 4 piles. If there are 76440 balls in all, how many in each pile?

99. Dividing by a Composite Number.—*When the divisor is a composite number, we may either divide by the whole at once or by its factors in turn.* The result will be the same. Division by a composite number may, therefore, be proved by dividing by its factors.

Example.—Divide 2232 by 24.

$$24 = 6 \times 4 \quad \text{or, } 8 \times 3 \quad \text{or, } 12 \times 2$$

24) 2232 (93	6) 2232	8) 2232	12) 2232
216	4) 372	3) 279	2) 186
72	93	93	93
72			

EXAMPLES FOR PRACTICE.

In these examples, first divide by the whole divisor; then prove the result by dividing by its factors:—

1. 63 gallons make a hogshead. How many hogsheads are there in 9828 gallons?

2. If 1184 barrels of flour are divided equally among sixteen boats, what is the load of each?

3. If a vessel sails 3168 miles in 32 days, what is her average rate per day?

4. Divide 681660 by 105 $(7 \times 3 \times 5)$.

5. Divide 160006 by 154 $(11 \times 2 \times 7)$.

6. Divide 793800 by 84; by 45.

7. Divide 4044425 by 121. Divide 11298 by 42.

8. Divide 2628528 by 56. Divide 33792 by 64.

9. Divide 22500525 by 75. Divide 28416 by 96.

100. The True Remainder.—In dividing by factors, two or more remainders may occur, from which we must find the true remainder. Remainders are always units of the same kind as the dividends from which they arise.

99. When the divisor is a composite number, what two modes of proceeding are there? How, then, may division by a composite number be proved?—100. When two or more remainders occur, in dividing by factors, how can we find the true remainder? Illustrate this process with the given example.

Example.—Divide 7464 by 385 $(11 \times 5 \times 7)$.

11) 7464		*Rem.*
5) 678	....	6
7) 135	...... $3 \times 11 =$	33
19	... $2 \times 5 \times 11 =$	110
	True rem.	149

Ans. 19, 149 *rem.*

Dividing by 11, we get 6 for the first remainder. Dividing by 11 makes the units in the quotient (678) 11 times greater than those of the original number. Hence 3, the remainder obtained on dividing this quotient, must be multiplied by 11 to make its units of the same kind as those of the former remainder. In like manner, the quotient 135 is made up of units 5 times 11, or 55, times greater than those of the original number. Hence 2, the remainder arising from this quotient, must be multiplied by 5×11. The three remainders being now of the same kind, we add them and get 149 for the true remainder. Hence,

To find the true remainder, add to the remainder arising from the first division, each subsequent remainder multiplied by all the divisors preceding the one that produced it.

EXAMPLES FOR PRACTICE.

First divide by the whole divisor; then prove the result by dividing by its factors, finding the true remainder:—

1. $223121 \div 27$. *Rem.* 20.
2. $258289 \div 35$. *Rem.* 24.
3. $333398 \div 48$. *Rem.* 38.
4. $324496 \div 54$. *Rem.* 10.
5. $459774 \div 64$. *Rem.* 62.
6. $715154 \div 77$. *Rem.* 55.
7. $264085 \div 98$ $(2 \times 7 \times 7)$.
8. $47484 \div 165$ $(3 \times 11 \times 5)$.
9. $89901 \div 242$ $(2 \times 11 \times 11)$.
10. $91189 \div 162$ $(2 \times 9 \times 9)$.
11. $57212 \div 198$ $(3 \times 6 \times 11)$.
12. $43937 \div 245$ $(5 \times 7 \times 7)$.

101. Naughts at the Right of the Divisor.—When there are naughts at the right of the divisor, the operation may be shortened.

Annexing a figure to a number, as we saw in § 25, throws its figures one place to the left, and thus multiplies it by 10. Consequently, cutting off a figure from the right of a number throws its remaining figures one place to the right, and thus divides it by 10. So, cutting off two figures divides by 100; cutting off three, by 1000, &c. Hence,

101. What is the effect of cutting off a figure from the right of a number? What is the effect of cutting off two figures? Three?

To divide a number by 10, 100, 1000, *&c., cut off as many figures at the right of the dividend as there are naughts in the divisor. The remaining figures are the quotient; those cut off, the remainder.*

$4200 \div 10 = 420$ $4200 \div 100 = 42$ $4200 \div 1000 = 4$, 200 rem.

102. The principle is the same in the case of any divisor ending with one or more naughts.

EXAMPLE.—Divide 9710 by 2400.

Divide by factors. $2400 = 100 \times 24$. To divide by 100, cut off two figures from the right of the dividend. Dividing the quotient thus arising by 24, and finding the true remainder, we get for our quotient 4 and 110 remainder. The result is the same as if we had cut off the two naughts of the divisor and two right-hand figures of the dividend, divided what remained, and annexed to the remainder the figures cut off from the dividend for a true remainder. Hence the following rule:—

	Quo.		*Rem.*
$9710 \div 100 =$	97	....	10
$97 \div 24 =$	4	... $1 \times 100 =$	100
		True rem.	110

Ans. 4, 110 *rem.*

$$
\begin{array}{l}
24\cancel{0}\cancel{0})\ 97\cancel{1}\cancel{0}\ (4 \textit{ quo.} \\
\underline{96} \\
110 \textit{ rem.}
\end{array}
$$

Cut off the naughts at the right of the divisor, and as many figures at the right of the dividend. Divide the remaining figures of the dividend by those of the divisor. If there is a remainder, annex to it the figures cut off from the dividend; if not, these figures are themselves the remainder.

$$
\begin{array}{l}
34\cancel{0})\ 1031\cancel{0}\ (30 \\
\underline{102} \\
11
\end{array}
\qquad
\begin{array}{l}
19\cancel{0}\cancel{0})\ 135\cancel{4}\cancel{7}\ (7 \\
\underline{133} \\
2
\end{array}
\qquad
\begin{array}{l}
9\cancel{0}\cancel{0}\cancel{0})\ 279\cancel{0}\cancel{4}\cancel{7} \\
\overline{\quad 31}
\end{array}
$$

Ans. 30, 110 *rem.* *Ans.* 7, 247 *rem.* *Ans.* 31, 47 *rem.*

Give the rule for dividing a number by 10, 100, 1000, &c.—102. Divide 9710 by 2400, using the factors of the divisor. What other way is there of arriving at the same result? Give the rule for dividing when the divisor ends with one or more naughts.

EXAMPLES FOR PRACTICE.

Find the quotient:—

1. 8875432÷10.
2. 493268÷100.
3. 84310006700÷10000.
4. 970000063002÷1000.
5. 3186200040÷10000.
6. 8800800800÷100000.
7. $\overline{\text{X}}$DCCCLXXX.÷X.
8. 843670÷560. *Rem.* 310.
9. 199301÷1200. *Rem.* 101.
10. 3315006÷850. *Rem.* 6.
11. 7294508÷900. *Rem.* 8.
12. 8400099÷280. *Rem.* 99.
13. 1733626÷550. *Rem.* 26.
14. $\overline{\text{V}}$CCCC.÷CL. *Ans.* 36.

MISCELLANEOUS QUESTIONS.—Name the four fundamental rules. *Ans.* Addition, Subtraction, Multiplication, Division; with these all calculations are performed. What is Addition? Subtraction? Multiplication? Division? What operation enables us to find a whole, when its parts are given? When the whole and one part are given, what operation enables us to find the other part? What is the converse of addition? Of multiplication?

What is the result of addition called? Name the three terms used in subtraction. *Ans.* Subtrahend, minuend, and difference. Define each of these terms. Name and define the three terms used in multiplication. Name and define the terms used in division. What is meant by the factors of a product? Which term in division corresponds with the product in multiplication? With what do the divisor and quotient correspond? At which side do we begin to add? To subtract? To multiply? To divide?

What does the sign minus denote? On which side of it must the subtrahend be placed? What does a horizontal line between two dots denote? On which side of this sign must the dividend be placed? What does plus denote? What does an oblique cross denote? What is the sign of equality? How is addition proved? Subtraction? Multiplication? Division? In what other way may multiplication be proved? *Ans.* By dividing the product by the multiplier; if the quotient equals the multiplicand, the work is right.

What is a composite number? Give an example of an abstract composite number; of a concrete composite number. How may we multiply or divide by a composite number? When we divide by factors, how do we find the true remainder? What is the shortest way of multiplying by 10, 100, &c.? How do we divide by 10, 100, &c.? When is division called Short, and when Long? What difference is there in the mode of performing the two operations?

MISCELLANEOUS EXAMPLES.

1. Find the sum, then the difference, then the product, of 343 and 8918; divide 8918 by 343.

2. How many times is 20000 contained in the difference between eleven million and eleven billion? *Ans.* 549450 times.

3. A United States senator receives $3000 a year. If he spends $8 a day, how much of his salary will he save in his six years' term, allowing 365 days to the year? *Ans.* $480.

4. If a person has an income of $3285 a year, how much is that a day?

5. A mile is 5280 feet. How many steps, of two feet each, will a boy take in walking 5 miles? *Ans.* 13200 steps.

6. Divide the sum of 168483 and 849717 by the difference between 97234 and 46324, and multiply the quotient by nine times nine. *Ans.* 1620.

7. If a man earns $1200 a year, and his yearly expenses are $860, how many years will it take him to lay up $5440? *Ans.* 16 years.

8. A farmer buys 75 tons of hay, at $32 a ton. He pays for it in wheat, at $2 a bushel. How many bushels of wheat must he give? *Ans.* 1200 bushels.

What was the whole cost of the hay? How much wheat, at $2 a bushel, will pay for it?

9. A merchant began business with $36000. At the end of 9 years he was worth $61875. How much a year had he made?

10. How many pounds of coffee, at 29 cents a pound, will pay for two hogsheads of sugar containing 1160 pounds each, at 19 cents a pound? *Ans.* 1520 pounds.

11. A person having $2879 in current bills, and $3997 in uncurrent, invests the whole in flour at $9 a barrel; how many barrels can he buy? *Ans.* 764 barrels.

12. Four partners commencing business put in respectively $8650, $9200, $7950, and $3000. At the end of a year the firm was worth $37875. Required their gain. *Ans.* $9075.

13. If a man buys 746 barrels of flour for $8206, what must

he sell the whole for, to gain $1 a barrel? How much is that a barrel? *Ans.* $12 a barrel.

14. A person willed $12000 to his wife, $300 to the poor, and the rest of his property to his six children in equal shares. If he was worth $71370, what was each child's share? *Ans.* $9845.

What was he worth in all? How much of this did he leave to his wife and the poor? How much remained? Into how many parts must this be divided?

15. A lady worth $48530 leaves her servant $550, her brother four times that amount, and divides the rest of her property equally among her four sons and three daughters. How much does each child receive? *Ans.* $6540.

How much does she leave to her servant? To her brother? How much to both? How much of her property is left? Among how many is this divided?

16. Three partners divide equally their yearly profit, amounting to $17064. One of them divides his share equally among his four children; what does each child get? *Ans.* $1422.

17. An army of 4525 men had 103075 days' rations. At the end of 21 days, 500 men were captured. How many days after that did the rations last? *Ans.* 2 days.

How many rations did 4525 men consume in 21 days? How many rations then remained? After the capture, how many men were left? How long would the rations left support these men?

18. A garrison of 842 men had 63472 days' rations. After 16 days a reënforcement of 158 men arrived. How long after their arrival did the rations last? *Ans.* 50 days.

19. A person bought 97 acres of land at $51 an acre, and 111 acres at $47 an acre. He paid $9539 cash, and for the balance gave 5 horses; what was each horse valued at? *Ans.* $125.

What was the cost of the first piece of land? Of the second? Of both? How much cash was paid? What remained due? If 5 horses were valued at this amount, what was each horse valued at?

20. A hogshead containing 63 gallons of molasses was bought for 67 cents a gallon. 7 gallons having leaked out, the rest was sold at 76 cents a gallon. What was the gain? *Ans.* 35 cents.

21. In an orchard containing 659 trees, 41 trees bear no fruit. If the income from the orchard is $4944, and the apples bring $4 a barrel, how many barrels on an average does each bearing tree produce? *Ans.* 2 barrels.

22. A railroad forty miles long cost a million of dollars, all but four hundred. What was the cost per mile? *Ans.* $24990.

23. The dividend of a sum in division is 4719, the quotient 96, the remainder 15. What is the divisor? *Ans.* 49.

Subtract the remainder from the dividend, and you have the product of the quotient and divisor; then proceed according to § 89.

24. On dividing 734062 by a certain number, I get 807 for the quotient, and 499 remainder. What is the divisor?

25. If 17 cows are worth $816, and each cow is worth as much as 6 sheep, what is the value of one sheep? *Ans.* $8.

26. An estate of $25101 was left to a family of four brothers and nine sisters. The brothers having given up their share to the sisters, how much did each of the latter receive?

27. A farmer had 100 hens, four of which died; if the remainder laid in one week four basketfuls of eggs, consisting of 120 each, what was the weekly average for each hen?

Relations of Dividend, Divisor, and Quotient.

103. The quotient depends on both dividend and divisor. If one of these is fixed, a change in the other changes the quotient. But, if both dividend and divisor are changed, these changes may neutralize each other, and the quotient remain the same. Thus:

$$24 \div 6 = 4$$

Keep the same divisor; then,	
Doubling dividend doubles quotient:	$48 \div 6 = 8$
Halving dividend halves quotient:	$12 \div 6 = 2$
Keep the same dividend; then,	
Doubling divisor halves quotient:	$24 \div 12 = 2$
Halving divisor doubles quotient:	$24 \div 3 = 8$
Doubling or halving both dividend and	$48 \div 12 = 4$
divisor makes no change in quotient:	$12 \div 3 = 4$

103. On what does the quotient depend? If either dividend or divisor is fixed, what is the effect of changing the other? If both dividend and divisor are changed, what may follow? With the same divisor, what is the effect of doubling the dividend? Of halving the dividend? With the same dividend, what is the effect of doubling the divisor? Of halving the divisor? What is the effect of doubling or halving both dividend and divisor?

104. From these examples we conclude that,

I. *With a fixed divisor, multiplying the dividend by any number multiplies the quotient by that number, and dividing the dividend divides the quotient.*

II. *With a fixed dividend, multiplying the divisor by any number divides the quotient by that number, and dividing the divisor multiplies the quotient.*

III. *Multiplying or dividing both dividend and divisor by the same number does not change the quotient.*

105. If we multiply one number by another, and then divide the product by the multiplier, we have the original number unchanged. Multiply 9 by 4; divide the product by 4, and we again have 9.

$9 \times 4 = 36$
$36 \div 4 = 9$

Prime and Composite Numbers.

106. Every number is either Prime or Composite.

A **Prime Number** is one that can not be divided by any number but itself or 1, without a remainder; as, 2, 11, 17.

A **Composite Number** is the product of two or more factors greater than 1, and is exactly divisible by each of its factors. 30 is a composite number $= 2 \times 3 \times 5$; it is, therefore, exactly divisible by 2, 3, and 5.

107. The first hundred prime numbers are as follows:—

1	29	71	113	173	229	281	349	409	463
2	31	73	127	179	233	283	353	419	467
3	37	79	131	181	239	293	359	421	479
5	41	83	137	191	241	307	367	431	487
7	43	89	139	193	251	311	373	433	491
11	47	97	149	197	257	313	379	439	499
13	53	101	151	199	263	317	383	443	503
17	59	103	157	211	269	331	389	449	509
19	61	107	163	223	271	337	397	457	521
23	67	109	167	227	277	347	401	461	523

104. State the principles deduced from these examples.—**105.** What is the effect of multiplying one number by another, and then dividing the product by the multiplier?—**106.** Into what two classes are all numbers divided? What is a Prime Number? What is a Composite Number?—**107.** Mention the first ten prime numbers.

108. An **Even Number** is one that can be divided by 2 without remainder; as, 2, 4, 6, &c.

An **Odd Number** is one that can not be divided by 2 without remainder; as, 1, 3, 5, &c.

109. A composite number is exactly divisible,

By 2, when its right-hand figure is 0, or is exactly divisible by 2; as, 30, 104.

By 3, when the sum of its figures is exactly divisible by 3; as, 456—the sum of its figures ($4+5+6=15$) being exactly divisible by 3.

By 4, when its two right-hand figures are naughts, or are exactly divisible by 4; as, 500, 324.

By 5, when it ends with 0 or 5; as, 10, 25.

By 6, when it is an even number and the sum of its figures is exactly divisible by 3; as, 744.

By 8, when its three right-hand figures are naughts, or are exactly divisible by 8; as, 17000, 3456.

By 9, when the sum of its figures is exactly divisible by 9; as, 790146.

By 10, when it ends with 0; as, 850.

EXERCISE.

Tell which of the following numbers are even, and which odd; which are prime and which composite. Select those that are exactly divisible by 2, by 3, by 4, by 5, by 6, by 8, by 9, by 10:—

1; 16; 325; 168; 450; 523; 2571; 62375; 9888; 19; 2967; 85; 29000; 401; 1000101; 8700; 347; 123; 7002; 75408; 6003; 10101001101201; 655; 10002; 1000.

Prime Factors.

110. The **Prime Factors** of a composite number are the prime numbers (other than 1) which multiplied together produce it. 2, 3, and 11, are the prime factors of 66, because $2 \times 3 \times 11 = 66$.

111. The prime factors of a composite number are found by successive divisions.

---

108. What is an Even Number? What is an Odd Number?—109. When is a composite number exactly divisible by 2? By 3? By 4? By 5? By 6? By 8? By 9? By 10?—110. What is meant by the Prime Factors of a composite number? Give an example.—111. How are the prime factors of a composite number found?

Example.—Find the prime factors of 5460.

As 5460 is an even number, we divide it by 2. The quotient, 2730, being an even number, we again divide by 2. The next quotient, 1365, is exactly divisible by 3, since the sum of its figures is exactly divisible by 3; we therefore divide it by 3. The next quotient, 455, is exactly divisible by 5, since it ends with 5; we therefore divide it by 5. The next quotient, 91, being exactly divisible by 7, we divide it by 7. The next quotient, 13, is a prime number. The prime factors required are the several divisors and the prime quotient—2, 2, 3, 5, 7, and 13.

```
2)5460
2)2730
3)1365
5) 455
7)  91
    13
```

Proof.—$2 \times 2 \times 3 \times 5 \times 7 \times 13 = 5460$.

112. Rule.—1. *To find the prime factors of a composite number, divide it by its smallest prime factor; treat the quotient in the same way, and continue thus dividing the successive quotients till a prime number is reached. The divisors and the last quotient are the prime factors required.*

2. *Prove by multiplying the prime factors, and seeing whether their product equals the given composite number.*

When a quotient is reached for which a divisor can not readily be found, look in the Table on page 64, to see whether it is prime. If it is, the work is done.

EXAMPLES FOR PRACTICE.

1. Find the prime factors of 6006. *Ans.* 2, 3, 7, 11, 13.
2. Find the prime factors of 16. Of 24. Of 36. Of 60.
3. Find the prime factors of 72. Of 90. Of 102. Of 111.
4. Find the prime factors of 125. Of 155. Of 178. Of 234.
5. Find the prime factors of 309. Of 404. Of 524.
6. Find the prime factors of 1040. Of 1324. Of 6276.
7. Resolve 7498 into its prime factors. *Ans.* 2, 23, 163.
8. Resolve 28055 into its prime factors. *Ans.* 5, 31, 181.
9. Resolve the following numbers into their prime factors: 1764; 14641; 78900; 6432; 49750; 390625.

Find the prime factors of 5460. Prove this example.—112. Recite the rule for finding prime factors. When a divisor can not readily be found, what should be done?

Cancellation.

113. When one set of factors is to be divided by another, the operation may often be shortened *by first rejecting equal factors.*

EXAMPLE.—Divide $6 \times 7 \times 9 \times 5$ by $5 \times 3 \times 9 \times 7$.

We may first multiply the factors of the dividend together, then those of the divisor, and then divide the first product by the second.

$$6 \times 7 \times 9 \times 5 = 1890$$
$$5 \times 3 \times 9 \times 7 = 945$$
$$1890 \div 945 = 2 \text{ } Ans.$$

But we save work *by setting the factors of the dividend above those of the divisor with a line between, rejecting equal factors from dividend and divisor, and dividing what remains above the line by what remains below.* Thus :—

$$\frac{6 \times 7 \times 9 \times 5}{5 \times 3 \times 9 \times 7}$$

$$\text{Rejecting 7, 9, and 5, } \frac{6 \times \cancel{7} \times \cancel{9} \times \cancel{5}}{\cancel{5} \times 3 \times \cancel{9} \times \cancel{7}}$$

$$6 \div 3 = 2 \text{ } Ans.$$

The answer must be the same as before, because rejecting a factor is dividing by that factor, and we learned in § 104 that *dividing both dividend and divisor by the same number does not change the quotient.*

114. On the same principle, the work may be shortened when the factors of dividend, or divisor, or both, are composite numbers.

EXAMPLE.—Divide 18 times 21 by 14.

Arrange as in the last example. Divide 18 and 14 by the common factor 2. Then divide 21 and 7 by the common factor 7. Multiplying the factors remaining in the dividend, we get 27, *Ans.*

$$\frac{\overset{9}{\cancel{18}} \times \overset{3}{\cancel{21}}}{\underset{\cancel{7}}{\cancel{14}}} = 27 \text{ } Ans.$$

113. When one set of factors is to be divided by another, how may the operation often be shortened? Illustrate this process with the given example.—114. In what other case may the work be similarly shortened? Show this with the given example.

115. The equal factors thus rejected from dividend and divisor are said to be *cancelled*, and the process is called **Cancellation.**

Since cancelling is dividing, 1 (*not* 0) takes the place of a cancelled factor.

When the divisor is all cancelled away, as in the last example, the product of the factors remaining in the dividend is the answer.

For every factor rejected from the dividend we must reject an equal factor from the divisor, and *only one* such equal factor. We must not, for instance, cancel two threes in the divisor for one three in the dividend.

The factors of the dividend, in stead of being placed above those of the divisor, may be set at their right with a vertical line between. Thus :—

$$\begin{array}{r|rr} \cancel{14} & \cancel{18} & 9 \\ \cancel{7} & \cancel{21} & 3 \\ \hline & & \end{array}$$

Ans. 27

EXAMPLES FOR PRACTICE.

Bring cancellation to bear in the following:—

1. Divide $2 \times 3 \times 8 \times 5 \times 7$ by $2 \times 4 \times 15$. *Ans.* 14.
2. Divide $25 \times 7 \times 11 \times 5$ by $55 \times 25 \times 7$. *Ans.* 1.
3. Divide $3 \times 7 \times 2 \times 11 \times 21$ by $7 \times 2 \times 3 \times 7$. *Ans.* 33.
4. 40×39 is how many times 10×13? *Ans.* 12.
5. Dividend, 121×6; divisor, 33×22; required the quotient.
6. How many times is 34×15 contained in $9 \times 17 \times 3 \times 5 \times 2$?
7. Divide $20 \times 36 \times 22 \times 60$ by $3 \times 11 \times 100$.
8. Divisor, 5 times 6 times 11; dividend, 6930; what is the quotient?
9. Divide $99 \times 360 \times 365$ by 11×73. *Ans.* 16200.
10. Divide the product of 17, 10, 16, and 14, by the product of 2, 5, 34, 7, and 2. *Ans.* 8.
11. How many boxes of raisins containing 12 pounds each, worth 20 cents a pound, will pay for 15 boxes of crackers, containing 16 pounds each, at 18 cents a pound? *Ans.* 18 boxes.
12. How many barrels of coal holding 3 bushels each, at 30 cents a bushel, must be given for 9 ten-pound boxes of soap, worth 12 cents a pound? *Ans.* 12 barrels.

---

115. What is said of the equal factors thus rejected? What is this process called? What takes the place of a cancelled factor? When the divisor is all cancelled away, what will the answer be? How many factors must be cancelled in the divisor for each factor rejected from the dividend? In what other way may the factors of the dividend and divisor be arranged?

CHAPTER VIII.

GREATEST COMMON DIVISOR.

116. When one number is contained in another without remainder, the former is called a **Divisor** or **Measure** of the latter; and the latter, a **Multiple** of the former. 6 is contained in 12 without remainder; hence 6 is a *divisor* or *measure* of 12, and 12 is a *multiple* of 6.

117. A **Common Divisor**, or **Common Measure**, of two or more numbers is any number that will divide each without remainder. Their **Greatest Common Divisor**, or **Measure**, is the greatest number that will divide each without remainder.

2, 4, 6, and 12, are common divisors of 24, 36, and 48. 12 is their greatest common divisor.

118. Numbers that have no common divisor except 1, are said to be *prime to each other.*

Numbers prime to each other are not necessarily prime numbers. 15 and 28 are prime to each other, yet are not prime numbers.

119. A divisor of any number is also a divisor of every multiple of that number. 3 is a divisor of 6; then it is also a divisor of 12, 18, 24, and every other multiple of 6.

120. A common divisor of two numbers is also a divisor of their sum and of their difference. 3 is a common divisor of 12 and 21; then it is also a divisor of their sum (33), and of their difference (9).

121. To find the greatest common divisor, when the numbers are small, *resolve them into their prime factors, and multiply together those factors that are common.*

---

116. When is one number called a Divisor or Measure of another? When is one number called a Multiple of another? Give examples.—117. What is a Common Divisor of two or more numbers? What is the Greatest Common Divisor of two or more numbers? Give examples.—118. When are numbers said to be prime to each other? Are numbers prime to each other necessarily prime numbers? Give an example.—119. Of what is a divisor of any number also a divisor? Give an example.—120. Of what is a common divisor of two numbers also a divisor? Give an example.—121. How may we find the greatest common divisor, when the numbers are small?

EXAMPLE.—Find the greatest common divisor of 72, 108, and 180.

$$\begin{aligned} 72 &= 2 \times 2 \times 2 \times 3 \times 3 \\ 108 &= 2 \times 2 \qquad \times 3 \times 3 \times 3 \\ 180 &= 2 \times 2 \qquad \times 3 \times 3 \times 5 \end{aligned}$$

The common factors are 2, 2, 3, and 3; and their product, 36, is the greatest common divisor.

EXAMPLES FOR PRACTICE.

Find the greatest common divisor of the following:—

1. 99 and 72. *Ans.* 9.
2. 54 and 90.
3. 147 and 189. *Ans.* 21.
4. 96 and 264. *Ans.* 24.
5. 120 and 180.
6. 144 and 192. *Ans.* 48.
7. 36, 108, and 252.
8. 66, 154, and 220.
9. 120, 135, and 255.
10. 48, 208, and 224.
11. 40, 60, 100, and 140.
12. 26, 104, 130, and 234.

122. When the numbers are large or not easily resolved into factors, we use a different method.

EXAMPLE.—What is the greatest common divisor of 475 and 589?

```
475) 589 (1
     475
     ---
     114) 475 (4
          456
          ---
           19) 114 (6
               114
               ---
                 0
```

Divide 589 by 475. If there were no remainder, 475 would exactly divide both, and would be the greatest common divisor. But, as there is a remainder, divide the last divisor by it. Again there is a remainder, 19. Divide the last divisor by it. There is now no remainder, and 19, the last divisor, is the greatest common divisor sought.

That 19 is a common divisor of 475 and 589, is proved by dividing those numbers by 19.

$$475 \div 19 = 25$$
$$589 \div 19 = 31$$

That 19 is the *greatest* common divisor is proved thus:—

Any number that is a divisor of 475 and 589,
is also a divisor of their difference, or 114 (§ 120),
also of 4 times 114, or 456 (§ 119);
and any number that is a divisor of 475 and 456,
is also a divisor of their difference, 19.

122. When do we use a different method? Illustrate this method with the given example. How is it proved that 19 is a common divisor of 475 and 589? How is it proved that 19 is their greatest common divisor?

Now, as the divisor of the original numbers must also be a divisor of 19, they can have no greater common divisor than 19.

123. RULE.—1. *To find the greatest common divisor of two numbers, divide the greater by the less; if there is a remainder, divide the last divisor by it, and so proceed till nothing remains. The last divisor is the greatest common divisor.*

2. *To find the greatest common divisor of more than two numbers, proceed as above with the two smallest first, then with the divisor thus found and the next largest, and so on till all the numbers are taken. The last common divisor is the one sought.*

EXAMPLES FOR PRACTICE.

Find the greatest common divisor of the following:—

1. 365 and 511.	*Ans.* 73.	9. 1242 and 2323.
2. 864 and 420.	*Ans.* 12.	10. 6409 and 7395.
3. 775 and 1800.	*Ans.* 25.	11. 10353 and 14877.
4. 2628 and 2484.	*Ans.* 36.	12. 285714 and 999999.
5. 2268 and 3444.	*Ans.* 84.	13. 505, 707, and 4343.
6. 14, 18, and 24.	*Ans.* 2.	14. 154, 28, 343, and 84.
7. 837, 1134, 1347.	*Ans.* 3.	15. 6914, 396, and 5184.
8. 78, 52, 13, 416.	*Ans.* 13.	16. 3885, 5550, and 6105.

17. A farmer wishes to bag 345 bushels of oats, 483 of barley, and 609 of corn, using the largest bags of equal size that will exactly hold each kind. How many bushels must each bag hold? How many bags will he need? *Ans.* 3 bu. 479 bags.

The number of bushels each bag must hold, will be the greatest common divisor of the given numbers. Then, how many bags holding 3 bushels each will it take to hold 345 bushels? How many, to hold 483 bushels? How many, to hold 609 bushels? How many bags will it take in all?

18. A man owning four farms, containing 45, 100, 55, and 115 acres, divides them into equal fields of the largest size that will allow each farm to form an exact number of fields. How many acres in each field? How many fields does he make?

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123. Recite the rule for finding the greatest common divisor of two numbers. How do you find the greatest common divisor of more than two numbers?

CHAPTER IX.

LEAST COMMON MULTIPLE.

124. A **Multiple** of a number is any number that it will exactly divide. 4, 6, 8, &c., are multiples of 2. Every number has an infinite number of multiples.

125. A **Common Multiple** of two or more numbers is any number that each will exactly divide. 12, 24, 36, &c., are common multiples of 3 and 4.

126. The **Least Common Multiple** of two or more numbers is the smallest number that each will exactly divide. 12 is the least common multiple of 3 and 4.

127. A common multiple of two or more numbers may always be obtained by multiplying them together. If the numbers are prime to each other, this product is their *least* common multiple.

128. A common multiple of several numbers must contain all the prime factors of each number taken separately. But a prime factor of one of the numbers may also appear in another; and factors thus repeated the *least* common multiple excludes. Hence, *the least common multiple is the product of the prime factors common to two or more of the numbers, and such factors of each as are not common.*

EXAMPLE.—Find the least common multiple of 12, 15, 18, and 24.

Write the numbers in a horizontal line. 2 is a prime factor of three of them, and will be a factor of the least common multiple; divide by it,

---

124. What is a Multiple of a number? How many multiples has every number? —125. What is a Common Multiple of two or more numbers?—126. What is the Least Common Multiple of two or more numbers? Give an example.—127. How may a common multiple of two or more numbers always be obtained? In what case will this product be their *least* common multiple?—128. Of what is the least common multiple of several numbers the product? Solve the given example, explaining each step.

setting down the quotients, and 15, which is not exactly divisible. 2 is a prime factor of two of the numbers in the second line; divide by it, setting down the quotients, and 15, which is not exactly divisible. 3 is a prime factor of all the numbers in the third line; divide by it, and set down the quotients. There is no need of dividing further, as no number will exactly divide more than one of the numbers in the fourth line. 5, 3, and 2, are the remaining factors of the original numbers; and the product of these and the divisors (which are the common factors) will be the least common multiple required. $2 \times 2 \times 3 \times 5 \times 3 \times 2 = 360$ *Ans.*

$$\begin{array}{r|rrrr}
2) & 12, & 15, & 18, & 24 \\ \hline
2) & 6, & 15, & 9, & 12 \\ \hline
3) & 3, & 15, & 9, & 6 \\ \hline
 & 1, & 5, & 3, & 2
\end{array}$$

$$2 \times 2 \times 3 \times 5 \times 3 \times 2 = 360 \text{ Ans.}$$

129. When one of the given numbers is a factor of another, any multiple of the latter must of course contain the former, and the former number may therefore be cancelled at the outset. Thus, in the last example, 12, being a factor of 24, may be cancelled. Proceeding as before, we get the same result with less work.

$$\begin{array}{r|rrrr}
2) & \cancel{12}, & 15, & 18, & 24 \\ \hline
 & 3) & 15, & 9, & 12 \\ \hline
 & & 5, & 3, & 4
\end{array}$$

$$2 \times 3 \times 5 \times 3 \times 4 = 360 \text{ Ans.}$$

130. RULE.—1. *Write the numbers in a horizontal line. Divide by any prime number that will divide two or more of them without remainder, placing the quotients and the numbers not exactly divisible in a line below.*

2. *Proceed with this second line as with the first; and so continue till there are no two numbers that have a common divisor greater than* 1. *The product of the divisors and the numbers in the lowest line will be the least common multiple.*

EXAMPLES FOR PRACTICE.

Find the least common multiple of the following:—

1. 37 and 41. (See § 127.)	5. 11, 77, and 88. *Ans.* 616.
2. 23 and 39. *Ans.* 897.	6. 24, 180, 45, 60. *Ans.* 360.
3. 19, 17, and 5.	7. 10, 20, 50, 25. *Ans.* 100.
4. 2, 4, 6, and 8. (See § 129.)	8. 48, 20, 21, 24. *Ans.* 1680.

129. When one of the given numbers is a factor of another, how may the operation be shortened?—130 Give the rule for finding the least common multiple.

9. 38, 209, 17, 19, 34.
10. 99, 13, 11, 26, and 100.
11. 34, 88, 75, and 99.
12. 375, 10, 8, 12, and 13.
13. 24, 20, 18, 16, 15, and 12.
14. 9, 15, 36, 135, and 162.
15. 144, 48, 80, and 36.
16. 125, 350, 150, and 75.
17. 9, 17, 12, 8, 21, 30, and 16.
18. 141, 235, 329. *Ans.* 4935.

19. What is the greatest number that will exactly divide 120 and 150? What is the smallest number they will exactly divide?

20. Find the smallest number that exactly contains 78, 156, and 390. Find the greatest number exactly contained in them.

21. Find the least common multiple of the first eight even numbers. *Ans.* 1680.

CHAPTER X.

COMMON FRACTIONS.

131. How Fractions arise.—When a whole is divided into *two* equal parts, each of these parts is called *one half.*

Half | Half

When a whole is divided into *three* equal parts, one of these parts is called *one third*; two are called *two thirds*; &c.

Third | Third | Third

When a whole is divided into *four* equal parts, one of these parts is called *one fourth* (or *quarter*); two are called *two fourths*; three, *three fourths*; &c.

Fourth | Fourth | Fourth | Fourth

In the same way we get *fifths*, *sixths*, *sevenths*, &c., by dividing a whole into *five*, *six*, *seven*, &c., equal parts. The name is taken from the number of equal parts into which the whole is divided.

131. How do we get halves? Thirds? Fourths? Fifths? Sixths? Sevenths? From what is the name taken?

132. The value of these equal parts varies according to their number. The more parts the whole is divided into, the smaller they must be. One half of a thing is greater than one third, one third than one fourth, as is shown by the above lines.

133. These equal parts into which a whole is divided, are called **Fractions.**

134. KINDS.—There are two kinds of Fractions, Common and Decimal. When we use the word *fraction* alone, we refer to a Common Fraction.

135. HOW COMMON FRACTIONS ARE WRITTEN.—Learn how common fractions are expressed in figures:—

One half	$\frac{1}{2}$	Five thirteenths	$\frac{5}{13}$
One third	$\frac{1}{3}$	Three twenty-seconds	$\frac{3}{22}$
One fourth (quarter)	$\frac{1}{4}$	Twenty sixty-firsts	$\frac{20}{61}$
One two-hundredth	$\frac{1}{200}$	Three thousandths	$\frac{3}{1000}$
One thousandth	$\frac{1}{1000}$	Six twelve-hundredths	$\frac{6}{1200}$

It will be seen that a common fraction, expressed in figures, consists of two numbers, one below the other, with a line between.

The number below the line is called the **Denominator.** It shows into how many equal parts the whole is divided, and therefore gives name to these parts.

The number above the line is called the **Numerator.** It shows how many of the equal parts denoted by the Denominator are taken.

The Numerator and the Denominator, taken together, are called the **Terms** of the fraction.

$\frac{5}{6}$ is a fraction. 5 and 6 are its Terms. 6 is the Denominator, and shows that the whole is divided into *six* equal parts, making each part one *sixth.* 5 is the Numerator, and shows that *five* of these equal parts are taken. In reading, name the Numerator first—*five sixths.*

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132. On what does the value of these equal parts depend? Which is greater, one half of a thing or one third? One third or one fourth?—133. What are the equal parts into which a whole is divided called?—134. How many kinds of fractions are there? What are they called?—135. Show by the given examples how common fractions are written. Of what does a common fraction, expressed in figures, consist? What is the number below the line called? What does it show? What is the number above the line called? What does it show? What are the numerator and denominator, taken together, called? Give examples of these definitions.

EXERCISE.

Read these fractions. Then name the numerator and the denominator, and tell what each shows:—

$\frac{6}{7}$; $\frac{11}{14}$; $\frac{73}{151}$; $\frac{472}{8002}$; $\frac{70}{10010}$; $\frac{12}{800000}$; $\frac{40203}{706007}$.

Write the following fractions in figures:—

1. Ten elevenths.
2. Thirteen halves.
3. Twenty millionths.
4. Seventy thousandths.
5. Eighty sixty-firsts.
6. Twelve billionths.
7. One hundredth.
8. Four twenty-seconds.
9. Seventy-three seventy-thirds.
10. One hundred and two fourteen-hundred-and-fifths.
11. Sixty-seven forty-thousand-five-hundredths.
12. Four hundred and two ten-thousandths.
13. Nineteen six-hundredths.

136. DEFINITIONS.—An **Integer** is a whole number; as, 1, 2.

A **Fraction** is one or more of the equal parts into which a whole is divided; as, $\frac{1}{3}$, $\frac{2}{3}$.

A **Proper Fraction** is one whose numerator is less than its denominator; as, $\frac{1}{3}$, $\frac{5}{6}$.

An **Improper Fraction** is one whose numerator is equal to or greater than its denominator; as, $\frac{3}{3}$, $\frac{7}{6}$.

A **Mixed Number** is one that consists of a whole number and a fraction; as, $7\frac{1}{2}$ (*seven and a half*). The whole number is called *the integral part.*

A **Compound Fraction** is a fraction of a fraction; as, $\frac{1}{2}$ of $\frac{2}{3}$, $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{7}{8}$.

A **Complex Fraction** is one that has a fraction in one or both of its terms; as,

$\dfrac{\frac{1}{2}}{9}$ *One half divided by nine.* $\dfrac{4\frac{2}{3}}{\frac{5}{6}}$ *Four and two thirds divided by five sixths.*

---

136. What is an Integer? What is a Fraction? What is a Proper Fraction? What is an Improper Fraction? What is a Mixed Number? What is meant by the *integral part* of a mixed number? What is a Compound Fraction? What is a Complex Fraction?

A fraction is said to be *inverted*, when its terms are interchanged; $\frac{4}{5}$ inverted becomes $\frac{5}{4}$.

137. PRINCIPLES.—A fraction indicates division (§ 87). The fractional line is the line used in the sign of division ÷. The numerator is the dividend, the denominator is the divisor, the value of the fraction is the quotient. Hence the same principles apply as in division (§ 104).

I. *Multiplying the numerator by any number multiplies the fraction by that number, and dividing the numerator divides the fraction.*

II. *Multiplying the denominator by any number divides the fraction by that number, and dividing the denominator multiplies the fraction.*

III. *Multiplying or dividing both numerator and denominator by the same number does not change the value of the fraction.*

138. A fraction indicates division. Hence, if numerator and denominator are equal, the value of the fraction is 1; because any quantity is contained in itself once. If the numerator is greater than the denominator, the value of the fraction is greater than 1; if less, less than 1.

Hence, the value of every improper fraction must be 1 or more than 1; that of every proper fraction, less than 1.

139. Any whole number may be thrown into a fractional form by giving it 1 for a denominator. $7 = \frac{7}{1}$. $9 = \frac{9}{1}$. It is clear that dividing a number by 1 does not alter its value.

EXERCISE.

Read the following. Tell what kind of fraction each is. In the third line, tell whether the value of each fraction is greater or less than 1:—

When is a fraction said to be *inverted*?—137. What does a fraction indicate? What corresponds with the dividend? What, with the divisor? What, with the quotient? To what, then, do the principles of division apply? Recite the three principles that apply to fractions.—138. When is the value of a fraction 1? When is it greater than 1? When is it less than 1? What must be the value of every improper fraction? Of every proper fraction?—139. How may any whole number be thrown into a fractional form?

$\frac{3}{8}$ of $\frac{5}{21}$. $\frac{7}{8}$ of $\frac{2}{10}$ of $\frac{4}{33}$. $8\frac{3}{601}$. $\frac{4}{5}$ of $\frac{12}{11}$. $12\frac{7}{8}$.

$\frac{6\frac{1}{4}}{5\frac{1}{4}}$. $\frac{8\frac{7}{8}}{7}$. $\frac{6004}{50\frac{1}{10}}$. $\frac{\frac{3}{8} \text{ of } \frac{4}{5}}{2170}$. $\frac{18\frac{5}{7}}{1\frac{4}{9}}$. $\frac{\frac{1}{3}}{\frac{1}{4} \text{ of } \frac{1}{5}}$.

$\frac{27}{27}$. $\frac{28}{27}$. $\frac{26}{27}$. $\frac{32}{32}$. $\frac{1}{3}$. $\frac{2}{1}$. $\frac{408}{60}$. $\frac{60}{408}$. $\frac{11}{11}$.

Throw 7 into a fractional form; 19; 871; 1002; 11; 6.

MENTAL EXERCISES ON FRACTIONS.

1. How many halves in 1 whole? How many thirds? How many fourths? How many tenths? How many fiftieths? How many thousandths?

2. How many halves in 1? In 2? In 3? In 4? In 10? In 100? In 1000? In 100000? How do you find how many halves there are in any number? *Ans.* By multiplying it by 2.

3. How many thirds in 1? In 2? In 3? In 5? In 12? In 100? In 400? In 5000? How do you find how many thirds there are in any number? *Ans.* By multiplying it by 3.

4. How many fourths in 1? In 2? In 6? In 8? In 11? In 20? In 200? How many fifths in 1? In 9? In 4? In 300? How do you find how many fourths there are in any number? How do you find how many fifths?

5. How many sixths in 1? In 5? In 8? In 10? In 12? How many sevenths in 1? In 4? In 6? In 7? In 11? How many eighths in 1? In 9? In 12? In 5? In 200? How many ninths in 1? In 9? In 12?

6. How many elevenths in 1? In 11? In 12? How many twelfths in 1? In 6? In 9? In 11? In 12? How many tenths in 1? In 11? In 17? In 176? In 84? In 71? How many hundredths in 1? In 5? In 12? In 33? In 45?

7. How do you get half of a thing? *Ans.* By cutting it into two equal parts. How do you find half of a number? *Ans.* By dividing it by 2. How much is half of 4? Of 6? Of 10? Of 18? Of 24? Of 7? (*Ans.* $3\frac{1}{2}$.) Of 9? Of 11?

8. How do you get one third of a thing? *Ans.* By cutting it into 3 equal parts. How do you find $\frac{1}{3}$ of a number? *Ans.* By dividing it by 3. What is $\frac{1}{3}$ of 9? Of 27? Of 11? (*Ans.* $3\frac{2}{3}$.)

9. How do you get $\frac{1}{4}$ of a thing? How do you find $\frac{1}{4}$ of a number? How much is $\frac{1}{4}$ of 24? Of 32? Of 36? Of 45? Of 19?

10. How do you find $\frac{1}{5}$ of a number? How do you find $\frac{1}{6}$? $\frac{1}{7}$? $\frac{1}{10}$? $\frac{1}{12}$? $\frac{1}{100}$?

11. How much is $\frac{1}{5}$ of 40? $\frac{1}{6}$ of 42? $\frac{1}{7}$ of 84? $\frac{1}{8}$ of 72? $\frac{1}{9}$ of 99? $\frac{1}{11}$ of 33? $\frac{1}{11}$ of 132? $\frac{1}{11}$ of 110? $\frac{1}{12}$ of 60? $\frac{1}{12}$ of 96? $\frac{1}{12}$ of 132? $\frac{1}{12}$ of 144? $\frac{1}{10}$ of 10? $\frac{1}{16}$ of 160? $\frac{1}{100}$ of 1700? $\frac{1}{100}$ of 4500?

12. How much is $\frac{3}{4}$ of 12? *Ans.* One fourth of 12 is 3; and three fourths are 3 times 3, or 9.

How much is $\frac{5}{6}$ of 6? $\frac{4}{7}$ of 14? $\frac{3}{5}$ of 25? $\frac{4}{9}$ of 18? $\frac{8}{11}$ of 44? $\frac{7}{12}$ of 24? $\frac{7}{10}$ of 40? $\frac{3}{4}$ of 48? $\frac{5}{6}$ of 36? $\frac{5}{8}$ of 72? $\frac{11}{12}$ of 132?

13. What part of 2 is 1? (*Ans.* $\frac{1}{2}$.) What part of 3 is 1? (*Ans.* $\frac{1}{3}$.) What part of 5 is 1? What part of 5 is 2? (*Ans.* $\frac{2}{5}$.) What part of 5 is 3? What part of 7 is 1? What part of 7 is 6?

14. How many half-pence in 9 pence?

15. How many quarters of beef in 12 oxen?

16. If I cut 10 oranges into sixths, how many pieces have I?

17. A vessel containing 48 passengers was wrecked. $\frac{5}{12}$ of the passengers escaped. How many escaped, and how many perished?

18. If a pound of coffee costs 40 cents, what will half a pound cost? $\frac{5}{8}$ of a pound? $\frac{3}{10}$ of a pound?

19. A boy having 60 marbles lost $\frac{5}{12}$ of them, gave $\frac{6}{12}$ away, and kept the rest. How many did he lose, give away, and keep?

20. If $\frac{1}{3}$ of a ton of coal costs \$3, what will a ton cost? Half a ton?

140. Fractions may be reduced, added, subtracted, multiplied, and divided.

Reduction of Fractions.

141. Reducing a fraction is changing its form without changing its value.

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140. What operations may be performed on fractions?—141. What is meant by reducing a fraction?

142. CASE I.—*To reduce a fraction to its lowest terms.*

A fraction is in its lowest terms when its numerator and denominator have no common divisor greater than 1.

EXAMPLE.—Reduce $\frac{45}{75}$ to its lowest terms.

Dividing both numerator and denominator by the same number does not alter the value of the fraction (§ 137). We therefore divide by their common factors in succession. Dividing by 5, we get $\frac{9}{15}$. Dividing the terms of this fraction by 3, we get $\frac{3}{5}$. This is the answer, since its terms have no common divisor but 1.

$$5 \,|\, \tfrac{45}{75} = \tfrac{9}{15}$$

$$3 \,|\, \tfrac{9}{15} = \tfrac{3}{5} \quad \textit{Ans.}$$

```
45) 75 (1
    45
    30) 45 (1
        30
        15) 30 (2
            30
```

$15 \,|\, \frac{45}{75} = \frac{3}{5}$ *Ans.*

In stead of dividing as above, we might have found the greatest common divisor (§ 123), and divided by it at once. This is the best method, when the numbers are large.

RULE.—*Divide numerator and denominator successively by every factor common to both. Or, divide them at once by their greatest common divisor.*

EXAMPLES FOR PRACTICE.

Reduce the following fractions to their lowest terms:—

1. $\frac{4}{6}$.	12. $\frac{42}{343}$.			23. $\frac{744}{786}$.	*Ans.* $\frac{124}{131}$.
2. $\frac{3}{9}$.	13. $\frac{276}{360}$.			24. $\frac{728}{808}$.	*Ans.* $\frac{91}{101}$.
3. $\frac{7}{21}$.	14. $\frac{308}{506}$.			25. $\frac{756}{1140}$.	*Ans.* $\frac{63}{95}$.
4. $\frac{24}{48}$.	15. $\frac{88}{132}$.			26. $\frac{3528}{4424}$.	*Ans.* $\frac{63}{79}$.
5. $\frac{32}{48}$.	16. $\frac{364}{404}$.			27. $\frac{1764}{2660}$.	*Ans.* $\frac{63}{95}$.
6. $\frac{60}{96}$.	17. $\frac{505}{707}$.	*Ans.* $\frac{5}{7}$.		28. $\frac{2323}{3131}$.	*Ans.* $\frac{23}{101}$.
7. $\frac{72}{144}$.	18. $\frac{315}{405}$.	*Ans.* $\frac{7}{9}$.		29. $\frac{1362}{1500}$.	*Ans.* $\frac{227}{250}$.
8. $\frac{120}{180}$.	19. $\frac{275}{440}$.	*Ans.* $\frac{5}{8}$.		30. $\frac{3178}{3500}$.	*Ans.* $\frac{227}{250}$.
9. $\frac{12}{72}$.	20. $\frac{592}{999}$.	*Ans.* $\frac{16}{27}$.		31. $\frac{1896}{2502}$.	*Ans.* $\frac{316}{417}$.
10. $\frac{81}{108}$.	21. $\frac{172}{1118}$.	*Ans.* $\frac{2}{13}$.		32. $\frac{834}{8834}$.	*Ans.* $\frac{417}{4417}$.
11. $\frac{144}{240}$.	22. $\frac{99}{9999}$.	*Ans.* $\frac{1}{101}$.		33. $\frac{4872}{9048}$.	*Ans.* $\frac{203}{377}$.

142. What is the first case of reduction of fractions? When is a fraction in its lowest terms? Solve the given example, explaining the steps. What other method is shown? Recite the rule for reducing a fraction to its lowest terms.

143. CASE II.—*To reduce an improper fraction to a whole or mixed number.*

A fraction indicates division. The numerator is the dividend, the denominator is the divisor. To find the quotient, that is the value of the fraction, we have only to divide, as indicated.

EXAMPLE 1.—Reduce $\frac{27}{9}$ to a whole or mixed number.

$$27 \div 9 = 3 \textit{ Ans.}$$

EXAMPLE 2.—Reduce $\frac{30}{9}$ to a whole or mixed number.

$$30 \div 9 = 3\tfrac{3}{9} = 3\tfrac{1}{3} \textit{ Ans.}$$

RULE.—*Divide the numerator by the denominator.*

If there is a remainder, the answer is a mixed number; if not, a whole number. If the answer is a mixed number, the fractional part must be reduced to its lowest terms.

EXAMPLES FOR PRACTICE.

Reduce these fractions to whole or mixed numbers:—

1. $\frac{9}{3}$.	15. $\frac{384}{37}$.	*Ans.* $10\frac{14}{37}$.	29. $\frac{5453}{303}$.
2. $\frac{63}{9}$.	16. $\frac{300}{16}$.	*Ans.* $18\frac{3}{4}$.	30. $\frac{72442}{441}$.
3. $\frac{26}{8}$.	17. $\frac{587}{13}$.	*Ans.* $45\frac{2}{13}$.	31. $\frac{21631}{187}$.
4. $\frac{144}{12}$.	18. $\frac{3410}{38}$.	*Ans.* $89\frac{28}{38}$.	32. $\frac{20835}{331}$.
5. $\frac{104}{13}$.	19. $\frac{3307}{97}$.	*Ans.* $34\frac{9}{97}$.	33. $\frac{18240}{760}$.
6. $\frac{192}{16}$.	20. $\frac{8997}{99}$.	*Ans.* $90\frac{87}{99}$.	34. $\frac{26351}{423}$.
7. $\frac{571}{35}$.	21. $\frac{5805}{63}$.	*Ans.* $92\frac{1}{7}$.	35. $\frac{15687}{98}$.
8. $\frac{971}{89}$.	22. $\frac{3313}{101}$.	*Ans.* $32\frac{81}{101}$.	36. $\frac{45835}{3635}$.
9. $\frac{60}{25}$.	23. $\frac{9000}{111}$.	*Ans.* $81\frac{9}{111}$.	37. $\frac{53400}{2800}$.
10. $\frac{30}{7}$.	24. $\frac{142641}{239}$.	*Ans.* $596\frac{197}{239}$.	38. $\frac{29036}{68}$.
11. $\frac{90}{12}$.	25. $\frac{6500}{250}$.	*Ans.* 26.	39. $\frac{34626}{597}$.
12. $\frac{56}{8}$.	26. $\frac{14022}{7}$.	*Ans.* $2003\frac{1}{7}$.	40. $\frac{99999}{16888}$.
13. $\frac{131}{7}$.	27. $\frac{11152}{13}$.	*Ans.* $857\frac{11}{13}$.	41. $\frac{427135}{8431}$.
14. $\frac{460}{44}$.	28. $\frac{2482}{43}$.	*Ans.* $57\frac{31}{43}$.	42. $\frac{848967}{326}$.

143. What is the second case of reduction of fractions? What does a fraction indicate? With what do the numerator and denominator correspond? How may we find the quotient,—that is, the value of the fraction? Give the rule for reducing an improper fraction to a whole or mixed number. Give examples.

144. CASE III.—*To reduce a mixed number to an improper fraction.*

EXAMPLE.—Reduce $9\frac{3}{5}$ to an improper fraction.

9	
5	
45	fifths.
3	fifths.
48	fifths.

The denominator of the fraction being 5, we reduce to fifths. In 1 there are 5 fifths, and in 9 nine times 5 fifths, or 45 fifths. 45 fifths and 3 fifths make 48 fifths. *Ans.* $\frac{48}{5}$.

Proof: $\frac{48}{5} = 48 \div 5 = 9\frac{3}{5}$

RULE.—1. *Multiply the whole number by the denominator of the fraction, add in the numerator, and set their sum over the denominator.*

2. *Prove by reducing the improper fraction obtained back to a mixed number.*

145. To reduce a whole number to an improper fraction with a given denominator, the process is the same, except that there is no numerator to add in. *Multiply the whole number by the given denominator, and set the product over the denominator.*

EXAMPLE.—Reduce 9 to fifths.

$$9 \times 5 = 45 \quad \textit{Ans.}\ \frac{45}{5}.$$

EXAMPLES FOR PRACTICE.

Reduce the following to improper fractions; prove each:—

1. $12\frac{4}{5}$. *Ans.* $\frac{64}{5}$.	5. $7\frac{7}{19}$.	9. $763\frac{51}{93}$.	13. $84\frac{260}{1231}$.
2. $16\frac{2}{3}$. *Ans.* $\frac{50}{3}$.	6. $77\frac{7}{11}$.	10. $3764\frac{17}{45}$.	14. $484\frac{26}{405}$.
3. $24\frac{7}{12}$.	7. $37\frac{31}{41}$.	11. $81\frac{353}{365}$.	15. $296\frac{12}{44}$.
4. $19\frac{273}{495}$.	8. $41\frac{65}{928}$.	12. $1234\frac{29}{31}$.	16. $8\frac{1317}{100000}$.

17. Reduce 13 to a fraction with 7 for its denominator. *Ans.* $\frac{91}{7}$.

18. How many 89ths in 746? In $29\frac{3}{89}$? In 450?

19. Reduce 26 to fortieths. To fiftieths. To sixtieths.

20. How many quarters of beef in 1225 oxen?

21. Reduce 387 to nineteenths. To eighty-fifths.

---

144. What is the third case of reduction of fractions? Solve and prove the given example. Recite the rule for reducing a mixed number to an improper fraction.—145. How does the operation differ, when a whole number is to be reduced to an improper fraction? Recite the rule. Give an example.

146. CASE IV.—*To reduce a fraction to higher terms.*
A fraction is reduced to *lower* terms (§ 142) by *division*, to *higher* terms by *multiplication*.

EXAMPLE.—Reduce $\frac{3}{4}$ to twenty-fourths.

$$24 \div 4 = 6$$

$$\frac{3 \times 6}{4 \times 6} = \frac{18}{24}$$

Ans. $\frac{18}{24}$.

Multiplying both numerator and denominator by the same number does not alter the value of the fraction. We therefore multiply both terms by such a number as will change fourths to twenty-fourths—that is, 6 (because $24 \div 4 = 6$). *Ans.* $\frac{18}{24}$.

RULE.—1. *Divide the given denominator by the denominator of the fraction, and multiply both terms by the quotient.*

2. *Prove by reducing the fraction back to its lowest terms.*

Mixed numbers must first be reduced to improper fractions.

147. A fraction can thus be reduced only to such higher terms as are multiples of the original terms. Thus, $\frac{3}{4}$ can be reduced to eighths, twelfths, sixteenths, &c., but not to fifths or sixths.

EXAMPLES FOR PRACTICE.

1. Reduce $\frac{3}{14}$ to seventieths. *Ans.* $\frac{15}{70}$.
2. Reduce the following to 36ths:—$\frac{5}{9}$; $\frac{7}{18}$; $\frac{3}{4}$; $\frac{1}{6}$.
3. Reduce to 288ths:—$\frac{7}{12}$; $\frac{5}{24}$; $\frac{19}{36}$; $\frac{29}{48}$; $\frac{31}{72}$; $\frac{109}{144}$; $\frac{79}{96}$; $\frac{1}{3}$.
4. Reduce $14\frac{3}{11}$ to twenty-seconds. *Ans.* $\frac{314}{22}$.
5. How many 840ths in $1\frac{7}{8}$? *Ans.* $\frac{1575}{840}$.
6. How many 360ths in $\frac{17}{90}$? In $4\frac{11}{18}$? In $2\frac{9}{20}$? In $\frac{49}{72}$?
7. How many seventy-seconds in $\frac{5}{36}$? In $2\frac{1}{24}$? In $4\frac{7}{8}$?
8. Reduce the following to 2400ths:—$\frac{17}{30}$; $\frac{19}{96}$; $\frac{321}{1200}$.

148. CASE V.—*To reduce two or more fractions to others having a common (that is, the same) denominator.*

EXAMPLE.—Reduce $\frac{3}{4}$, $\frac{1}{2}$, and $\frac{5}{6}$, to fractions that have a common denominator.

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146. What is the fourth case of reduction of fractions? How is a fraction reduced to lower terms? How, to higher terms? Reduce $\frac{3}{4}$ to twenty-fourths, explaining the steps. Recite the rule. What must first be done with mixed numbers?—147. To what higher terms alone can a fraction thus be reduced?—148. What is the fifth case of reduction of fractions? Work out and explain the given example.

The denominators are 4, 2, and 6. Now, a product is the same, in whatever order the factors are taken. Hence, if we multiply each denominator by the other two, we shall get a common multiple of all three, and this will be the common denominator.

$$\left.\begin{array}{l} 4 \times 2 \times 6 = 48 \\ 2 \times 4 \times 6 = 48 \\ 6 \times 4 \times 2 = 48 \end{array}\right\} \text{Common denom.}$$

$$\left.\begin{array}{l} \frac{3 \times 2 \times 6}{4 \times 2 \times 6} = \frac{36}{48} \\ \frac{1 \times 4 \times 6}{2 \times 4 \times 6} = \frac{24}{48} \\ \frac{5 \times 4 \times 2}{6 \times 4 \times 2} = \frac{40}{48} \end{array}\right\} \textit{Ans.}$$

But the value of the fractions must not be changed. We must, therefore, multiply each numerator by the same multipliers as its denominator. Hence the rule:—

RULE.—*Multiply both terms of each fraction by all the denominators except its own.*

Whole numbers must first be reduced to a fractional form, and mixed numbers to improper fractions.

EXAMPLES FOR PRACTICE.

Reduce the following to equivalent fractions having a common denominator:—

1. Reduce $\frac{3}{4}$ and $\frac{5}{6}$. *Ans.* $\frac{18}{24}$, $\frac{20}{24}$.
2. Reduce $\frac{10}{11}$ and $\frac{11}{12}$. *Ans.* $\frac{120}{132}$, $\frac{121}{132}$.
3. Reduce $\frac{17}{18}$ and $\frac{19}{20}$. *Ans.* $\frac{340}{360}$, $\frac{342}{360}$.
4. Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$. *Ans.* $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$.
5. Reduce $\frac{1}{3}$, $\frac{1}{5}$, and $\frac{1}{7}$. *Ans.* $\frac{35}{105}$, $\frac{21}{105}$, $\frac{15}{105}$.
6. Reduce $\frac{18}{20}$, $\frac{6}{7}$, and $\frac{4}{5}$. *Ans.* $\frac{630}{700}$, $\frac{600}{700}$, $\frac{560}{700}$.
7. Reduce $\frac{12}{13}$, $\frac{13}{14}$, and $\frac{14}{15}$. *Ans.* $\frac{2520}{2730}$, $\frac{2535}{2730}$, $\frac{2548}{2730}$.
8. Reduce 4, $1\frac{1}{3}$, 12, and $\frac{5}{9}$. *Ans.* $\frac{108}{27}$, $\frac{36}{27}$, $\frac{324}{27}$, $\frac{15}{27}$.
9. Reduce $2\frac{2}{25}$, $\frac{1}{6}$, and 18. *Ans.* $\frac{312}{150}$, $\frac{25}{150}$, $\frac{2700}{150}$.
10. Reduce 3, 6, $2\frac{1}{4}$, and $8\frac{5}{8}$. *Common Denom.* 32.
11. Reduce $\frac{2}{3}$, $\frac{4}{15}$, $\frac{1}{2}$, and $2\frac{1}{7}$. *Common Denom.* 630.
12. Reduce $3\frac{1}{3}$, 4, $\frac{5}{7}$, $8\frac{1}{9}$, and $\frac{3}{2}$. *Common Denom.* 378.
13. Reduce $\frac{63}{50}$, 17, $\frac{4}{9}$, 5, and $\frac{7}{2}$. *Common Denom.* 900.
14. Reduce $\frac{7}{12}$, $1\frac{5}{7}$, $4\frac{3}{5}$, and 220. *Common Denom.* 420.
15. Reduce $3\frac{1}{6}$, $\frac{2}{3}$, $\frac{2}{7}$, $3\frac{3}{4}$, and 100. *Common Denom.* 504.
16. Reduce $\frac{9}{14}$, $\frac{13}{8}$, $5\frac{1}{3}$, 7, and $\frac{9}{10}$. *Common Denom.* 840.
17. Reduce $\frac{11}{18}$, $\frac{24}{7}$, $\frac{3}{10}$, 15, and $2\frac{5}{6}$. *Common Denom.* 7560.

What is the rule for reducing two or more fractions to others having a common denominator? What must first be done with whole and mixed numbers?

149. CASE VI.—*To reduce two or more fractions to others having the least common denominator.*

Under the last Case, we found that the common denominator was a common multiple of the several denominators. The *least* common denominator is the *least* common multiple of the denominators. Find this least common multiple, therefore; and then reduce the given fractions to others that have this least common multiple for their denominator, according to § 146.

EXAMPLE.—Reduce $\frac{3}{4}$, $\frac{1}{2}$, and $\frac{5}{6}$, to fractions having the least common denominator.

$$\begin{array}{r|l} 2 & 4,\ 2,\ 6 \\ \hline & 2,\ 1,\ 3 \end{array}$$

$2 \times 2 \times 3 = 12$ *L. C. D.*
$12 \div 4 \times 3 = 9$
$12 \div 2 \times 1 = 6$
$12 \div 6 \times 5 = 10$

Ans. $\frac{9}{12}$, $\frac{6}{12}$, $\frac{10}{12}$.

The least common multiple of the denominators is 12, which is therefore the least common denominator. To find the several numerators, divide this least common denominator by the denominator of each fraction, and multiply the quotient by its numerator.

150. RULE.—1. *For the least common denominator, find the least common multiple of the given denominators.*

2. *For the new numerators, divide this least common denominator by the denominator of each fraction, and multiply the quotient by its numerator.*

First reduce the fractions to their lowest terms, and whole or mixed numbers to improper fractions.

EXAMPLES FOR PRACTICE.

Reduce the following to equivalent fractions having the least common denominator:—

1. $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$. *Ans.* $\frac{45}{60}$, $\frac{48}{60}$, $\frac{50}{60}$.
2. $3\frac{1}{2}$, $4\frac{1}{3}$, and $\frac{12}{10}$. *Ans.* $\frac{105}{30}$, $\frac{130}{30}$, $\frac{36}{30}$.
3. $\frac{8}{9}$, $\frac{6}{8}$, and $\frac{5}{7}$. *Ans.* $\frac{224}{252}$, $\frac{189}{252}$, $\frac{180}{252}$.

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149. What is the sixth case of reduction of fractions? Under the last case, what did we find the common denominator to be? What, then, will the least common denominator be? How, therefore, must we proceed? Solve and explain the given example.—150. Recite the rule for reducing fractions to others having the least common denominator. What should first be done with the fractions? With whole or mixed numbers?

4. $\frac{5}{12}$, $\frac{7}{15}$, and $\frac{11}{24}$. *Ans.* $\frac{50}{120}$, $\frac{56}{120}$, $\frac{55}{120}$.
5. $\frac{8}{9}$, $\frac{13}{20}$, and $\frac{7}{15}$. *Ans.* $\frac{160}{180}$, $\frac{117}{180}$, $\frac{84}{180}$.
6. $\frac{4}{15}$, $\frac{5}{11}$, and $6\frac{3}{22}$. *Ans.* $\frac{88}{330}$, $\frac{150}{330}$, $\frac{2025}{330}$.
7. $\frac{1}{2}$, $\frac{2}{3}$, $8\frac{1}{4}$ and $\frac{3}{10}$. *Ans.* $\frac{30}{60}$, $\frac{40}{60}$, $\frac{125}{60}$, $\frac{18}{60}$.
8. $\frac{1}{10}$, $\frac{1}{3}$, $\frac{1}{7}$, and $\frac{4}{21}$. *Ans.* $\frac{21}{210}$, $\frac{70}{210}$, $\frac{30}{210}$, $\frac{40}{210}$.
9. $\frac{4}{3}$, 4, $\frac{5}{9}$, and $\frac{7}{12}$. *Ans.* $\frac{48}{36}$, $\frac{144}{36}$, $\frac{20}{36}$, $\frac{21}{36}$.
10. $3\frac{2}{5}$, $\frac{7}{10}$, $\frac{10}{40}$, and 50. *Ans.* $\frac{68}{20}$, $\frac{14}{20}$, $\frac{5}{20}$, $\frac{1000}{20}$.
11. $\frac{5}{36}$, $\frac{7}{48}$, $\frac{8}{72}$, and $1\frac{5}{8}$. *Least Com. Den.* 144.
12. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{9}{20}$. *Least Com. Den.* 60.
13. $\frac{5}{6}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{9}{10}$, and $\frac{7}{120}$.
14. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, and $\frac{1}{9}$.
15. $\frac{2}{7}$, $\frac{1}{4}$, $\frac{9}{10}$, $\frac{7}{16}$, $\frac{3}{8}$, $1\frac{1}{8}$, and $\frac{5}{8}$.
16. $2\frac{1}{12}$, $1\frac{7}{18}$, 5, $3\frac{29}{72}$, and $4\frac{5}{8}$.

Addition of Fractions.

151. Like parts, such as halves and halves, thirds and thirds, can be added, just as we can add pears and pears, dollars and dollars. Unlike parts, such as halves and thirds, can not be thus *directly* added, any more than we can add pears and dollars.

EXAMPLE 1.—Add 5 sixths and 3 sixths. *Ans.* 8 sixths.

The denominators being the same, we add the numerators, and place their sum over the common denominator. $\frac{5}{6}+\frac{3}{6}=\frac{8}{6}$

EXAMPLE 2.—Add 5 sixths and 3 fourths.

$$\frac{5}{6}=\frac{10}{12}$$
$$\frac{3}{4}=\frac{9}{12}$$
$$Ans.\ \frac{19}{12}=1\frac{7}{12}$$

The denominators being different, we can not add the numerators, and call the sum 8 sixths or 8 fourths. But, if we reduce the fractions to others having a common denominator, we can then add, as in Ex. 1. 12 being the least common denominator, reduce the given fractions to twelfths.

EXAMPLE 3.—Add together $\frac{7}{10}$, $2\frac{1}{4}$, $4\frac{2}{5}$, and 1.

Add the fractions, as in Ex. 2:	$\frac{7}{10}+\frac{1}{4}+\frac{2}{5}=1\frac{7}{20}$
Add the whole numbers:	$2+4+1=7$
Add these two sums:	*Ans.* $8\frac{7}{20}$

151. Can we add like parts, such as halves and halves, directly? Can we add unlike parts, such as halves and thirds, directly? Add $\frac{5}{6}$ and $\frac{3}{6}$. Add $\frac{5}{6}$ and $\frac{3}{4}$. Solve Example 3.

152. Rule.—1. *When the fractions have a common denominator, place over it the sum of their numerators. When not, after reducing them to their lowest terms, change them to equivalent fractions having the least common denominator, and add as above. Reduce the result to its lowest terms, or to a whole or mixed number, as may be necessary.*

2. *To add mixed numbers, find the sum of the fractions and whole numbers separately, and add the results.*

EXAMPLES FOR PRACTICE.

Find the sum of the following fractions:—

1. $\frac{1}{9}+\frac{4}{9}+\frac{7}{9}$. *Ans.* $1\frac{1}{3}$.
2. $\frac{17}{27}+\frac{19}{27}$.
3. $\frac{7}{30}+\frac{11}{30}+\frac{17}{30}$.
4. $\frac{1}{44}+\frac{15}{44}+\frac{21}{44}+1\frac{29}{44}$.
5. $2\frac{3}{50}+3+99+\frac{47}{50}$.
6. $\frac{1}{8}+\frac{2}{8}+\frac{3}{8}+\frac{4}{8}+\frac{5}{8}+\frac{7}{8}$.
7. $\frac{2}{3}+\frac{3}{4}+\frac{11}{12}$. *Ans.* $2\frac{1}{3}$.
8. $\frac{5}{6}+\frac{2}{3}+\frac{1}{3}$.
9. $\frac{2}{3}+\frac{1}{6}+\frac{3}{5}+\frac{1}{10}$.
10. $\frac{2}{3}+\frac{5}{8}+\frac{4}{6}+\frac{3}{4}$.
11. $\frac{5}{15}+\frac{10}{15}+\frac{20}{15}+\frac{2}{3}$.
12. $\frac{3}{12}+\frac{2}{8}+\frac{12}{16}+\frac{6}{24}$.

13. Add together $\frac{5}{18}$, $\frac{22}{24}$, $\frac{19}{20}$, and $\frac{7}{8}$. *Ans.* $2\frac{307}{360}$.

14. Add together 2, $\frac{17}{14}$, $\frac{5}{18}$, and $\frac{20}{27}$. *Ans.* $4\frac{44}{189}$.

15. Find the sum of $\frac{3}{10}$, $10\frac{1}{3}$, $12\frac{3}{4}$, and $2\frac{11}{15}$. *Ans.* $26\frac{7}{60}$.

16. Find the sum of $4\frac{7}{36}$, $4\frac{7}{48}$, $4\frac{7}{72}$, and $\frac{5}{18}$. *Ans.* $12\frac{103}{144}$.

17. What is the value of $3\frac{1}{3}+6\frac{1}{6}+9\frac{1}{9}+\frac{11}{12}$? *Ans.* $19\frac{19}{36}$.

18. What is the value of $3\frac{7}{12}+\frac{5}{8}+1\frac{2}{3}+\frac{5}{6}+1\frac{1}{4}$? *Ans.* $7\frac{23}{24}$.

19. Add $\frac{5}{11}$ and $\frac{7}{12}$. Add $\frac{2}{3}$ and $\frac{1}{4}$. Add $\frac{5}{6}$ and $\frac{3}{8}$.

20. What is the cost of four fields, containing respectively $4\frac{1}{9}$, $2\frac{1}{2}$, $3\frac{4}{9}$, and $1\frac{17}{18}$ acres, at \$25 an acre? *Ans.* \$300.

21. Bought \$$10\frac{1}{4}$ worth of cloth, \$$5\frac{3}{8}$ worth of lace, \$$18\frac{25}{100}$ worth of velvet, and \$$9\frac{1}{8}$ worth of muslin. How much change must I receive for a \$50 bill? *Ans.* \$7.

22. How many times can four baskets, holding respectively $3\frac{7}{15}$, $2\frac{1}{3}$, $1\frac{5}{12}$, and $2\frac{3}{4}$ pecks, be filled from a pile containing 20 pecks of potatoes? *Ans.* Twice.

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152. Recite the rule for the addition of fractions. Recite the rule for the addition of mixed numbers.

Subtraction of Fractions.

153. Case I.—*To subtract a fraction from a fraction.* As in addition, so in subtraction, if the fractions have not a common denominator, they must be reduced to equivalent fractions that have.

Example 1.—From 5 sixths take 4 sixths. $\frac{5}{6} - \frac{4}{6} = \frac{1}{6}$ *Ans.*

4 sixths from 5 sixths leave 1 sixth. *Ans.* $\frac{1}{6}$.

Example 2.—From 5 sixths take 3 fourths.

We can not *directly* take *fourths* from *sixths*; but (12 being the least common multiple of the denominators) we can reduce both to *twelfths*, and then subtract.

$$\frac{5}{6} = \frac{10}{12}$$
$$\frac{3}{4} = \frac{9}{12}$$
Ans. $\frac{1}{12}$

154. Rule.—*When the fractions have a common denominator, place over it the difference of their numerators. When not, reduce them to equivalent fractions having the least common denominator, and proceed as above.*

EXAMPLES FOR PRACTICE.

Find the value of the following:—

1. $\frac{8}{9} - \frac{5}{9}$. *Ans.* $\frac{1}{3}$.
2. $\frac{5}{6} - \frac{1}{6}$.
3. $\frac{11}{15} - \frac{1}{15}$.
4. $\frac{49}{90} - \frac{4}{90}$.
5. $\frac{99}{144} - \frac{27}{144}$.
6. $\frac{1}{5} - \frac{1}{8}$. *Ans.* $\frac{3}{40}$.
7. $\frac{9}{14} - \frac{4}{21}$. *Ans.* $\frac{19}{42}$.
8. $\frac{3}{5} - \frac{9}{36}$. *Ans.* $\frac{7}{20}$.
9. $\frac{7}{20} - \frac{1}{10}$. *Ans.* $\frac{1}{4}$.
10. $\frac{17}{18} - \frac{5}{12}$. *Ans.* $\frac{19}{36}$.
11. $\frac{4}{5} - \frac{3}{8}$.
12. $\frac{5}{9} - \frac{2}{11}$.
13. $\frac{9}{16} - \frac{1}{12}$.
14. $\frac{11}{15} - \frac{3}{10}$.
15. $\frac{5}{28} - \frac{1}{42}$.

155. Case II.—*To subtract a fraction from a whole number.*

Example.—From 3 take $\frac{2}{9}$.

Take 1 of the 3 units, and reduce it to ninths. From the $\frac{9}{9}$ thus obtained subtract $\frac{2}{9}$, and bring down the 2 units. *Ans.* $2\frac{7}{9}$.

$$3 = 2\frac{9}{9}$$
$$\frac{2}{9}$$
Ans. $2\frac{7}{9}$

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153. In subtraction of fractions, what is it necessary to do if the fractions have not a common denominator? From $\frac{5}{6}$ take $\frac{4}{6}$. From $\frac{5}{6}$ take $\frac{3}{4}$.—154. Recite the rule for subtraction of fractions.—155. From 3 take $\frac{2}{9}$.

RULE.—*Reduce* 1 *to a fraction having the same denominator as the given fraction. From this subtract the given fraction, and annex the remainder to the given whole number less* 1.

EXAMPLES FOR PRACTICE.

1. $2-\frac{3}{4}$. *Ans.* $1\frac{1}{4}$.	5. $1-\frac{6}{7}$. *Ans.* $\frac{1}{7}$.	9. $2-\frac{15}{43}$.
2. $14-\frac{1}{14}$.	6. $8-\frac{53}{100}$. *Ans.* $7\frac{47}{100}$.	10. $1-\frac{1008}{1360}$.
3. $7-\frac{10}{13}$.	7. $5-\frac{31}{35}$. *Ans.* $4\frac{4}{35}$.	11. $19-\frac{347}{1843}$.
4. $10-\frac{9}{10}$.	8. $11-\frac{5}{47}$. *Ans.* $10\frac{42}{47}$.	12. $28-\frac{146}{5061}$.

156. CASE III.—*To subtract one mixed number from another.*

EXAMPLE 1.—From $4\frac{2}{3}$ take $1\frac{1}{5}$.

$$\frac{2}{3}=\frac{10}{15} \quad \frac{1}{5}=\frac{3}{15}$$
$$\frac{10}{15}-\frac{3}{15}=\frac{7}{15}$$
$$4-1=3$$
$$\textit{Ans. } 3\frac{7}{15}$$

$\frac{1}{5}$, the fraction of the subtrahend, being less than $\frac{2}{3}$, the fraction of the minuend, we subtract fraction from fraction, and whole number from whole number, and combine the results.

EXAMPLE 2.—From $4\frac{1}{5}$ subtract $1\frac{2}{3}$.

$$4\frac{1}{5}=4\frac{3}{15}=$$
$$3+1\frac{3}{15}=3\frac{18}{15}$$
$$1\frac{2}{3}=1\frac{10}{15}$$
$$\frac{18}{15}-\frac{10}{15}=\frac{8}{15}$$
$$3-1=2$$
$$\textit{Ans. } 2\frac{8}{15}.$$

Reducing the given fractions to others having a common denominator, we get $\frac{3}{15}$ and $\frac{10}{15}$. The numerator of the fraction in the subtrahend being the greater, we can not proceed as in the last Example.

From 4, the whole number of the minuend, we take 1, and, reducing it to fifteenths, add the result to the $\frac{3}{15}$ of the minuend. From $\frac{18}{15}$, thus obtained, subtracting $\frac{10}{15}$, the fraction of the subtrahend, we have $\frac{8}{15}$ for the remainder.

Then, proceeding to the whole numbers, 1 from 3 leaves 2. Combining the results, we have $2\frac{8}{15}$. *Ans.*

157. RULE.—*Reduce the fractions, if necessary, to others having a common denominator. If the numerator of the fraction in the minuend is equal to, or greater than, that in the subtrahend, subtract fraction from fraction, and whole number from whole number. If not, take* 1 *from the whole number of the minuend, and reduce it with the fraction of the minuend to an improper fraction. Then subtract as above.*

Recite the rule for subtracting a fraction from a whole number.—156. From $4\frac{2}{3}$ subtract $1\frac{1}{5}$. From $4\frac{1}{5}$ take $1\frac{2}{3}$.—157. Recite the rule for subtracting one mixed number from another.

EXAMPLES FOR PRACTICE

1. $6\frac{6}{7}-2\frac{3}{14}$.	*Ans.* $4\frac{9}{14}$.	5. $8\frac{1}{5}-2\frac{2}{9}$.	*Ans.* $5\frac{44}{45}$.
2. $2\frac{1}{3}-1\frac{2}{5}$.	*Ans.* $\frac{14}{15}$.	6. $5\frac{1}{9}-5\frac{1}{10}$.	*Ans.* $\frac{1}{90}$.
3. $4\frac{2}{11}-\frac{7}{22}$.	*Ans.* $3\frac{19}{22}$.	7. $3\frac{15}{64}-2\frac{11}{24}$.	*Ans.* $\frac{149}{192}$.
4. $200\frac{11}{36}-98\frac{3}{16}$.		8. $47\frac{19}{24}-31\frac{7}{18}$.	*Ans.* $16\frac{29}{72}$.

9. From $5\frac{1}{4}+9\frac{2}{3}$ take $4\frac{5}{6}$. *Ans.* $10\frac{1}{12}$.

10. Take $\frac{2}{3}+\frac{17}{24}$ from $\frac{5}{7}+4$. *Ans.* $3\frac{19}{56}$.

11. Take $4\frac{1}{5}+3\frac{1}{12}$ from $6\frac{1}{8}+7\frac{1}{10}$. *Ans.* $5\frac{113}{120}$.

12. From $\frac{2}{3}+1\frac{1}{3}+\frac{5}{4}+1\frac{1}{5}$ take $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}$. *Ans.* $3\frac{1}{6}$.

13. From $\frac{1}{20}+\frac{1}{40}$ take $\frac{1}{30}+\frac{1}{50}$. *Ans.* $\frac{13}{600}$.

14. Take the sum of $1\frac{1}{3}$ and $2\frac{1}{4}$ from $2\frac{1}{2}+3\frac{1}{3}$. *Ans.* $2\frac{1}{4}$.

15. From $181\div20$ subtract $\frac{9}{10}$. *Ans.* $8\frac{3}{20}$.

16. From $440\div80$ subtract $\frac{101}{102}$. *Ans.* $4\frac{26}{51}$.

Multiplication of Fractions.

158. CASE I.—*To multiply a fraction by a whole number.*

We found in § 137, that *multiplying the numerator or dividing the denominator by any number multiplies the fraction by that number.* Hence the rule:—

RULE.—*Divide the denominator of the fraction by the whole number, when it can be done without a remainder; when not, multiply its numerator.*

EXAMPLE 1.—Multiply $\frac{3}{25}$ by 5.

25 is exactly divisible by 5. Divide it. *Ans.* $\frac{3}{5}$.

EXAMPLE 2.—Multiply $\frac{3}{25}$ by 6.

25 is not exactly divisible by 6. Multiply the numerator. *Ans.* $\frac{18}{25}$.

It is best to divide the denominator when it can be done, because the answer is thus found in its lowest terms.

Dividing the denominator increases the *size* of the parts as many times as there are units in the divisor. Multiplying the numerator increases the *number* of parts as many times as there are units in the multiplier.

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158. Recite the rule for multiplying a fraction by a whole number. Solve the examples given. Why is it best to divide the denominator when it can be done? What is the effect of dividing the denominator? What is the effect of multiplying the numerator?

159. Multiplying a fraction by its own denominator gives the numerator. Thus: $\frac{7}{9} \times 9 = \frac{7}{1} = 7$ *Ans.*

EXAMPLES FOR PRACTICE.

Find the value of the following :—

1. $\frac{17}{24} \times 24$. *Ans.* 17.	6. $\frac{5}{16} \times 8$.	11. $\frac{21}{144} \times 12$.
2. $\frac{7}{6} \times 3$. *Ans.* $3\frac{1}{2}$.	7. $\frac{3}{42} \times 14$.	12. $\frac{1}{360} \times 18$.
3. $\frac{4}{9} \times 5$. *Ans.* $2\frac{2}{9}$.	8. $\frac{5}{13} \times 13$.	13. $\frac{3}{500} \times 125$.
4. $\frac{8}{21} \times 14$. *Ans.* $5\frac{1}{3}$.	9. $\frac{4}{7} \times 10$.	14. $\frac{233}{288} \times 288$.
5. $\frac{13}{98} \times 49$. *Ans.* $6\frac{1}{2}$.	10. $\frac{1}{5} \times 4$.	15. $\frac{4}{51} \times 15$.

160. CASE II.—*To multiply a mixed by a whole number.*

RULE.—*Multiply the fractional and the integral part separately, and add the products.*

EXAMPLE.—Multiply $3\frac{5}{6}$ by 7.

Multiply the fractional part:	$\frac{5}{6} \times 7 = \frac{35}{6} =$	$5\frac{5}{6}$
Multiply the integral part:	$3 \times 7 =$	21
Add the products:		$26\frac{5}{6}$ *Ans.*

EXAMPLES FOR PRACTICE.

1. Multiply $4\frac{4}{7}$ by 3. By 4. By 5. By 7. By 14.
2. What cost 8 dolls, at $\$1\frac{1}{16}$ each? *Ans.* $\$8\frac{1}{2}$.
3. Multiply $2\frac{1}{4} + 3\frac{5}{12}$ by 4. *Ans.* $22\frac{2}{3}$.
4. At $\$6\frac{2}{3}$ apiece, what cost five coats? *Ans.* $\$33\frac{1}{3}$.
5. Multiply $6\frac{3}{20} - 2\frac{9}{10}$ by 7. *Ans.* $22\frac{3}{4}$.
6. Multiply 12 times $4\frac{7}{8}$ by 10. *Ans.* 585.
7. How much cloth in 4 pieces, each containing $39\frac{5}{8}$ yards?

161. CASE III.—*To multiply a whole number by a fraction.*

Multiplying by $\frac{1}{2}$ is *taking* $\frac{1}{2}$ (or dividing by 2); *multiplying by* $\frac{1}{3}$ is *taking* $\frac{1}{3}$ (or dividing by 3); and gene-

159. What is obtained, if we multiply a fraction by its own denominator?—160. Give the rule for multiplying a mixed number by a whole number. Multiply $3\frac{5}{6}$ by 7. —161. What is meant by multiplying by one half? By one third?

rally, *multiplying by a fraction is taking such a part as is denoted by the fraction.*

EXAMPLE.—Multiply 19 by $\frac{2}{3}$.

$$\begin{array}{r} 3)\underline{19} \\ 6\frac{1}{3} \\ \underline{2} \\ Ans.\ 12\frac{2}{3} \end{array}$$

Multiplying 19 by $\frac{2}{3}$ is taking $\frac{2}{3}$ of 19. One third of 19 is $6\frac{1}{3}$, and two thirds are twice $6\frac{1}{3}$, or $12\frac{2}{3}$. *Ans.* $12\frac{2}{3}$.

Here we have divided the whole number by the denominator 3, and then multiplied by the numerator 2; but the result is the same if we multiply first and then divide, and it often saves trouble to do so. Hence the rule:—

$$\begin{array}{r} 19 \\ \underline{2} \\ 3)\underline{38} \\ Ans.\ 12\frac{2}{3} \end{array}$$

RULE.—*Multiply the whole number by the numerator of the fraction, and divide by its denominator.*

First see that the fraction is in its lowest terms.

EXAMPLES FOR PRACTICE.

Find the value of the following:—

1. $47 \times \frac{8}{9}$.	*Ans.* $41\frac{7}{9}$.	4. $221 \times \frac{7}{12}$.	7. $49 \times \frac{22}{35}$.
2. $93 \times \frac{5}{6}$.	*Ans.* $77\frac{1}{2}$.	5. $453 \times \frac{3}{14}$.	8. $2846 \times \frac{14}{33}$.
3. $69 \times \frac{7}{3}$.	*Ans.* 161.	6. $598 \times \frac{11}{13}$.	9. $6789 \times \frac{15}{37}$.

10. Multiply four billion by $\frac{100}{909}$. *Ans.* $440044004\frac{364}{909}$.

11. Find the product of 19 million and $\frac{88}{99}$. *Ans.* $16888888\frac{8}{9}$.

12. A century is 100 years. How many years in $\frac{3}{4}$ of 10 centuries? In $\frac{4}{5}$ of 20 centuries?

13. How many feet in $\frac{7}{8}$ of a mile, there being 5280 feet in a mile? How many feet in $\frac{9}{10}$ of a mile?

14. A merchant owes \$20000. How much is his property worth, if it amounts to $\frac{3}{7}$ of his debts? *Ans.* $\$8571\frac{3}{7}$.

15. The moon is 240000 miles from the earth. If it were but $\frac{9}{16}$ of that distance, how far from the earth would it be?

162. CASE IV.—*To multiply a whole by a mixed number.*

RULE.—*Multiply the fractional part and the whole part separately, and add the products.*

In general, what is multiplying by a fraction? Multiply 19 by $\frac{2}{3}$, in both the ways shown above. Recite the rule for multiplying a whole number by a fraction. —162. Recite the rule for multiplying a whole number by a mixed number.

EXAMPLE.—Multiply 458 by $9\frac{3}{4}$.

	458
	$9\frac{3}{4}$
Multiply 458 by $\frac{3}{4}$ (§ 161):	$343\frac{1}{2}$
Multiply 458 by 9:	4122
Add the products:	$4465\frac{1}{2}$ *Ans.*

EXAMPLES FOR PRACTICE.

1. $19\times4\frac{7}{20}$. *Ans.* $82\frac{13}{20}$.
2. $62\times12\frac{1}{7}$. *Ans.* $752\frac{6}{7}$.
3. $86\times7\frac{5}{16}$. *Ans.* $628\frac{7}{8}$.
4. $875\times63\frac{4}{5}$.
5. $741\times8\frac{15}{38}$.
6. $219\times9\frac{3}{73}$.
7. $34\times2\frac{1}{34}$.
8. $9080\times5\frac{4}{5}$.
9. $79\times37\frac{21}{43}$.
10. Multiply 45 thousand by $81\frac{3}{17}$. *Ans.* $3652941\frac{3}{17}$.
11. How many feet in 320 rods, there being $16\frac{1}{2}$ feet in one rod?
12. At \$75 an acre, what is the cost of three lots containing respectively $3\frac{1}{3}$, $4\frac{1}{4}$, and $5\frac{1}{5}$ acres? *Ans.* \$$958\frac{3}{4}$.

163. CASE V.—*To multiply a fraction by a fraction, or to reduce a compound fraction to a simple one.*

Multiplying by a fraction, we learned in § 161, is equivalent to taking such a part as is denoted by the fraction. Multiplying $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{7}{9}$ together, is equivalent to taking $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{7}{9}$. The same process is therefore used in multiplying fractions together and in reducing compound fractions to simple ones.

EXAMPLE 1.—Multiply $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{7}{9}$ together.

These fractions indicate division. The numerators are the dividends; the denominators, the divisors. Multiply the numerators together to find the total dividend, and the denominators to find the total divisor. Then set the former product over the latter in the form of a fraction.

$$\frac{2}{3}\times\frac{4}{5}\times\frac{7}{9}=\frac{56}{135}\ \textit{Ans.}$$

164. As in division (§ 113), cancelling often shortens the operation. By first cancelling the equal factors common to any numerator and denominator, we get the answer at once in its lowest terms.

Solve the given example.—163. To what is multiplying by a fraction equivalent? In what two operations, therefore, is the same process used? Explain Example 1.—164. How may the operation often be shortened? What do we gain by first cancelling equal factors?

Ex. 2.—Reduce $\frac{5}{6}$ of $\frac{3}{49}$ of $\frac{2}{9}$ of $\frac{14}{5}$ to a simple fraction.

Cancel 5 and 5. Cancel 3 in the second numerator and first denominator. Cancel the 2 then remaining in the first denominator, and 2 in the third numerator. Cancel 7 in the fourth numerator and second denominator. Then multiply the remaining factors, as in the last Example.

$$\frac{\cancel{5}}{\underset{\cancel{2}}{\cancel{6}}} \text{ of } \frac{\cancel{3}}{\underset{7}{\cancel{49}}} \text{ of } \frac{\cancel{2}}{9} \text{ of } \frac{\overset{2}{\cancel{14}}}{\cancel{5}} = \frac{2}{63} \textit{ Ans.}$$

Ex. 3.—Multiply together $2\frac{1}{8}$, $1\frac{29}{34}$, $\frac{4}{9}$, and 11.

Reduce the mixed numbers to improper fractions. Throw the whole number into a fractional form, by giving it 1 for its denominator. Then proceed as in Example 2.

$$\frac{\cancel{17}}{\underset{2}{\cancel{8}}} \times \frac{\overset{7}{\cancel{63}}}{\underset{2}{\cancel{34}}} \times \frac{\cancel{4}}{\cancel{9}} \times \frac{11}{1} = \frac{77}{4} = 19\tfrac{1}{4} \textit{ Ans.}$$

Cancel 17 in the first numerator and second denominator. Cancel 4 in the third numerator and first denominator. Cancel 9 in the second numerator and third denominator. Multiply the remaining factors. Reduce the improper fraction obtained to a mixed number.

165. Rule.—1. *Cancel factors common to any numerator and denominator. Then multiply the numerators together for a new numerator, and the denominators for a new denominator.*

2. *Whole numbers must first be reduced to a fractional form, and mixed numbers to improper fractions. Reduce the result, when necessary, to a whole or mixed number.*

EXAMPLES FOR PRACTICE.

Find the value of the following:—

1. $\frac{8}{11} \times \frac{55}{56}$.	*Ans.* $\frac{5}{7}$.	5. $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$.	*Ans.* $\frac{2}{5}$.
2. $\frac{13}{14} \times \frac{28}{120}$.	*Ans.* $\frac{13}{60}$.	6. $\frac{5}{6} \times \frac{4}{7} \times \frac{6}{11}$.	*Ans.* $\frac{20}{77}$.
3. $\frac{68}{121} \times \frac{11}{17}$.	*Ans.* $\frac{4}{11}$.	7. $\frac{5}{8} \times \frac{9}{14} \times \frac{16}{70}$.	*Ans.* $\frac{9}{98}$.
4. $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}$.		8. $\frac{1}{5} \times \frac{22}{8} \times 40$.	

9. Reduce to a simple fraction $\frac{3}{8}$ of $\frac{8}{9}$ of $\frac{3}{4}$ of $\frac{1}{7}$. *Ans.* $\frac{1}{28}$.

Go through Example 2. Explain Example 3.—165. Recite the rule for multiplying a fraction by a fraction, or reducing a compound fraction to a simple one.

10. Reduce $\frac{1}{4}$ of $\frac{5}{8}$ of $\frac{10}{3}$ of $\frac{15}{36}$ to its simplest form. *Ans.* $1\frac{7}{8}$.
11. Find the product of $\frac{4}{7}$ of $\frac{11}{12}$ and $\frac{2}{3}$ of $\frac{12}{14}$. *Ans.* $\frac{44}{147}$.
12. Multiply $\frac{2}{4}$ of $\frac{7}{8}$ of $\frac{3}{5}$ by $\frac{7}{10}$ of $\frac{11}{12}$. *Ans.* $\frac{539}{3200}$.
13. Multiply $\frac{1}{5}$ of $4\frac{1}{6}$ by $3\frac{1}{5}$. *Ans.* $2\frac{2}{3}$.
14. Multiply $7\frac{1}{2}$ by $3\frac{1}{3}$. $14\frac{1}{2}$ by 5. Add the products.
15. Multiply $6\frac{1}{5}$ by $2\frac{1}{11}$. $\frac{1}{35}$ by $8\frac{3}{7}$. Add the products.
16. Find the difference between $3\frac{1}{4}\times\frac{8}{26}$ and $5\frac{5}{6}\times2\frac{2}{35}$. *Ans.* 11.
17. Find the value of $\frac{3}{8}\times\frac{1}{51}\times4\frac{2}{25}\times\frac{5}{6}\times7$. *Ans.* $\frac{7}{40}$.
18. Reduce $\frac{3}{7}$ of $\frac{7}{8}$ of $\frac{5}{6}$ of $\frac{4}{11}$ of $\frac{11}{12}$ of $\frac{8}{7}$ of $\frac{2}{3}$. *Ans.* $\frac{5}{63}$.
19. How much more is 6 times $\frac{5}{9}$ than 18 times $\frac{1}{27}$? *Ans.* $2\frac{2}{3}$.
20. Multiply $3\frac{1}{2}+3\frac{2}{3}+3\frac{7}{12}$ by $1\frac{1}{2}+\frac{17}{43}$. *Ans.* $20\frac{3}{8}$.
21. Reduce $\frac{18}{25}$ of $\frac{18}{20}$ of $\frac{70}{8}$ of $\frac{15}{76}$ of $1\frac{21}{54}$. *Ans.* $4\frac{3}{8}$.
22. Reduce $\frac{18}{19}$ of $\frac{1}{11}$ of $\frac{5}{7}$ of $\frac{14}{15}$ of $3\frac{1}{6}$. $\Big\{$ *Same ans.*
23. Multiply $\frac{2}{21}\times3\times\frac{5}{11}\times\frac{1}{5}$ by 7. *for both.*

Division of Fractions,
Reduction of Complex Fractions.

166. A fraction divided by a fraction may be expressed in two ways: with the sign of division, or in the form of a complex fraction. Whichever way the division is expressed, the operation is the same. Hence, to reduce a complex fraction to a simple one, *take the denominator as a divisor, and proceed as in division of fractions.*

Three fourths divided by five sixths. $\left\{ \frac{3}{4} \div \frac{5}{6} \quad \text{Or, } \frac{\frac{3}{4}}{\frac{5}{6}} \right.$

167. CASE I.—*To divide a fraction by a whole number.*

We found in § 137 that *dividing the numerator or multiplying the denominator by any number divides the fraction by that number.* Hence the rule:—

RULE.—*Divide the numerator of the fraction by the whole number when it can be done without a remainder; when not, multiply its denominator.*

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166. In what two ways may a fraction divided by a fraction be expressed?—167. What is the first case of division of fractions? Recite the rule for dividing a fraction by a whole number.

Dividing the numerator diminishes the *number* of parts as many times as there are units in the divisor. Multiplying the denominator diminishes the *size* of the parts as many times as there are units in the multiplier.

EXAMPLE 1.—Divide $\frac{36}{7}$ by 6.

36 is exactly divisible by 6. Divide it. $\frac{36}{7} \div 6 = \frac{6}{7}$ *Ans.*

EXAMPLE 2.—Reduce $\frac{\frac{36}{7}}{5}$ to a simple fraction.

36 is not exactly divisible by 5. Multiply the denominator. $\frac{36}{7} \div 5 = \frac{36}{35}$ *Ans.*

EXAMPLES FOR PRACTICE.

Find the value of the following:—

1. $\frac{72}{111} \div 9$.	7. $\frac{56841}{1111} \div 10$.	13. Reduce $\frac{\frac{144}{51}}{36}$. *Ans.* $\frac{4}{51}$.
2. $\frac{72}{111} \div 8$.	8. $\frac{22885}{45668} \div 5$.	
3. $\frac{72}{111} \div 5$.	9. $\frac{15910}{74223} \div 43$.	14. Reduce $\frac{\frac{112}{5}}{31}$. *Ans.* $\frac{112}{155}$.
4. $\frac{72}{111} \div 7$.	10. $\frac{22512}{849} \div 67$.	
5. $\frac{11988}{471} \div 12$.	11. $\frac{16744}{16203} \div 91$.	15. Reduce $\frac{\frac{42}{5}}{6}$; $\frac{\frac{121}{17}}{11}$; $\frac{\frac{22}{15}}{7}$.
6. $\frac{40160}{393} \div 10$.	12. $\frac{24886}{9875} \div 86$.	

168. CASE II.—*To divide a mixed by a whole number.*

Ex. 1.—Divide $819\frac{2}{3}$ by 9.

Divide the integral part:	$819 \div 9 = 91$
Divide the fractional part:	$\frac{2}{3} \div 9 = \frac{2}{27}$
Combine the quotients:	$91\frac{2}{27}$ *Ans.*

Ex. 2.—Reduce $\frac{84673\frac{2}{5}}{8}$ to a simple fraction.

The numerator of the complex fraction is the dividend, the denominator the divisor. Divide 84673, the integral part of the dividend, by 8. 1 remains, which prefixed to the fraction makes $1\frac{2}{5}$, or $\frac{7}{5}$. Dividing $\frac{7}{5}$ by 8, we have $\frac{7}{40}$. Combining the quotients, we get $10584\frac{7}{40}$ *Ans.*

$$8\underline{)\,84673}$$
$$10584,\ 1 \text{ rem.}$$

$$1\tfrac{2}{5} = \tfrac{7}{5} \qquad \tfrac{7}{5} \div 8 = \tfrac{7}{40}$$

Ans. $10584\frac{7}{40}$

RULE.—1. *Divide the integral and the fractional part separately, and combine the quotients.*

What is the effect of dividing the numerator? Of multiplying the denominator? Solve the examples.—168. What is the second case of division of fractions? Explain the given examples. Recite the rule for dividing a mixed by a whole number.

2. *If, on dividing the integral part, there is a remainder, prefix it to the fractional part, reduce to an improper fraction, divide as in Case I., and combine this quotient with that obtained by dividing the integral part.*

EXAMPLES FOR PRACTICE.

1. Divide $8\frac{4}{7}$ by 20. *Ans.* $\frac{3}{7}$.
2. Divide $3\frac{9}{11}$ by 8. *Ans.* $\frac{21}{44}$.
3. Divide $6\frac{5}{8}$ by 9. *Ans.* $\frac{53}{72}$.
4. Divide $9\frac{3}{11}$ by 3. *Ans.* $3\frac{1}{11}$.
5. Divide $8\frac{7}{29}$ by 2. *Ans.* $4\frac{7}{58}$.
6. Divide $9\frac{3}{7}$ by 7. *Ans.* $1\frac{17}{49}$.
7. Reduce $\frac{87\frac{5}{6}}{5}$. *Ans.* $17\frac{17}{30}$.
8. Reduce $\frac{73\frac{2}{19}}{3}$. *Ans.* $24\frac{7}{19}$.
9. $5940\frac{3}{15} \div 12$. *Ans.* $495\frac{1}{60}$.
10. $8991\frac{17}{63} \div 25$. *Ans.* $359\frac{41}{63}$.
11. $9509\frac{15}{17} \div 52$. *Ans.* $182\frac{15}{17}$.
12. $1001\frac{3}{4} \div 10$. *Ans.* $100\frac{7}{40}$.
13. $9107\frac{1}{5} \div 64$. *Ans.* $142\frac{3}{10}$.
14. $7841\frac{14}{15} \div 38$. *Ans.* $206\frac{11}{30}$.
15. Reduce $\frac{77\frac{11}{17}}{40}$. *Ans.* $1\frac{16}{17}$.
16. Reduce $\frac{84\frac{6}{25}}{39}$. *Ans.* $2\frac{4}{25}$.
17. Reduce $\frac{4\frac{1}{4}}{6}$; $\frac{25\frac{3}{8}}{19}$; $\frac{10\frac{3}{7}}{36}$; $\frac{144\frac{8}{11}}{24}$; $\frac{288\frac{2}{13}}{36}$; $\frac{451\frac{3}{13}}{74}$; $\frac{2671\frac{3}{50}}{33}$.

169. CASE III.—*To divide a fraction, whole, or mixed number, by a fraction or mixed number.*

Ex. 1.—How many times is $\frac{2}{7}$ contained in $\frac{3}{5}$?

$\frac{1}{7}$ is contained in 1, 7 times. In $\frac{3}{5}$ it is contained $\frac{3}{5}$ of 7 times, or $\frac{21}{5}$ times.

$$\frac{3}{5} \times 7 = \frac{21}{5}$$

But $\frac{2}{7}$ is twice as great as $\frac{1}{7}$, and hence is contained only half as many times. $\frac{1}{2}$ of $\frac{21}{5} = \frac{21}{10} = 2\frac{1}{10}$ *Ans.*

$$\frac{21}{5 \times 2} = \frac{21}{10}$$

Now, what have we done to the dividend $\frac{3}{5}$, to produce the quotient $\frac{21}{10}$? We have multiplied it by the divisor inverted. Hence the rule:—

$$\frac{3}{5} \times \frac{7}{2} = \frac{21}{10}$$

RULE.—1. *Multiply the dividend by the divisor inverted.*

2. *Whole and mixed numbers must first be reduced to improper fractions.*

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169. What is the third case of division of fractions? How many times is $\frac{2}{7}$ contained in $\frac{3}{5}$? What have we done to the dividend, to produce the quotient? Recite the rule for dividing one fraction by another

5

EXAMPLE 2.—Reduce $\frac{3\frac{1}{4}}{2\frac{4}{11}}$ to its simplest form.

Reduce the numerator to an improper fraction: $3\frac{1}{4} = \frac{13}{4}$

Reduce the denominator to an improper fraction: $2\frac{4}{11} = \frac{26}{11}$

Multiply the dividend by the divisor inverted, cancelling common factors. Reduce the result to a mixed number.

$$\frac{\cancel{13}}{4} \times \frac{11}{\underset{2}{\cancel{26}}} = \frac{11}{8} = 1\tfrac{3}{8} \textit{ Ans.}$$

EXAMPLE 3.—Divide $\frac{4}{7}$ by $\frac{2}{7}$.

The denominators, being the same, are cancelled when the divisor is inverted, and we have only to divide 4, the numerator of the dividend, by 2, the numerator of the divisor. Hence, *When the fractions have a common denominator, reject it, and divide the numerator of the dividend by that of the divisor.*

$$\frac{4}{7} \div \frac{2}{7} = \frac{4}{7} \times \frac{7}{2}$$

$$\frac{4}{\cancel{7}} \times \frac{\cancel{7}}{2} = 2 \textit{ Ans.}$$

EXAMPLES FOR PRACTICE.

Find the value of the following:—

1. $\frac{1}{2} \div \frac{1}{6}$. *Ans.* 3.
2. $\frac{3}{4} \div \frac{1}{3}$. *Ans.* $2\frac{1}{4}$.
3. $\frac{7}{7} \div \frac{7}{11}$. *Ans.* $1\frac{4}{7}$.
4. Divide $\frac{7}{15}$ by $\frac{12}{20}$.
5. Divide $\frac{46}{97}$ by $\frac{23}{97}$.
6. Divide $\frac{88}{89}$ by $\frac{11}{89}$.
7. $\frac{133}{15} \div \frac{7}{15}$.
8. $\frac{18}{19} \div \frac{36}{57}$.
9. $\frac{117}{120} \div \frac{9}{10}$.

Reduce the following to their simplest forms:—

10. Reduce $\frac{2\frac{1}{2}}{3\frac{1}{2}}$. *Ans.* $\frac{5}{7}$.
11. Reduce $\frac{8}{4\frac{1}{3}}$. *Ans.* $1\frac{11}{13}$.
12. Reduce $\frac{6\frac{1}{9}}{\frac{1}{5}}$. *Ans.* $30\frac{5}{9}$.
13. Reduce $\frac{6\frac{12}{28}}{19\frac{2}{7}}$. *Ans.* $\frac{1}{3}$.
14. Reduce $\frac{\frac{1}{2} \text{ of } \frac{2}{3}}{6\frac{1}{3}}$. *Ans.* $\frac{1}{19}$.
15. Reduce $\frac{7\frac{1}{8}}{\frac{3}{4} \text{ of } \frac{9}{10}}$. *Ans.* $10\frac{5}{9}$.
16. Reduce $\frac{\frac{3}{7}+\frac{1}{14}}{4\frac{1}{2}}$. *Ans.* $\frac{1}{9}$.
17. Reduce $\frac{\frac{5}{18}-\frac{1}{24}}{1\frac{15}{36}}$. *Ans.* $\frac{1}{6}$.

18. How many times can a pitcher holding $1\frac{7}{8}$ quarts be filled from a pail containing $5\frac{5}{8}$ quarts?

Solve and explain the given examples. When may cancellation be brought to bear? When the fractions have a common denominator, what is the best mode of proceeding?

19. What is the rate per hour of a boat that goes $230\frac{13}{20}$ miles in $18\frac{3}{8}$ hours? *Ans.* $12\frac{5}{13}$ miles.

20. If $37\frac{5}{8}$ yards of calico are used in cutting three dresses of equal size, how many yards are there in each dress?

21. Five and a half yards make a rod. How many rods in $83\frac{3}{8}$ yards?

22. If a man makes $\$1\frac{13}{20}$ on every table he sells, how many tables must he sell to make $\$27\frac{3}{4}$?

MISCELLANEOUS QUESTIONS ON FRACTIONS.—What does the word *fraction* come from? *Ans.* From the Latin word *fractus, broken,* because a fraction indicates the *breaking* up or dividing of a unit into equal parts. What is meant by the terms of a fraction? With what do they correspond in division? What is the difference between a proper and an improper fraction? Which is the greater? Which is greater, a proper fraction or a mixed number? Which is greater, $\frac{1}{5}$ or $\frac{1}{6}$? When we increase the denominator of a fraction, do we increase or diminish its value? Which is greater, $\frac{1}{5}$ or $\frac{2}{5}$? When we increase the numerator of a fraction, do we increase or diminish its value? Which is greater, $\frac{4}{4}$ or $\frac{7}{7}$? What kind of fractions are these?

What is meant by reducing a fraction? Mention all the cases of reduction of fractions that you can remember. How do you reduce a fraction to its lowest terms? How do you reduce an improper fraction to a whole or mixed number? How do you reduce a mixed number to an improper fraction? How do you reduce a compound fraction to its simplest form? How do you reduce a complex fraction to its simplest form?

When we take $\frac{1}{3}$ of a number, do we multiply or divide by $\frac{1}{3}$? What is dividing by $\frac{1}{3}$ equivalent to? Dividing by 3 is equivalent to multiplying by what? Multiplying by 3 is equivalent to dividing by what? Does multiplying a number by a proper fraction increase or diminish it?

How may addition of fractions be proved? *Ans.* By subtracting one of the given fractions from the sum obtained, and seeing whether the remainder equals the sum of the remaining fractions. How may subtraction of fractions be proved? *Ans.* By adding subtrahend and remainder, and seeing whether their sum equals the minuend. How may multiplication of fractions be proved? *Ans.* By dividing the product by the multiplier, and seeing whether the quotient equals the multiplicand. How may division of fractions be proved? *Ans.* By multiplying divisor and quotient, and seeing whether their product equals the dividend.

170. Miscellaneous Examples.

1. Find the sum, then the difference, and then the product of $8\frac{5}{8}$ and $1\frac{7}{24}$. Divide $8\frac{5}{8}$ by $1\frac{7}{24}$.

2. From a piece of cloth, $\frac{1}{3}$ and $\frac{3}{5}$ of itself were cut off. What part remained? *Ans.* $\frac{1}{15}$.

3. One third of a piece of cloth was cut off, and then $\frac{3}{5}$ of what remained. How much of it was left? *Ans.* $\frac{4}{15}$.

4. A and B together have 1477 sheep, of which A owns $\frac{4}{7}$, and B $\frac{3}{7}$. How many belong to each?

5. A person owning $\frac{8}{9}$ of a farm gives $\frac{1}{3}$ of his share to his sister, who divides it equally among her four sons. What part of the whole does each son receive? *Ans.* $\frac{2}{27}$.

6. A owns $\frac{8}{11}$ of a ship worth \$15422; he sells B $\frac{3}{8}$ of his share. What part of the whole does A then have? What part has B? What is the value of A's part? Of B's part?

7. 88 is $\frac{11}{12}$ of what number?

88 is $\frac{11}{12}$; then 1 twelfth is $\frac{1}{11}$ of 88, or 8; and 12 twelfths, or the whole number, is 12 times 8, or 96. *Ans.* 96.—We divide 88 by the numerator, and multiply by the denominator.

8. 1200 is $\frac{200}{443}$ of what number? *Ans.* 2658.

9. 1552 is $\frac{97}{1003}$ of what number? 76 is $\frac{19}{98}$ of what number?

10. 14042 is $\frac{14}{15}$ of what number? 85 is $\frac{17}{78}$ of what number?

11. $\frac{9}{10}$ of 1200 is $\frac{2}{3}$ of what number? *Ans.* 1620.

Find how much $\frac{9}{10}$ of 1200 is; then proceed as in Example 7.

12. $\frac{4}{21}$ of 1743 is $\frac{166}{209}$ of what number? *Ans.* 418.

13. $\frac{5}{6}$ of 126 is $\frac{21}{40}$ of what number?

14. $\frac{12}{25}$ of 3000 is $\frac{4}{11}$ of what number?

15. A farmer kept his sheep in two pastures; half of his flock was in one, and 87 sheep in the other. How many sheep had he?

16. A sum of money is divided between A and B. A gets $\frac{1}{4}$ of it, and B gets \$360. How many dollars does A receive?

If A gets $\frac{1}{4}$, how many fourths does B get? If \$360 is *three* fourths, what will *one* fourth be?

17. A sum of money is divided among A, B, and C. A gets $\frac{1}{3}$, B $\frac{1}{4}$, and C \$70. What was the amount divided? *Ans.* \$168.

How much do A and B together receive? What fraction is left for C? If \$70 equals this fraction, what will the whole be?

18. How many sheep has a farmer, who keeps $\frac{1}{5}$ of his flock in one field, $\frac{1}{6}$ in a second, and the rest, numbering 779, in a third? *Ans.* 1230 sheep.

19. A merchant paid \$272 for flour, at \$$9\frac{5}{7}$ a barrel. How many barrels did he buy?

20. What fraction is 335 of 603? *Ans.* $\frac{335}{603} = \frac{5}{9}$.

21. Paul has read 96 pages in a volume that contains 144 pages. What fraction of the book has he read through?

22. A certain school is composed of 67 boys and 59 girls. What fraction of the whole do the boys form, and what the girls?

23. Having a large cake, I divide half of it into five equal parts, and give three of these parts away. What portion of the whole cake have I left? *Ans.* $\frac{7}{10}$.

24. A lady divides \$300 among her three sons, giving the first \$75, the second \$125, and the third the rest. What fraction of the whole does each receive?

25. What number must be added to $4\frac{4}{7}$ to make $6\frac{2}{3}$?

26. What number taken from $8\frac{1}{3}$ leaves $3\frac{3}{5}$? *Ans.* $4\frac{11}{15}$.

27. The sum of two numbers is $47\frac{13}{20}$; the less is $14\frac{3}{5}$. What is the greater?

28. Given, the quotient $2\frac{17}{60}$, the divisor $\frac{18}{274}$; required the dividend. *Ans.* $\frac{3}{20}$.

29. Subtrahend, $5\frac{1}{3}$; remainder, $2\frac{11}{15}$; what is the minuend?

30. The product of two factors is $132\frac{96}{101}$; one of the factors is 12, what is the other? *Ans.* $11\frac{8}{101}$.

31. What is the difference between $\frac{2}{21} + \frac{3}{35}$ and $\frac{4}{7}$ of $\frac{28}{135}$?

32. A bank paid out half of its money, then half of what remained, and again half of what then remained. What fraction of the whole was left? *Ans.* $\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{8}$.

33. A bank paid out $\frac{1}{3}$ of its money, then $\frac{1}{3}$ of what remained, and again $\frac{1}{3}$ of what then remained. What part of its money was left? *Ans.* $\frac{8}{27}$.

34. C walks $3\frac{1}{4}$ miles an hour; D, $4\frac{1}{4}$. How many hours will it take C to walk 15 miles, and how many D? If they walk towards each other from two points 15 miles apart, how long before they will meet? *Last ans.* 2 hours.

35. What number added to $\frac{5}{16}+\frac{3}{40}+\frac{11}{24}$ will make 3?

36. What number must be multiplied into $\frac{5}{6}$ of $\frac{21}{25}$ of $1\frac{3}{7}$, to produce $3\frac{1}{4}$? *Ans.* $3\frac{1}{4}$.

37. F can mow a field in 8 hours, and G in 9 hours. What part can F do in one hour, and what part G? What part can both do in one hour? *Ans.* F, $\frac{1}{8}$; G, $\frac{1}{9}$; both, $\frac{17}{72}$.

38. P can dig a trench in 12 hours, Q in 15 hours. What part can each do in one hour, and what part both?

39. A and B can mow a field in 14 hours; B alone can mow it in 24 hours; how long will it take A to do it? *Ans.* $33\frac{3}{5}$ hours.

How much can A and B together do in 1 hour? How much can B alone do in 1 hour? How much is left for A to do in 1 hour? If he does this fraction in 1 hour, how long will it take him to do the whole?

40. If A can dig a cellar in 20 hours, and B in 24, how long will it take both, working together, to do it? *Ans.* $10\frac{10}{11}$ hours.

41. Reduce $\frac{5}{12}$ to ninety-sixths (§ 146). *Ans.* $\frac{40}{96}$.

42. How many forty-fourths in 7? In $3\frac{3}{22}$?

43. What is the smallest fraction which added to the sum of $\frac{29}{31}$ and $\frac{1}{3}$ will make the result a whole number?

44. A family consume $1\frac{3}{4}$ tons of coal in the parlor, $2\frac{5}{6}$ tons in the kitchen, and $\frac{1}{2}$ of a ton in each of their five bed-rooms. How much do they use in all? *Ans.* $7\frac{1}{12}$ tons.

45. Four men agreed to share their earnings for one month equally. The first earned $\$60\frac{1}{2}$; the second, $\$40\frac{5}{8}$; the third, $\$50\frac{3}{10}$; the fourth, $\$72\frac{3}{20}$. What did each receive?

46. Will you increase or diminish the fraction $\frac{3}{10}$, if you add 4 to each of its terms, and how much? *Ans.* Inc. $\frac{1}{5}$.

47. Will you increase or diminish the fraction $\frac{11}{12}$, if you subtract 2 from each term, and how much? *Ans.* Dim. $\frac{1}{60}$.

48. Sold a house and lot for $\$4250\frac{1}{2}$. The house cost $\$3759\frac{7}{8}$; the lot, $\$846\frac{3}{4}$. How much was gained or lost? *Ans.* $\$356\frac{1}{8}$ lost.

49. A man who has a journey of $78\frac{5}{16}$ miles to make, goes $15\frac{19}{20}$ miles the first day, and $23\frac{1}{15}$ miles the next. How far has he then to go? *Ans.* $39\frac{71}{240}$ miles.

50. The difference between two fractions is $\frac{7}{15}$; if the smaller fraction is $\frac{3}{50}$, what is the greater?

CHAPTER XI.

DECIMAL FRACTIONS.

171. A **Decimal Fraction** is one whose denominator is 10, or 10 multiplied into itself one or more times. Its numerator only is written, with a dot (.) called the *decimal point* or *separa'trix* before it. Thus:—

$\frac{9}{10}$ is written .9	$\frac{889}{1000}$ is written .889
$\frac{11}{100}$ is written .11	$\frac{7777}{10000}$ is written .7777

Decimal Fractions are briefly called Decimals. The term comes from the Latin word *decem*, *ten*.

172. Decimals arise from successive divisions by *ten*. If a unit is divided into *ten* equal parts, each part is called *one tenth*. If one of these tenths is subdivided into *ten* equal parts, each of these subdivisions is *one hundredth* ($\frac{1}{10}$ of $\frac{1}{10} = \frac{1}{100}$). So, from further divisions by 10, we get *thousandths*, *ten-thousandths*, *hundred-thousandths*, *millionths*, &c.

When tenths, hundredths, thousandths, &c., are expressed with both numerator and denominator, they are common fractions; when with the numerator alone, preceded by a dot, they are decimals. As common fractions, they may be added, &c., according to the rules already given; but they are operated on much more easily as decimals.

Notation of Decimals.

173. In writing integers, we found that the value of each figure depends on the place it occupies, being *ten times* as great as if it stood one place further to the *right*, and *one tenth* of what it would be in the next place to the left. Continuing this notation on the right of the

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171. What is a Decimal Fraction, and how is it written? Give examples. What are decimal fractions briefly called? What does the term *decimal* come from?—172. Show how decimals arise. When are tenths, hundredths, thousandths, &c., common fractions, and when decimals? In which form are they most easily operated on?—173. In writing integers, what did we find with respect to the value of each figure? Continuing this notation on the right of the units' place, what do we obtain?

units' place, we obtain decimal orders, each of which, as in the case of integers, invests its figure with a value ten times as great as the next order on its right.

Decimals are therefore expressed according to the same system as integers. Hence they may be written beside integers (with the *separatrix* to *separate* them), and may be added, subtracted, multiplied, and divided, in the same way as integers.

In the expression 2222.2222, each 2, whether integral or decimal, has a value ten times as great as the 2 next on its right. The integral twos represent collections of units; the decimal twos, parts of a unit.

174. Table.—The names of the places on the right of the units' place resemble those on the left. They may be learned from the following Table:—

&c., &c.	Ten-billions.	Billions.	Hundred-millions.	Ten-millions.	Millions.	Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.	. *Decimal point.*	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.	Ten-millionths.	Hundred-millionths.	Billionths.	Ten-billionths.	&c., &c.
ORDERS OF INTEGERS.													ORDERS OF DECIMALS.										

Observe that in going from *tens* to *hundreds*, *thousands*, &c., we pass to *higher* orders; but in going from *tenths* to *hundredths*, *thousandths*, &c., we pass to *lower* orders.

Observe that *two* figures are required to express *tens* (10), *one* to express *tenths* (.1); *three* for *hundreds*, *two* for *hundredths*; and generally, one less figure for a decimal order than for an order of integers of similar name.

175. General Principles.—As in whole numbers, so in decimals, we give a figure a certain value by writing it in a certain place. Thus we express

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Where, then, may decimals be written, and how may they be added, &c.?—174. What resemblance may be noticed in the names of the decimal orders? Name the orders of decimals, going to the right from the decimal point. Name the orders of integers, going to the left. In which case do we pass to higher orders, and in which to lower? How many figures are required, to express tens? To express tenths? To express hundreds? To express hundredths? What general principle is deduced from this?—175. How do we give a decimal figure a certain value? Give examples.

$\frac{9}{10}$ by writing 9 in the place of *tenths* .9;
$\frac{9}{100}$ by writing 9 in the place of *hundredths* .09;
$\frac{9}{1000}$ by writing 9 in the place of *thousandths* .009, &c.

176. From the above examples we see that *vacant decimal places on the left must be filled with naughts.* By leaving out the naughts in *nine hundredths* and *nine thousandths*, as written above, we would change them to *nine tenths.*

177. We also see that *to express a decimal we must use as many figures as there are naughts in its denominator.* There is *one* naught in 10; *one* decimal figure expresses tenths. There are *two* naughts in 100; *two* decimal figures express hundredths, &c.

178. It follows that *every decimal has for its denominator* 1 *with as many naughts as there are figures in the numerator.*

179. A naught prefixed to a whole number does not change its value; every naught annexed multiplies it by 10. With decimals it is not so.

A naught prefixed to a decimal (on the right of the separatrix) *throws its figures one place to the right, and thus divides it by* 10: .3 is ten times as great as .03.

A naught annexed to a decimal does not change its value, because denominator as well as numerator is multiplied by 10. .3 = .30 ($\frac{3}{10} = \frac{30}{100}$).

180. RULE.—*To express a decimal in figures, write its numerator as a whole number. If it contains fewer figures than the denominator contains naughts, prefix naughts to supply the deficiency. Finally, prefix the decimal point.*

EXAMPLE.—Write forty-two millionths as a decimal.

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176. How must vacant decimal places on the left be filled?—177. How many figures must we use, to express a decimal?—178. What does every decimal have for its denominator?—179. What is the effect of prefixing a naught to a whole number? To a decimal? What is the effect of annexing a naught to a whole number? To a decimal?—180. Recite the rule for expressing a decimal in figures. Give examples.

Write the numerator as a whole number, 42. The denominator contains six naughts; hence, as the numerator contains but two figures, we must prefix to it four naughts. *Ans.* .000042.

So, four and 357 millionths,	4.000357
Ten and nineteen ten-thousandths,	10.0019
Twenty and eighty-nine billionths,	20.000000089

EXERCISE.

1. How many figures are required, to express thousandths (§177)? To express millionths? Billionths? Hundredths? Ten-thousandths? Hundred-trillionths? Ten-millionths?

2. Give the denominators of the following decimals:— .001; .00001; .19; 4.1; .000003; 15.62; .3333; 5.162.

3. Write the following as decimals, letting the decimal points range in line:—37 thousandths; 8 hundredths; 48 millionths; 95 hundred-millionths; 490 hundred-thousandths; 1240 ten-millionths; 10000004 hundred-millionths; 96 billionths; 9301 hundred-millionths; 2711 trillionths.

4. Eight hundred and forty-one thousand ten-millionths.

5. Eighty-thousand,* four hundred and two millionths.

6. Seventy-one million three thousand and four billionths.

7. Eight hundred and ninety-six thousand hundred-millionths.

8. Forty-nine thousand,* and seven hundred-thousandths.

9. Sixty billion and fourteen thousand trillionths.

10. Eight hundred million and ninety-nine ten-billionths.

11. Seventeen thousand and forty-one ten-trillionths.

12. $\frac{487}{100000}$; $\frac{7}{10}$; $\frac{81}{1000}$; $\frac{3241}{10000000}$; $\frac{9}{100}$; $\frac{6857}{1000000}$.

Numeration of Decimals.

181. RULE.—*Read the numerator first as a whole number, then name the denominator, as in common fractions.*

.09 is read	Nine hundredths.
.090018	Ninety-thousand and eighteen millionths.
70.000000401	Seventy, and four hundred and one billionths.

181. Recite the rule for reading decimals.

* The comma is here used to show that what precedes it is whole number.

EXERCISE.—Read the following:—

.8	.010010101	46.0017	1.2
.90	.900909	3.123456789	.463
.407	.0043200203	20.0020001	2.0303
.6945	.00076	.1110111111	5.231231123
.12003	.00002007	19.20200202	0.047
.005007	.0509	5.0000004	.0004000045
.36709	.4000059	99.199099199	8.0019019

182. Addition of Decimals.

Ex.—Add 9.0421, .42386, 881, .03, and 23.5945.

That we may unite things of the same kind, we set the numbers down with the decimal points ranging in line, which brings figures of the same order in the same column. Add as in whole numbers, and place a decimal point in the result under the points in the numbers added.

```
      9.0421
       .42386
    881.
       .03
     23.5945
Ans. 914.09046
```

RULE.—1. *Write the numbers with their decimal points ranging in line. Add as in whole numbers. Place the decimal point in the result under the points in the numbers added.*

2. *Prove by adding from the top downward.*

EXAMPLES FOR PRACTICE.

1. Add .123, .11496, 4.01, .06784, and 9.0342. *Ans.* 13.35.

2. What is the value of .4397+219.31+3.00067+.043851 +5675.159+99.0004759+.15006342? *Ans.* 5997.10376032.

3. 8.8+450.329+.988927+87.71+.9+.272073.

4. .999+999+9.887706+.07809+88.199+.4.

5. 7.71+.853+9.6+96+.96+.096+960+.54.

6. 105.501+0.105+8.648301+.19+.776655432+.3.

7. Find the sum of 2063 millionths; 3064 ten-thousandths; 99 hundredths; 500, and 6009 hundred-thousandths; seven, and 12 millionths; and 863003 billionths. *Ans.* 508.359428003.

---

182. Set down the given example in addition of decimals, and perform the operation. Give the rule.

8. Add five thousandths; nineteen, and eighteen millionths; five hundred and twenty hundred-thousandths; forty, and seven tenths; 87 hundredths; 919 ten-thousandths. *Ans.* 60.672118.

9. Required the sum of nineteen tenths; four hundred, and two hundredths; ninety-three thousandths; one hundred-thousandth.

10. Add, as decimals, $8\frac{3}{100}$; $10\frac{3}{1000}$; $\frac{427}{1000000}$; $\frac{5617}{10000}$; $2\frac{53}{10000000}$.

183. Subtraction of Decimals.

EXAMPLE.—From 4.19 subtract .000001.

Set the subtrahend under the minuend, with the decimal points in line. Write naughts in the vacant places of the minuend (or supply them mentally), and subtract as in whole numbers. Place a decimal point in the remainder under the other points.

	4.190000
	.000001
Ans.	4.189999

RULE.—1. *Write the subtrahend under the minuend, with their decimal points ranging in line. Subtract as in whole numbers. Place the decimal point in the remainder under the other decimal points.*

2. *Prove by adding subtrahend and remainder.*

EXAMPLES FOR PRACTICE.

	(1)	(2)	(3)	(4)
From	11.	8.00042	2.31400	23.56
Take	.897	.875	.401006	1.0941875

5. Subtract 47.99999 from 831.012. *Ans.* 783.01201.

6. From .8754321 take .0006. *Ans.* .8748321.

7. From 9.3 take the sum of .47 and 2.961. *Ans.* 5.869.

8. Subtrahend, .88637; minuend, 312.42; required, the remainder.

9. From one thousand take five thousandths. *Ans.* 999.995.

10. Take 11 hundred-thousandths from 117 thousandths.

11. From three million and one millionth, subtract one tenth.

12. Find the value of $2.4 + .009 + .73 - 1.8$.

13. From eight and three tenths take eighty-four hundredths.

---

183. Set down the given example in subtraction of decimals. Perform the operation. Give the rule.

14. From 83.1 subtract .8176; from the remainder take $\frac{33}{1000}$.

15. Find the difference between $\frac{822}{1000}$ and $\frac{487}{1000000}$, first as common fractions, then as decimals. Do the results agree?

184. Multiplication of Decimals.

EXAMPLE 1.—Multiply .324 by .03.

Write the given decimals as common fractions, and multiply. $\frac{324}{1000} \times \frac{3}{100} = \frac{972}{100000}$, which expressed decimally is .00972. The same result is obtained by multiplying the given decimals together, and prefixing two naughts and the decimal point to the product.

$$\begin{array}{r} .324 \\ \underline{.03} \\ \textit{Ans.}\ .00972 \end{array}$$

Why prefix two naughts?—The multiplicand containing 3 figures, its denominator contains 3 naughts (§ 178). The multiplier containing 2 figures, its denominator contains 2 naughts. Hence the product of their denominators contains 3 + 2 naughts; and the product of their numerators must contain 3 + 2 figures (§ 177). As it has but three figures, we prefix two naughts.—The product of two decimals must therefore contain as many decimal places as both factors contain.

RULE.—1. *Multiply as in whole numbers. From the right of the product point off as many figures for decimals as there are decimal places in both factors. If there are not so many, prefix naughts to supply the deficiency.*

2. *Prove by multiplying multiplier by multiplicand.*

EXAMPLE 2.—Multiply 3.8 by .97.

Multiply as in whole numbers. There being 1 decimal place in the multiplicand, and 2 in the multiplier, point off 1 + 2, or 3, figures from the right of the product.

$$\begin{array}{rr} 3.8 & \qquad \textit{Proof}\ .97 \\ \underline{.97} & \underline{3.8} \\ 266 & 776 \\ \underline{342} & \underline{291} \\ \textit{Ans.}\ 3.686 & 3.686 \end{array}$$

185. To multiply a decimal by 10, 100, 1000, &c., *remove the decimal point as many places to the right as there are naughts in the multiplier.* If there are not figures enough for this, annex naughts to supply the deficiency.

$$\begin{aligned} .015 \times 10 &= .15 \\ .015 \times 1000 &= 15 \\ .015 \times 10000 &= 150 \end{aligned}$$

---

184. Multiply .324 by .03 in the two ways shown above. Why do we prefix two naughts to the decimal product? Recite the rule for the multiplication of decimals. —185. How may we multiply a decimal by 10, 100, 1000, &c.? Give examples.

EXAMPLES FOR PRACTICE.

	(1)	(2)	(3)	(4)
Multiply	81.009	.008765	7.91365	3256.9
By	4.067	.0495	8.401	4.0008

5. Multiply together 6.321, .987, and 1000. *Ans.* 6238.827.
6. Multiply .4639721 by .00832. *Ans.* .003860247872.
7. Multiply 5.432 by 21; by .21; by 9.8; by .00008.
8. Multiply by 100 the following: .1; .003; .00007; 1.14.
9. Find the product of one billionth and one billion.
10. Multiply 73 thousandths by 19 hundredths.
11. Multiply ninety-seven millionths by ten thousand.
12. Multiply .00468 by 3.0009. Multiply 14.7 by .0908006.
13. Find the product of $\frac{7}{100}$, $\frac{19}{100}$, and $\frac{3}{1000}$, first as common fractions, then as decimals. Do the results agree?
14. Multiply the sum of nineteen hundredths and eighteen thousandths by seventeen ten-thousandths. *Ans.* .0003536.
15. Multiply the difference between two ten-thousandths and two hundred-thousandths by nine tenths. *Ans.* .000162.

Division of Decimals.

186. Division is the converse of multiplication. The dividend corresponds with the product, the divisor and quotient with the factors.

Now, in multiplication of decimals, we found that the product contains as many decimal places as both factors together. Hence, in division of decimals, the dividend must contain as many decimal places as divisor and quotient together; and the quotient, as many as the decimal places in the dividend exceed those in the divisor.

187. Rule.—1. *Divide as in whole numbers. Point off from the right of the quotient as many figures as the decimal places in the dividend exceed those in the divisor.*

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186. In division of decimals, how many decimal places must the dividend contain? How many must the quotient contain? How does this follow from the mode of pointing in multiplication of decimals?—187. Give the rule for division of decimals.

If there are not so many, prefix naughts to supply the deficiency.

2. *Prove by multiplying divisor by quotient.*

EXAMPLE 1.—Divide 84.0065 by .05.

Divide as in whole numbers. There being 4 decimal places in the dividend, and 2 in the divisor, point off 4−2, that is 2, figures from the right of the quotient.

```
.05) 84.0065
Ans. 1680.13
```

188. Annexing naughts to a decimal does not change its value. Hence, *when the dividend contains fewer decimal places than the divisor, annex naughts to it till its decimal places equal those of the divisor; then divide, and the quotient will be a whole number.*

EXAMPLE 2.—Divide 7240.5 by .0009.

Annex 3 naughts to the dividend, to make its decimal places equal those of the divisor. The quotient is a whole number.

```
.0009) 7240.5000
  Ans. 8045000
```

189. When there is a remainder, after using all the figures of the dividend, naughts may be annexed to the dividend and the division continued. *In pointing off the quotient, these naughts must be counted as decimal figures of the dividend.* The sign + is annexed to a quotient, to show that the division does not terminate.

EXAMPLE 3.—Divide .075 by 4.3.

```
        00
4.3).075(174
     43
     320
     301
      190
      172
       18
Ans. .0174+
```

After using all the figures of the dividend, we annex naughts (placed above it), to continue the division, which may thus be carried out as far as desired. Using 2 naughts, we have 5 decimal places in the dividend, and 1 in the divisor. We must therefore point off 5−1, or 4, figures from the right of the quotient, which requires us to prefix to it a naught. *Ans.* .0174+

190. To divide a decimal by 10, 100, 1000, &c., *remove the decimal point as*

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188. When the divisor contains more decimal places than the dividend, how must we proceed? Divide 7240.5 by .0009.—189. When there is a remainder after using all the figures of the dividend, what may be done? In pointing off, how must we consider these annexed naughts? What does the sign + annexed to a quotient show? Apply this rule in Ex. 3.—190. How may we divide a decimal by 10, 100, 1000, &c.? Give examples.

many places to the left as there are naughts in the divisor. If there are not figures enough for this, prefix naughts to supply the deficiency.

$$
\begin{aligned}
156.3 \div 10 &= 15.63 \\
156.3 \div 1000 &= .1563 \\
156.3 \div 100000 &= .001563
\end{aligned}
$$

EXAMPLES FOR PRACTICE.

Find the value of the following; prove each example:—

1. $.144 \div .36$ *Ans.* .4
2. $.49 \div 700$ *Ans.* .0007
3. $132 \div .11$ *Ans.* 1200.
4. $.8 \div 7.3$ *Ans.* .109589+
5. Divide .75 by .7500.
6. Divide 10000 by .01.
7. Divide .24 by 60.
8. Divide 8437 by 1.8.
9. Divide 1210 by .11.
10. Divide .00001 by 1001.
11. Divide .00063 by 9.
12. Divide .5 by 50000.
13. Divide .491 by .00007.
14. Divide 810 by .000009.
15. Divide .0001 by .001.
16. Divide 2880 by .0036.
17. Divide 19 by 42.96.
18. Divide .120 by 100000.
19. Divide 64000 by .0016.
20. Divide $\frac{12}{1000}$ by $\frac{67}{1000000}$.
21. Divide 639.521 by 10000; by 100; by 10000000; by 10.
22. Divide one million by one ten-thousandth.
23. Divide the sum of 941 thousandths and 38 hundredths by one thousand. *Ans.* .001321
24. Divide the difference between eight tenths and one millionth by seventy-nine hundredths. *Ans.* 1.0126569+
25. Divide the product of one hundredth and one thousandth by one ten-billionth.
26. Divide 7 tenths by 3.5, and the quotient by 20000.
27. Divide $\frac{1682}{1000000}$ by $\frac{47}{10000}$, first as common fractions, then as decimals. Do the results agree?
28. Divide the sum of five thousand and two thousandths by two hundredths. *Ans.* 250000.1
29. Divide the difference between 200 and 2 hundredths by 9 hundredths. *Ans.* 2222.
30. Divide by 100 the following: 26.83; .2683; 268.3; .02683.

Reduction of Decimals.

191. CASE I.—*To reduce a decimal to a common fraction.*

RULE.— *Write the given decimal, with its point omitted, over its denominator, and reduce this common fraction to its lowest terms.*

EXAMPLE.—Reduce .125 to a common fraction.

$$.125 = \frac{125}{1000} = \frac{1}{8} \textit{ Ans.}$$

EXAMPLES FOR PRACTICE.

Reduce the following to common fractions:—

1. .375; .5; .15; .25; .88.
2. .225; .435; .575; .656.
3. .00375 *Ans.* $\frac{3}{800}$.
4. .000225 *Ans.* $\frac{9}{40000}$.
5. .36984 *Ans.* $\frac{4623}{12500}$.
6. .0982 *Ans.* $\frac{491}{5000}$.
7. .46; .046; .0046; .11.
8. .076; .075; .0075 .13.
9. .00764 *Ans.* $\frac{191}{25000}$.
10. .00025 *Ans.* $\frac{1}{4000}$.
11. .000155 *Ans.* $\frac{31}{200000}$.
12. .01250505 *Ans.* $\frac{250101}{20000000}$.

192. CASE II.—*To reduce a common fraction to a decimal.*

EXAMPLE.—Reduce $\frac{1}{8}$ to a decimal.

$$8\overline{)1.000}$$
$$\textit{Ans. } .125$$

$\frac{1}{8}$ is 1 divided by 8. To perform the division, annex decimal naughts to the dividend 1, and divide by 8. Point off three figures from the right of the quotient, because there are three decimal places in the dividend and none in the divisor. *Ans.* .125.

To prove the result, reduce .125 back to a common fraction (§ 191), and see whether it produces $\frac{1}{8}$.

RULE.—1. *Annex decimal naughts to the numerator, and divide by the denominator. Point off the quotient as in division of decimals.*

2. *Prove by reducing the decimal obtained back to a common fraction.*

Compound fractions must first be reduced to simple ones.

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191. Recite the rule for reducing a decimal to a common fraction. Give an example.—192. Solve and explain the given example. Recite the rule for reducing a common fraction to a decimal. What must first be done with compound fractions?

EXAMPLES FOR PRACTICE.

Find the value of the following in decimals :—

1. $\frac{1}{2}$; $\frac{3}{4}$; $\frac{4}{5}$; $\frac{7}{8}$; $\frac{7}{10}$; $\frac{9}{20}$.
2. $\frac{9}{16}$; $\frac{3}{125}$; $\frac{9}{32}$; $\frac{23}{64}$.
3. $\frac{3}{4}$ of $\frac{3}{5}$ of $\frac{1}{2}$. *Ans.* .225
4. $\frac{1}{5}$ of $\frac{3}{50}$ of $\frac{5}{8}$ of $1\frac{1}{2}$.
5. $\frac{1}{2}$ of $\frac{1}{4}$ of $\frac{1}{8}$. *Ans.* .015625
6. $\frac{1}{5}$ of $\frac{1}{5}$ of $\frac{1}{5}$. *Ans.* .008
7. $\frac{5}{6} \times \frac{1}{10} \times \frac{12}{13} \times 4\frac{1}{3}$. *Ans.* .33+
8. $\frac{3}{5}$; $\frac{3}{8}$; $\frac{9}{10}$; $\frac{6}{80}$; $\frac{1}{240}$.
9. $\frac{7}{40}$; $\frac{1}{4}$; $\frac{3}{20}$; $\frac{11}{250}$.
10. $\frac{1}{2} - \frac{43}{100}$. *Ans.* .07
11. $\frac{1}{5} + \frac{3}{4}$. *Ans.* .95
12. $\frac{1}{2} + \frac{1}{4} + \frac{3}{16}$. *Ans.* .9375
13. $\frac{35}{72} \div \frac{7}{9}$. *Ans.* .625
14. $\frac{2}{27} \div \frac{22}{135}$. *Ans.* .4545+

Circulating Decimals.

193. Sometimes (as in Example 7 and 14 above) no decimal can be obtained exactly equivalent to a common fraction. This is because the division does not terminate, but the same figure or set of figures keeps recurring in the quotient. In such cases, the further the division is carried out, the more nearly correct will the answer be.

194. A decimal in which one or more figures are constantly repeated, is called a **Circulating Decimal.** The repeated figure or figures are called the **Repetend.**

195. A repetend is denoted by a dot placed over it, if it is a single figure,—or over its first and last figure, if it contains more than one. Thus: $.\dot{3} = .333$, &c. $.\dot{4}\dot{5} = .454545$, &c. $.2\dot{1}4\dot{8} = .2148148$, &c.

196. A **Pure Circulating Decimal** is one that consists wholly of a repetend; as, $.\dot{3}$, $.\dot{2}4\dot{3}$.

A **Mixed Circulating Decimal** is one in which the repetend is preceded by one or more decimal figures, which form what is called the **Finite Part**: as, $.2\dot{3}$; .2 is the finite part.

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193. Why, in some cases, can not a decimal be obtained equivalent to a common fraction?—194. What is a Circulating Decimal? What are the repeated figure or figures called?—195. How is a repetend denoted?—196. What is a Pure Circulating Decimal? What is a Mixed Circulating Decimal? Give examples.

197. Reduction of Circulating Decimals.—Reducing according to the rule in § 192, we find

$\frac{1}{9} = .111111$, &c. or, $.\dot{1}$	Hence, reasoning backward,	$.\dot{1} = \frac{1}{9}$
$\frac{2}{9} = .222222$, &c. or, $.\dot{2}$	" " "	$.\dot{2} = \frac{2}{9}$
$\frac{3}{9} = .333333$, &c. or, $.\dot{3}$	" " "	$.\dot{3} = \frac{3}{9}$
$\frac{1}{99} = .010101$, &c. or, $.\dot{0}\dot{1}$	" " "	$.\dot{0}\dot{1} = \frac{1}{99}$
$\frac{2}{99} = .020202$, &c. or, $.\dot{0}\dot{2}$	" " "	$.\dot{0}\dot{2} = \frac{2}{99}$
$\frac{3}{99} = .030303$, &c. or, $.\dot{0}\dot{3}$	" " "	$.\dot{0}\dot{3} = \frac{3}{99}$
$\frac{1}{999} = .001001$, &c. or, $.\dot{0}0\dot{1}$	" " "	$.\dot{0}0\dot{1} = \frac{1}{999}$

It will be seen from the above that the denominator of a repetend consists of as many nines as it contains figures. Hence the following rule:—

198. Rule I.—*To reduce a repetend to a common fraction, write under it for a denominator as many nines as it contains figures.*

199. Rule II.—*To reduce a mixed circulating decimal, reduce the repetend to a common fraction, as above, annex it to the finite part, and place the whole over the denominator of the finite part. Reduce the complex fraction thus formed to a simple one.*

Example. Reduce $.23\dot{3}\dot{6}$ to a common fraction.

Reduce $\dot{3}\dot{6}$ to a common fraction:	$\frac{36}{99} = \frac{4}{11}$
Annex the fraction to the finite part:	$.23\frac{4}{11}$
Place the whole over the denom. of the finite part:	$\dfrac{23\frac{4}{11}}{100}$
Reduce the complex fraction thus formed:	$\frac{257}{1100}$ *Ans.*

EXAMPLES FOR PRACTICE.

1. Write as circulating decimals: .3333+ ($.\dot{3}$); .263263263+ ($.\dot{2}6\dot{3}$); .10471047+; .246666+ ($.24\dot{6}$); 1492121+; 4.9871871+; 3.2300300+; .12345671234567+; .2424+.

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197. Of what does the denominator of a repetend consist? How is this shown?—198. Recite the rule for reducing a repetend to a common fraction.—199. Recite the rule for reducing a mixed circulating decimal to a common fraction. Apply this rule in the given example.

2. Write as circulating decimals (§ 197): $\frac{7}{9}$ $(.\dot{7})$; $\frac{7}{99}$ $(.\dot{0}\dot{7})$; $\frac{7}{999}$ $(.\dot{0}0\dot{7})$; $\frac{7}{9999}$; $\frac{7}{99999}$; $\frac{14}{99}$; $\frac{5}{9999}$; $\frac{181}{99999}$; $\frac{16}{9999999}$; $\frac{2}{9}$; $\frac{2}{99}$; $\frac{2}{999}$; $\frac{2}{9999}$; $\frac{41}{99}$; $\frac{1947}{9999}$; $\frac{35643}{99999}$.

3. Reduce to common fractions (§ 198) in their lowest terms: $.\dot{2}\dot{5}$; $.\dot{0}8\dot{1}$; $.\dot{3}1\dot{5}$; $.\dot{1}00\dot{0}$. *Ans.* $\frac{25}{99}$; $\frac{9}{111}$; $\frac{35}{111}$; $\frac{1000}{9999}$.

4. Reduce to common fractions (§ 199) in their lowest terms: $.0\dot{6}$; $.24\dot{3}$; $.63\dot{2}1\dot{9}$; $.8\dot{1}00\dot{3}$. *Ans.* $\frac{1}{15}$; $\frac{73}{300}$; $\frac{5263}{8325}$; $\frac{16199}{19998}$.

5. How much more is $.\dot{8}$ than $.8$? *Ans.* $\frac{4}{45}$.

6. How much more is $.\dot{2}\dot{1}$ than $.21$? *Ans.* $\frac{7}{3300}$.

7. How much less is $.72$ than $.\dot{7}\dot{2}$?

8. Which is the greater, $.48$ or $.\dot{4}\dot{8}$, and how much?

9. Reduce $.\dot{1}2\dot{3}$. *Ans.* $\frac{41}{333}$.

10. Reduce $.\dot{3}2\dot{1}$. *Ans.* $\frac{107}{333}$.

11, Reduce $.27\dot{6}\dot{3}$. *Ans.* $\frac{76}{275}$.

12. Reduce $.0\dot{0}4\dot{5}$. *Ans.* $\frac{1}{222}$.

13. Reduce $.\dot{1}\dot{2}$ and $.189\dot{4}$.

14. Reduce $.08\dot{3}$ and $.4\dot{8}9\dot{6}$.

15. Reduce $.1\dot{3}\dot{5}$ and $.\dot{0}39\dot{8}$.

16. Reduce $.\dot{1}3\dot{5}$ and $.\dot{6}34\dot{5}$.

CHAPTER XII.

FEDERAL MONEY.

200. A **Coin** is a stamped piece of metal used as money.

201. By the **Currency** of a country is meant its money, consisting of coins, bank bills, government notes, &c.

202. Different countries have different currencies. The currency of the United States is called **Federal Money.**

TABLE OF FEDERAL MONEY.

10 mills (m.) make	1 cent, . . .	c., ct.
10 cents,	1 dime, . .	d.
10 dimes,	1 dollar, . .	$
10 dollars,	1 eagle, . .	E.

200. What is a Coin?—**201.** What is meant by the Currency of a country?—**202.** What is the currency of the United States called? Recite the Table of Federal Money.

The *mill*, one thousandth part of a dollar, takes its name from the Latin word *mille*, a thousand; the *cent*, one hundredth of a dollar, from the Latin *centum*, a hundred; the *dime*, one tenth of a dollar, from the French *dime*, a tithe or tenth. The word *dollar* comes from the German *thaler*. The dollar-mark $ is supposed to have originated from the letters U. S. (for United States) written one upon the other.

203. UNITED STATES COINS.—The coins of the United States represent all the denominations of the above Table except mills, as well as other values. They are as follows:—

GOLD.	Double eagle,	worth	$20.	SILVER.	Dollar,	worth	$1.
	Eagle,	"	$10.		Half-dollar,	"	50 c.
	Half-eagle,	"	$ 5.		Quarter-dollar,	"	25 c.
	Three-dollar piece,	"	$ 3.		Dime,	"	10 c.
	Quarter-eagle,	"	$ 2½.		Half-dime,	"	5 c.
	Dollar,	"	$ 1.		Three-cent piece,	"	3 c.

COPPER.	Three-cent piece,	3 c.
	Two-cent piece,	2 c.
	Cent,	1 c.

The gold and silver coins are nine tenths pure metal, the former being alloyed with one tenth of silver and copper, and the latter with one tenth of copper. The copper coins consist of 88 parts of copper to 12 of nickel.

204. WRITING AND READING FEDERAL MONEY.—In passing from mills to cents, from cents to dimes, and from dimes to dollars, we go each time to a denomination ten times greater, just as we do in passing from thousandths to hundredths, from hundredths to tenths. Federal Money is therefore a decimal currency, and may be written and operated on in all respects like decimals.

205. In writing and reading Federal Money, the only denominations used are dollars, cents, and mills. The dollar is the unit or integer, and is separated by the deci-

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From what does the mill take its name? The cent? The dime? The dollar? How is the dollar-mark supposed to have originated?—203. Name the gold coins of the United States, and their value. The silver coins. The copper coins. What proportion of the gold and silver coins is pure metal? With what are they alloyed? Of what do the copper coins consist?—204. What kind of a currency is Federal Money? How may it be written and operated on?—205. What denominations are used in writing and reading Federal Money? Which of these is the integer? How is it separated from cents?

mal point from cents, which occupy the first two places on the right of the point, mills occupying the third. Hence the rules.

Cents occupy two places,—that of dimes and their own,—because we do not recognize dimes in reading. Cents are sometimes written in the form of a common fraction, as hundredths of a dollar; as, $\$5\frac{32}{100}$.

206. Rule I.—*Write Federal Money decimally, the dollars as integer, the cents as hundredths, the mills as thousandths.*

Examples.—Twelve dollars, six cents,	\$12.06
Twelve dollars, sixty cents,	\$12.60
Twelve dollars, six cents, five mills,	\$12.065
Twelve dollars, five mills,	\$12.005

207. Rule II.—*In reading Federal Money, call the integer dollars, the hundredths cents, the thousandths mills.*

EXAMPLES FOR PRACTICE.

1. Write eleven dollars, eleven cents.
2. Write six hundred dollars, three mills.
3. Write ninety-eight dollars, seven cents.
4. Write one thousand dollars, ten cents, nine mills.
5. Write six dollars, seventeen cents, eight mills.
6. Write ninety-nine cents, nine mills.
7. Write a million dollars, one cent, one mill.
8. Read

\$840.268	\$560.005	\$ .009
\$23.01	\$5.90	\$1.00
\$14296.30	\$280.09	\$11.111

208. Operations in Federal Money.—*To add, subtract, multiply, or divide Federal Money, express the given amounts decimally, and proceed as in decimals.*

Represent $\frac{1}{2}$ cent as 5 mills, $\frac{1}{4}$ cent as 25 ten thousandths of a dollar. Thus: $37\frac{1}{2}$ cents = \$.375 $6\frac{1}{4}$ cents = \$.0625

How many places do cents occupy? Why? How are cents sometimes written? —206. Recite the rule for writing Federal Money.—207. Recite the rule for reading Federal Money.—208. Give the rule for adding, subtracting, multiplying, or dividing Federal Money. How is $\frac{1}{2}$ cent represented? $\frac{1}{4}$ cent?

As there are no mills coined, less than 5 mills in a result is disregarded in business dealings, and 5 mills or more are called an additional cent.

EXAMPLE 1.—Add together \$4.83, \$10.005, \$480, \$.37½, and \$3.36.

Follow the rule for the addition of decimals, § 182. Set the items down with their decimal points in line, representing the half cent as 5 mills. Add, and bring down the decimal point under the points in the items added. *Ans.* \$498.57.

	\$4.83
	10.005
	480.00
	.375
	3.36
Ans.	\$498.570

EXAMPLE 2.—A person having \$14 spent \$9.83; how much had he left?

\$14.00	
9.83	
\$ 4.17	*Ans.*

He had left the difference between \$14 and \$9.83. Proceed as in subtraction of decimals, § 183. *Ans.* \$4.17.

EXAMPLE 3.—What will 12 coats cost, at \$14.75 each?

If 1 coat costs \$14.75, 12 coats will cost 12 times \$14.75. Proceed as in multiplication of decimals, § 184. Point off two figures at the right of the product, because there are two decimal places in multiplicand and multiplier. *Ans.* \$177.

	\$14.75
	12
Ans.	\$177.00

EXAMPLE 4.—How many photographs, at 12½ cents apiece, can be bought for \$6.25?

```
.125) 6.250 (50
      625
       00
```

As many as 12½ cents is contained times in \$6.25. Proceed as in division of decimals, § 187. Annex a naught to the dividend, to make its decimal places equal those of the divisor, and the quotient will be a whole number. *Ans.* 50 photographs.

EXAMPLES FOR PRACTICE.

1. Add together forty-three dollars; seven dollars, twenty cents; nineteen cents, nine mills; twenty dollars, three mills; four cents, six mills; and seventy-five cents. *Ans.* \$71.198.

2. Take forty-three cents from thirty dollars.

3. From nine dollars, nine cents, subtract eight dollars, eighty cents, eight mills.

4. A farmer received \$41.60 for poultry, \$125 for a horse, \$3.12 for eggs, and \$5.55 for cheese. What was the sum total?

How is less than 5 mills in a result regarded? How are 5 mills or more regarded?

5. A father divides twenty thousand dollars equally among his 7 children; how much does each get? *Ans.* $2857.142 +

To get cents and mills in the answer, annex decimal naughts to the dividend, and continue the division.

6. How much is four and a half times $13.13?

In such a case express the fraction of the multiplier decimally. $13.13 × 4.5

7. If 23 acres are worth $724.50, how much is $\frac{3}{4}$ of an acre worth? *Ans.* $23.625.

First find how much 1 acre is worth; then $\frac{3}{4}$ of an acre.

8. A owes B $75.93, and borrows of him $37.50 more. If A then pays B $100.75, how much will still remain due?

9. What cost 185 pounds of coffee, at $.298 a pound?

10. Bought 8.375 cords of wood, at $5.50 a cord. What did it cost? *Ans.* $46.06.

11. The Erie Canal is 363 miles long, and cost $7143789. What was the average expense per mile?

12. A farmer sold his butter for 27 cents a pound, receiving $982.935. How many pounds did he sell? *Ans.* 3640.5 pounds.

13. What cost 16 sofas, at $43.75 apiece?

14. The butter made in one year from the milk of 53 cows, having been sold for 30 cents a pound, brought $2369.10. How many pounds were sold, and what was the average amount produced by each cow? *Ans.* Average, 149 pounds.

15. A man having 7 sons and 4 daughters, divides $100 among his sons, and $75 among his daughters. By how much does each daughter's share exceed each son's share? *Ans.* 4.46\frac{3}{7}$.

How much is each son's share ($\frac{1}{7}$ of $100)? How much is each daughter's share ($\frac{1}{4}$ of $75)? Find the difference between these two amounts.

16. How much will a man waste on segars in 50 years, if he smokes four daily, averaging 4$\frac{1}{2}$ cents each, allowing 365 days to the year? *Ans.* $3285.

17. A person who earns $1050 a year, spends in January, $98.41; in February, $81.33; in March, $102.28; in April, $125.26; in May, $74.38; in June, $73.47; in July, $65.98; in August, $87.21; in September, $70.34; in October, $122.08; in November, $79.68; in December, $52.77. How much has he left at the end of the year? *Ans.* $16.81.

18. How many pounds of cheese will be made from 46 cows in 30 days, if each cow averages 2.5 gallons of milk daily, and each gallon produces 1.1 pounds of cheese? What will the whole bring at 18½ cents a pound? *Ans.* $702.075.

How much milk is produced daily by 46 cows? How much cheese is produced daily? How much cheese, then, is produced in 30 days? What will this bring at $.185 a pound?

19. If I lay in eleven tons of coal, at $9.75 a ton; two barrels of charcoal, at 95 cents a barrel, and three loads of wood at $4.25 a load, and pay $3.80 for sawing and splitting, what does my fuel cost me? *Ans.* $125.70.

20. Suppose that a man buys two glasses of liquor a day, at ten cents a glass; how many volumes costing $1.50 each, could he purchase with the sum that he would thus spend in 30 years, allowing 365 days to the year? *Ans.* 1460 volumes.

21. A farmer buys 23¼ (23.25) yards of cloth at $3.75 a yard; if he pays for it with butter at 30 cents a pound, how much butter must he give? *Ans.* 290.625 pounds.

22. What will 24 copy-books cost, at 12 cents apiece?

23. The expenses of a family for May are as follows:—fuel, $10.25; table, $47.90; clothing, $13; rent, $31.25; sundries, $9.53. The next month they diminish their expenses one half; what does it cost them to live in June? *Ans.* $55.965.

24. A ferry-master who received $5.26 one morning, and $7.93 in the afternoon, found that he had taken a counterfeit dollar-bill and two bad quarter-dollars. How much good money did he take that day? *Ans.* $11.69.

25. The Welland Canal, 36 miles long, cost $7000000. What was the average cost per mile?

26. On the debtor (Dr.) side of an account are the following items: $1050, $241.71, $99.88, $760, $437.75. On the creditor (Cr.) side are the following: $69.95, $860, $.875, $43.17. What is the balance?

The balance is found by adding the items on the debtor side, then those on the creditor side, and taking the less sum from the greater.

27. What is the value of 42 bales of cotton, containing 425½ pounds each, at 56 cents a pound? *Ans.* $10007.76.

28. A farmer exchanges 9 tons of hay, worth $33.75 a ton, for wheat at $2.10 a bushel. How many bushels should he receive?

29. What cost 144 paper-cutters, at 37½ cents each?

30. Three partners bought some land for $9375. They sold it for $1100 cash, $973.50 worth of produce, and notes to the amount of $8000. What was each partner's profit? *Ans.* $232.83⅓.

How much did they receive for the land in all—cash, produce, and notes? What was the whole profit? Divide this among three.

31. C and D bought 30 acres of land each. C sold his so as to gain $1.75 an acre. D sold his so as to gain four times as much as C. How much did D make on his 30 acres?

How much did D make on 1 acre? How much on 30 acres?

32. A man buys 9 chairs, at $2.75 each. He sells them at a profit of 50 c. each. What does he get for the whole? *Ans.* $29.25.

33. The profits of a certain firm for one year are $8961. One of the partners, who receives ¼ of the profits, divides his share equally among his five sons. How much does each son receive?

34. If I buy some lace for $2.62½ a yard, and sell it for $3.10, do I gain or lose, and how much?

35. If I receive $3.15 a barrel for apples that cost me $3.87½, do I gain or lose, and how much?

36. F's account at a certain store stands as follows:—Debits, $49.75, $63, $3.75, $15.375, $304.05, $27, $199.875. Credits, $415, $88.80, $42. What is the balance? *Ans.* $117.

37. Find the balance of each of the two following accounts:—

Dr.	Cr.	Dr.	Cr.
$84.09	$149.68	$2264.00	$1860.43
63.86	19.94	185.76	10249.75
69.88	286.41	12458.63	76.49
2726.45	59.88	5289.21	3486.91
765.50	1147.31	18.75	590.43
338.88	6.66	451.39	1751.52
47.67	999.88	865.56	8064.63
4827.33	1428.72	4157.88	591.27
695.63	2379.64	386.27	7203.48
5820.94	822.56	21234.33	2457.87
967.19	1865.63	13642.85	3599.19
Ans. Balance	$7241.11	*Ans.* Balance	$21022.66

209. Aliquot Parts.—**Aliquot Parts** of a number are either whole or mixed numbers that will divide it without remainder. $4\frac{1}{2}$, 3, and $2\frac{1}{4}$, are aliquot parts of 9.

When an aliquot part is a whole number, it is a factor; 3 is both an aliquot part and a factor of 9.

210. The aliquot parts of a dollar most frequently used, are as follows:—

50 cents = $\frac{1}{2}$ of $1.	$12\frac{1}{2}$ cents = $\frac{1}{8}$ of $1.
$33\frac{1}{3}$ cents = $\frac{1}{3}$ of $1.	10 cents = $\frac{1}{10}$ of $1.
25 cents = $\frac{1}{4}$ of $1.	$6\frac{1}{4}$ cents = $\frac{1}{16}$ of $1.
20 cents = $\frac{1}{5}$ of $1.	5 cents = $\frac{1}{20}$ of $1.

211. When the cost of a number of articles is required, and the price of one is an aliquot part of $1, we may save work by operating with it as the fraction of $1.

Ex.—What will 600 Spellers cost, at $33\frac{1}{3}$ cents each?

At $1 each, 600 Spellers would cost $600. But $33\frac{1}{3}$ cents are $\frac{1}{3}$ of $1; therefore, at $33\frac{1}{3}$ cents, they will cost $\frac{1}{3}$ of $600, or $200.

$\frac{1}{3}$ of \$600 = \$200 *Ans.*

Rule.—*Take such a part of the given number as the price is of* $1, *and the result will be the answer in dollars.*

212. In like manner, *to divide by an aliquot part of a dollar, divide by the fraction that represents it.*

Example.—How many pass-books, at $6\frac{1}{4}$ cents each, can be bought for $2?

As many as $6\frac{1}{4}$ cents is contained times in $2. $6\frac{1}{4}$ cents is $\frac{1}{16}$ of a dollar. $\frac{1}{16}$ of $1 is contained in $2 32 times. *Ans.* 32 pass-books.

$6\frac{1}{4}$ cents = \$$\frac{1}{16}$

$2 \div \frac{1}{16} = 2 \times \frac{16}{1} = 32$ *Ans.*

EXAMPLES FOR PRACTICE.

1. What cost 144 pencils, at $6\frac{1}{4}$ cents each? *Ans.* $9.
2. What cost 500 Primers, at 20 cents each? *Ans.* $100.

209. What is meant by Aliquot Parts? When is an aliquot part a factor, and when not?—210. Mention the aliquot parts of a dollar most frequently used.—211. What is the rule for finding the cost of a number of articles, when the price of one is an aliquot part of $1?—212. What is the rule for dividing by an aliquot part of $1?

3. What cost 1600 pounds of sugar, at 25 cents?
4. What cost 1728 bottles of ink, at 12½ cents?
5. At 33⅓ cents a pound, what cost 150 pounds of candles?
6. What cost 2500 pounds of soap, at 12½ c.? *Ans.* $312.50.
7. What must I give for 85 rulers, at 50 cents each?
8. At 6¼ cents apiece, what cost 1000 cabbages?
9. If 1 yard of muslin costs 33⅓ cents, what cost 96 yards?
10. How many mackerel at 33⅓ cents, can be bought for $15?
11. How many dozen eggs, at 25 c. a dozen, can I buy for $6?
12. How much sugar, at 20 c. a pound, can be bought for $20?
13. How many baskets of berries, at 6¼ c., will $12 buy?
14. At 12½ c. a quart, how many quarts of nuts will $5 buy?

213. ARTICLES SOLD BY THE 100 OR 1000.—The price of articles is sometimes given by the 100 or 1000.

EXAMPLE.—What will 1550 envelopes cost, at $3.25 a thousand?

$$\begin{array}{r} 1550 \\ 3.25 \\ \hline 7750 \\ 3100 \\ 4650 \\ \hline 5037.50 \end{array}$$

Ans. $5.0375

If we multiply the number of articles and the price per thousand together, we get a result 1000 times too great. We must therefore divide the product by 1000,—that is, point off three *additional* figures. Hence the rule:—

RULE.—*To find the cost of a number of articles whose price is given by the* 100 *or* 1000, *multiply the given number and price together, and point off two additional figures in the product if the price is per hundred, or three if it is per thousand.*

The price of lumber (boards, plank, logs, &c.) is generally given by the thousand feet. *Per C.* (for *centum*) means *a hundred; per M.* (for *mille*) means *a thousand.*

EXAMPLES FOR PRACTICE.

1. What cost 525 oysters, at 95 c. a hundred? *Ans.* $4.9875.
2. At $4.20 a thousand, what cost 5625 envelopes?
3. What cost 1750 pounds of dried codfish, at $9.50 a hundred?

213. Solve the given example. Recite the rule for finding the cost of a number of articles whose price is given by the 100 or 1000. How is the price of lumber generally given? What does *per C.* mean? *Per M.*?

4. What must I pay for 1300 feet of boards, at $9.75 per M.?

5. What cost 425 feet of white-oak logs, at $65 per M.?

6. At $14.50 a hundred, what cost 9870 pounds of lead?

7. What cost 5000 laths, at 24 cents per C.?

8. What is the freight on 962 pounds, at $1.25 per 100?

9. What must be paid for laying 1275 bricks, at $8 per 1000?

10. Required the cost of 90422 bricks, at $7.75 a thousand?

11. How much must be paid for planing 4976280 feet of boards, at 84 cents per 1000 feet? *Ans.* $4180.075.

12. If a man carts 575 loads of bricks, averaging 1800 to the load, and is paid at the rate of 95 c. a thousand, how much will he receive? *Ans.* $983.25.

13. Sold, at $11¼ a hundred, three cargoes of pine-apples, the first consisting of 840, the second of 970, the third of 724. How much did they all bring? *Ans.* $285.075.

14. Bought 2000 feet of boards, at $9.10 per M., and 675 feet of pine stuff, at $1.80 per C. What was the whole cost? *Ans.* $30.35.

Making out Bills.

214. A **Bill** is a statement of what one party owes another for goods bought or services rendered. It may consist of several items, which are added or "footed up", to find the whole amount. Specimens of bills follow.

215. On the first line (see Bill 1, page 126) stand the name of the place and the date. On the second line is the name of the party owing the bill, and on the third that of the party to whom it is owed. *To* ——— *Dr.* means *To* ——— *Debtor*, or *indebted. B't of* ———, which is another form, used in Bill 2, means *Bought of* ———.

Then come the items, each with its date if the dates are different, as in Bill 4. @ before the price means *at.*

When a bill is presented and not paid, it is left without signature, like Bill 1. When it is paid, the party receiving it signs his name under the words *Received Payment*, as in Bill 2, and the person paying it retains it as evidence that it is paid. A clerk or collector signs his name for his principal, as shown in Bills 3 and 5.

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214. What is a Bill?—215. What is found on the first line of a bill? On the second line? On the third line? What is meant by *To* —— *Dr.*? *B't of*? What is the meaning of the sign @ in the items? When is a bill left without signature? When a bill is paid, what does the party receiving it do? Give the forms in which a clerk or collector signs his name.

If the party indebted gives his note, or written promise to pay, in stead of cash, it may be mentioned after the words *Received Payment*, as in Bill 4.

The person indebted may have paid something on account, or may have charges against the party rendering the bill. Such amounts are called *credits*. They are placed below the items of the bill, and are denoted by the letters *Cr.*, as in Bill 5. The balance is obtained by finding the difference between the sum of the credits and the sum of the debits.

EXAMPLES FOR PRACTICE.

Copy each bill, learn the forms, find the cost of each item, insert it in its proper place on the right, add the several amounts, and see whether their sum agrees with the given answer:—

(1)

N. Y., Feb. 23, 1865.

MR. HENRY ROE,

To TERRY & BROWN, Dr.

To 75 yards carpeting, @ \$2.50 . . .	\$
" 42 yards drugget, @ \$1.87½ . . .	
" 6 mats, @ \$3.25	
" 18 rugs, @ \$22.30	
" 81 yards oil-cloth, @ \$1.10 . . .	
	\$776.25

Received Payment,

(2)

Philadelphia, March 1, 1865.

MRS. H. S. SKINNER,

B't of R. J. JAMES.

3 yards linen, @ \$1.25	\$
12 pair hose, @ .75	
4 pair gloves, @ .95	
6 skeins silk, @ .05	
1 piece muslin, 44 yards, @ 55 c. . .	
	\$41.05

Received Payment,

R. J. JAMES.

If the party indebted gives his note, in stead of cash, where may it be mentioned? What is meant by *credits*? Where are they placed? How are they denoted? How is the balance obtained?

(3)

Columbus, May 1, 1865.

MESSRS. PLUMB & NIXON,

To ROBERT SUTTER, Dr.

To 7 ink-stands, @ 15 c.	$
" 9 boxes steel pens, @ $87\frac{1}{2}$ c. . . .	
" 8 reams foolscap paper, @ $4 . . .	
" 5 dozen copy-books, @ 96 c. a dozen	
" 3 rosewood writing-desks, @ $7.50 .	
	$68.23

Received Payment,

CALEB HUNT,

for ROBERT SUTTER.

(4)

Trenton, April 1, 1865.

MR. BENJAMIN STARK,

Bot. of DUDLEY STARR & Co.

Jan.	10	8 boxes raisins, @ $6.25	$
"	12	50 pounds sugar, @ 20 c.	
Feb.	16	48 pounds currants, @ $33\frac{1}{3}$ c.	
"	17	120 pounds tallow, @ $16\frac{2}{3}$ c.	
Mar.	23	14 barrels flour, @ $10.75	
"	25	30 gallons kerosene, @ 85 c.	
			$272.00

Received Payment, by note,

DUDLEY STARR & Co.

(5)

Concord, May 10, 1865.

MR. RICHARD FOOT,

To DR. W. S. CRANE, Dr.

To professional services to date		$68.00
" medicines to date		4.35
		$72.35
CR. By cash	$15.	
" 5 cords wood, @ $6	30.	
		$45.00
Balance . . .		$27.35

Received Payment,

W. S. CRANE,

by ASA GREEN.

6. John Cox bought of Philip Brady, of Boston, the following articles:—Jan. 2, 1865, 6 pair of gloves, at $1.25; Feb. 1, 12 shirts, at $3.75; Feb. 9, 18 pair socks, at 33⅓ c.; Feb. 13, 1 overcoat, at $40; Feb. 20, 2 vests, at $8.75, and 2 umbrellas, at $3.30. Make out Cox's bill, and receipt it. *Ans.* $122.60.

7. James Ray, of Detroit, sold George Mott the following articles:—25 pounds beef, at 19 c.; 8 pair fowls, 40 pounds, at 21 c.; 50 pounds sausage-meat, at 12½ c.; 25 bushels potatoes, at 50 c.; 45 pounds of lard, at 16⅔ c. Mott paid on account $15. Make out his bill, and find the balance due. *Ans.* $24.40.

8. H. S. Fair, of Hartford, sold N. T. Wright, 4960 feet of scantling, at $8.25 per M.; 3575 feet hemlock boards, at $11 per M.; 2240 feet white-oak plank, at $60 per M.; 4785 feet pine boards, at $7.25 per M. The same party bought of N. T. Wright, 37 yards carpeting, at $1.50; 24½ yards matting, at 90 c.; 40½ yards oil-cloth, at $1.40. Make out Fair's bill against Wright, showing the balance due. *Ans.* $115.44.

9. William Haight bought of Hiram See, of N. O., 42 boxes of oranges, at $8.12; 7640 pounds coffee, at 33⅓ c.; 2400 gallons molasses, at 92 c.; 875 pounds rice, at 8½ c.; 1250 pounds sugar, at 20 c. See credits Haight with 400 barrels of flour, at $9.75, and takes Haight's note to balance account. Make out bill.

Ans. Balance, $1518.63.

10. Mrs. Stewart, of Newark, presents her bill to P. S. Howard for board, &c., as follows:—6 weeks' board, at $8.25 a week; fuel 6 weeks, at $1.20; gas 6 weeks, at 50 c.; washing, 7 dozen pieces, at $1 a dozen. Make out the bill. *Ans.* $66.70.

11. Suppose you buy of D. Appleton & Co. 5 reams of note paper, at $3.25; 4500 envelopes, at $4.75 a thousand; 24 boxes steel pens, at $1.12½; 6 French Dictionaries, at $1.50; 3 Photographic Albums, at $5.75. Make out your bill. *Ans.* $90.88.

12. M. Stagg, of Baltimore, sold James Quinn the following articles:—April 1, 1865, 24 yd. black silk, at $2.25; April 3, 2 pieces French calico, 40 yd. each, at 30 c.; May 2, 4 dress patterns, at $6.75; May 9, 22½ yd. linen, at $1.12. Quinn paid $55 on account. Make out his bill, showing balance due. *Ans.* $75.20.

Miscellaneous Questions on Decimals and Federal Money.—What does the word decimal come from? How do decimals arise? How can you find the denominator of a decimal? Does annexing a naught to a decimal increase or diminish its value? When adding decimals, where do you place the decimal point in the result? When subtracting? When multiplying? When dividing? In division of decimals, when will the quotient be a whole number? How do we multiply a decimal by 10, 100, 1000, &c.? How do we divide a decimal by 10, 100, 1000, &c.?

What is a circulating decimal? What is a repetend? How do you reduce a decimal to a common fraction? How do you reduce a repetend to a common fraction? How do you reduce a common fraction to a decimal?

What is federal money? How do you write federal money? Why are two places appropriated to cents? How do you read federal money? How do you add, subtract, multiply, and divide federal money? What is a bill? When are credits to be entered in a bill? When must a bill be receipted? What are the forms to be used by a clerk in receipting a bill?

CHAPTER XIII.

REDUCTION.

216. How many cents in five dollars?

100 cents make $1; in $5, therefore, there are 100 times 5 cents, or 500 cents. *Ans.* 500 cents.

We have here changed the denomination from dollars to cents, without changing the value. This process is called Reduction. We have *reduced* dollars to cents.

217. Reduction is the process of changing the denomination of a number without changing its value.

218. There are two kinds of Reduction:—

1. **Reduction Descending**, in which we change a higher denomination to a lower, as dollars to cents. Here we must multiply.

216. How many cents in $5? What have we here done? What is this process called?—217. What is Reduction?—218. How many kinds of Reduction are there? What are they called? What is Reduction Descending?

2. **Reduction Ascending**, in which we change a lower denomination to a higher, as cents to dollars. Here we must divide.

219. Reduction Descending.

EXAMPLE 1.—Reduce $41 to mills.

100 cents make $1; in $41, therefore, there are 100 times 41 cents, or 4100 cents.

10 mills make 1 cent; in 4100 cents, therefore, there are 10 times 4100 mills, or 41000 mills.

$$\begin{array}{r} \$41 \\ \underline{100} \\ 4100 \text{ c.} \\ \underline{10} \\ \textit{Ans. } 41000 \text{ m.} \end{array}$$

EXAMPLE 2.—Reduce $41.375 to mills.

Reduce $41 to cents:	$41 \times 100 = 4100$ c.
Add in 37 cents:	$4100 + 37 = 4137$ c.
Reduce 4137 cents to mills:	$4137 \times 10 = 41370$ m.
Add in 5 mills:	$41370 + 5 = 41375$ m. *Ans.*

220. RULE FOR REDUCTION DESCENDING.—*Multiply the highest given denomination by the number that it takes of the next lower to make one of this higher, and add in the number belonging to such lower denomination, if any be given. Go on thus with each denomination in turn, till the one required is reached.*

221. Reduction Ascending.

EXAMPLE 3.—Reduce 41375 mills to dollars.

10 mills make 1 cent; therefore in 41375 mills there are as many cents as 10 is contained times in 41375, or 4137 cents, and 5 mills over.

100 cents make 1 dollar; therefore in 4137 cents there are as many dollars as 100 is contained times in 4137, or $41, and 37 cents over.

$$\begin{array}{l} 10\overline{)41375} \text{ m.} \\ 100\overline{)4137} \text{ c., } 5 \text{ m} \\ \quad \textit{Ans. } \$41.375 \end{array}$$

The last quotient and the two remainders form the answer—$41, 37 cents, 5 mills, or $41.375.

222. RULE FOR REDUCTION ASCENDING.—*Divide the given denomination by the number that it takes of it to*

What is Reduction Ascending?—219. Solve the given examples, explaining the several steps.—220. What is the rule for Reduction Descending?—221. Reduce 41375 mills to dollars.—222. What is the rule for Reduction Ascending?

make one of the next higher. Divide the quotient in the same way, and go on thus till the required denomination is reached. The last quotient and the several remainders form the answer.

223. In Example 2 we reduced $41.375, and obtained 41375 mills. In Example 3, we reduced 41375 mills, and obtained $41.375. Thus it will be seen that Reduction Descending and Reduction Ascending prove each other.

224. Reduction of Federal Money.

In Example 1, § 219, we reduced dollars to cents by annexing two naughts, cents to mills by annexing one naught.

In Example 2, § 219, comparing the result, 41375 mills, with $41.375, the amount to be reduced, we find it is the same, with the dollar-mark and decimal point omitted.

In Example 3, § 221, comparing the result, $41.375, with 41375 mills, the amount to be reduced, we find that we have simply pointed off three figures from the right, and inserted the dollar-mark. Hence the following rules:—

RULES FOR THE REDUCTION OF FEDERAL MONEY.—1. *To reduce dollars to mills, annex three naughts; to reduce dollars to cents, two; to reduce cents to mills, one.*

2. *To reduce dollars and cents to cents, or dollars, cents, and mills, to mills, simply remove the dollar-mark and the decimal point.*

3. *To reduce mills to dollars, point off three figures from the right; to reduce cents to dollars, two; to reduce mills to cents, one.*

EXAMPLES FOR PRACTICE.

Reduce the following:—

1. $63.47 to cents.	6. $.059 to mills.
2. $5.485 to mills.	7. $2.85 to cents.
3. $2480 to mills.	8. $5000 to mills.
4. $56.90 to mills.	9. 2468 mills to cents.
5. $4283 to cents.	10. 2570 mills to dollars.

223. How is Reduction Descending proved? Reduction Ascending?—224. Recite the rules for the reduction of federal money.

11. 8620 cents to dollars.
12. 490000 mills to cents.
13. 56000 cents to dollars.
14. 56000 cents to mills.
15. 8705 cents to dollars.
16. $87.05 to mills.
17. How many cents in 4 eagles? (4 eagles = $40) *Ans.* 4000 c.
18. How many cents is a double eagle worth? *Ans.* 2000 c.
19. How many eagles are 8000 cents worth? 2000 cents?
20. Reduce 423756890 mills to dollars.
21. How many cents in $89½? In $102¼? In $4¼?
22. How many mills in 37½ cents? In $5.62½?
23. How many quarter-dollars equal a double eagle?
24. How many dimes in $1? In $15? In $30? In $49?
25. How many cents in 1 dime? In 5 dimes? In 20 dimes?
26. How many dimes are equal to 10 cents? To 150 cents?
27. How many half-dollars ought I to receive in change for an eagle? For two double eagles?
28. How many cents is a quarter-eagle worth? A half-eagle? A three-dollar piece? A half-dollar? Five dimes?
29. Reduce each of the following to cents, and add the results: 2 eagles; 5 half-dollars; 15 dollars; 1 double eagle; 3 quarter-dollars; 12 dimes; 120 mills. *Ans.* 5957 cents.

Compound Numbers.

225. A **Compound Number** is one consisting of different denominations; as, 3 dollars, 19 cents.

226. Compound numbers may be reduced, added, subtracted, multiplied, and divided.

227. To show the relations that different denominations bear to each other, Tables are constructed. These are now presented in turn, with examples in Reduction under each; they should be thoroughly committed to memory. For convenience of reference, these Tables are reproduced together on the last page of the book.

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225. What is a Compound Number?—226. What operations may be performed on Compound Numbers?—227. For what purpose have Tables been constructed in connection with Compound Numbers?

ENGLISH OR STERLING MONEY.

228. English or **Sterling Money** is the currency of Great Britain.

TABLE.

4	farthings (far., qr.),	1 penny,	. . . d.
12	pence,	1 shilling,	. . . s.
20	shillings,	1 pound,	. . . £.
21	shillings,	1 guinea,	. . . guin.

guin.		£		s.		d.		far.
						1	=	4
				1	=	12	=	48
		1	=	20	=	240	=	960
1	=	$1\frac{1}{20}$	=	21	=	252	=	1008

The pound mark £ is a capital *l*, standing for the Latin word *libra*, a pound; it always precedes the number, as £2. *S.* stands for the Latin *solidus*, a shilling; *d.* for *denarius*, a penny; *qr.* for *quadrans*, a farthing.

Shillings are sometimes written at the left of an inclined line, and pence at the right: $^2/-$ = 2s. $-/_6$ = 6d. $^2/_6$ = 2s. 6d. Farthings are sometimes written as the fraction of a penny, 1 far. as $\frac{1}{4}$d., 2 far. as $\frac{1}{2}$d., 3 far. as $\frac{3}{4}$d.

The pound is simply a denomination; a gold coin called the Sovereign represents it. The Sovereign is worth $4.84. The English shilling is worth $24\frac{1}{5}$ cents, and the English penny about 2 cents.

Guineas, originally made of gold brought from Guinea, are no longer coined. The Crown is a silver coin, worth 5 shillings.

229. In the twelfth century, some traders from the Baltic coasts, called by the people Easterlings because coming from regions farther east, were employed to regulate the coinage of England. From these Easterlings the currency took the name of *Sterling* Money.

EXAMPLES FOR PRACTICE.

230. Recite the rules for Reduction, § 220, 222.

EXAMPLE 1.—Reduce £5 19s. 3 far. to farthings.

---

228. What is English or Sterling Money? Recite the Table of Sterling Money. What is the pound mark, and where does it stand? What do *s.*, *d.*, and *qr.*, stand for? How are shillings sometimes written? How are farthings sometimes written? Is the pound a denomination or a coin? What represents it? What is the sovereign worth? The English shilling? The English penny? Why were guineas so called? What is the Crown?—229. From whom did sterling money receive its name?—230. Go through and explain the given examples in Reduction.

This is a case of Reduction Descending. Multiply the £5 by 20, to reduce them to shillings, because 20 shillings make a pound. Add in the 19 shillings.

Multiply 119s., thus obtained, by 12, to reduce them to pence, because 12 pence make a shilling. There are no pence in the given number to add in.

Multiply the 1428d., thus obtained, by 4, to reduce them to farthings, because 4 farthings make a penny. Add in the 3 farthings. *Ans.* 5715 far.

```
£5  19s.  3 far.
20
----
119s.
 12
----
1428d.
   4
----
5715 far. Ans.
```

EXAMPLE 2.—Reduce 15383 far. to pounds, shillings, &c.

```
4)15383 far.
  --------
12)3845d. 3 far.
   -------
2|0)32|0s. 5d.
   ------
   £16
Ans. £16 5¾d.
```

This is a case of Reduction Ascending. Divide 15383 far. by 4, to reduce them to pence, because 4 farthings make a penny.

Divide the quotient, 3845d., by 12, to reduce it to shillings, because 12 pence make a shilling.

Divide the quotient, 320s., by 20, to reduce it to pounds, because 20 shillings make a pound. The last quotient and the several remainders form the answer.—Always mark the denominations throughout, as in these examples.

EXAMPLE 3.—Reduce £457 to farthings.

We may here proceed as above, or we may somewhat shorten the operation. Looking under the Table on page 133, we find £1 = 960 far. Then in £457 there are 960 times 457 farthings. When, then, the number to be reduced has but one denomination, we may multiply at once by the number that connects it with the denomination required.

```
 £457              £1 = 960 far.
   20
 -------
 9140s.              £457
   12                 960
 -------             ------
 109680d.            27420
     4               4113
 -------             ------
 438720 far.         438720 far.
```

4. Reduce £7 5s. 10d. 3 far. to farthings. *Ans.* 7003 far.

5. Reduce £47 5s. 2d. 1 far. to farthings. *Ans.* 45369 far.

6. Reduce £1 5¼d. to farthings. *Ans.* 981 far.

7. In 18s. 3 far. how many farthings? *Ans.* 867 far.

8. Reduce 4963 far. to pounds, &c. *Ans.* £5 3s. 4d. 3 far.

9. Reduce $^{10}/_{6}$ to farthings. Reduce $^{6}/_{8}$ to pence.

Prove by reducing the answers obtained back to shillings.

10. In £8000 how many pence?

11. How many farthings in $^{3}/_{6}$? In $^{11}/-$? In 8s. 3½d.?

12. How many sovereigns are 12480 pennies worth?

13. How many pence are 840 sovereigns worth?

14. Reduce 560 guineas to farthings. *Ans.* 564480 far.

15. Reduce 118567 far. to pounds, &c. *Ans.* £123 10s. 1¾d.

16. Reduce £3 10s. to pence. Reduce 18s. 9d. to pence.

17. How many pounds, &c., in 15199 pence? In 189s.?

18. How many crowns are £25 equal to?

1 crown = 5s. How many crowns in £1, or 20s.? How many in £25?

19. How many pounds are 100 guineas equal to?

20. Reduce 7643s. to pounds; to guineas.

21. Reduce £1000 to farthings.

22. Reduce 4800000 far. to pounds, &c.

23. In 24000 far. how many crowns? *Ans.* 100 crowns.

24. A subscribes £500 for the poor; B, 500 guineas. Which subscribes the most, and how much? *Ans.* B £25.

TROY WEIGHT.

231. To express weight, three different scales are used, called Troy, Apothecaries', and Avoirdupois Weight.

232. Troy Weight is used in weighing gold, silver, coins, and precious stones; also in philosophical experiments.

TABLE.

24 grains (gr.) make	1 pennyweight,	. .	pwt.
20 pennyweights,	1 ounce,	. . . .	oz.
12 ounces,	1 pound,	. . . .	lb.

lb.		oz.		pwt.		gr.
				1	=	24
		1	=	20	=	480
1	=	12	=	240	=	5760

The Troy pound is the standard unit of weight of the United States and Great Britain. It is equal to the weight of 22.794377 cubic inches of distilled water, at its greatest density.

233. The denominations *grain* and *pennyweight* take their name from the fact that silver pennies were once coined, required by law to equal in weight 32 grains of wheat from the middle of the ear, well dried. The value of the penny being afterwards reduced, the number of grains in the

231. Name the different scales used to express weight.—232. For what is Troy Weight used? Recite the Table of Troy Weight. What is the standard unit of weight of the United States? To what is the Troy pound equal?—233. Why are the grain and pennyweight so called?

pennyweight was also reduced to 24.—*Oz.* stands for the Spanish word *onza*, an ounce.

234. Troy Weight takes its name from Troyes, a town of France, whence it was carried to England by goldsmiths; or, according to others, from Troy Novant, an old name applied to London.

EXAMPLES FOR PRACTICE.

1. Reduce 30 lb. 3 oz. to pwt.

Which kind of Reduction does this fall under? Recite the rule (§ 220).—Multiply the 30 lb. by 12, to reduce them to ounces; add in the 3 oz. Multiply the ounces thus obtained by 20, to reduce them to pwt.; there being no pwt. to add in, this result is the answer.

30 lb. 3 oz.
12
363 oz.
20
Ans. 7260 pwt.

2. Reduce 7681 pwt. to pounds, &c.

Which kind of Reduction does this fall under? Recite the rule (§ 222).—As 20 pwt. make an ounce, divide the given pennyweights by 20, to reduce them to ounces. Divide the ounces thus obtained by 12, to reduce them to pounds. The last quotient and the remainder form the answer.

2|0) 768|1 pwt.
12) 384 oz. 1 pwt.
32 lb.
Ans. 32 lb. 1 pwt.

3. Reduce 6 lb. 4 oz. 8 pwt. 5 gr. to grains. *Ans.* 36557 gr.
4. How many grains in 11 oz. 19 pwt. 23 gr.? *Ans.* 5759 gr.
5. In 1200¾ lb. how many pennyweights? *Ans.* 288180 pwt.
6. Reduce 9999 gr. to pounds. *Ans.* 1 lb. 8 oz. 16 pwt. 15 gr.
7. Reduce 999 pwt. to pounds, &c. *Ans.* 4 lb. 1 oz. 19 pwt.
8. Reduce 1561 oz. to pounds, &c. *Ans.* 130 lb. 1 oz.
9. Reduce 18 pwt. 4 gr. to grains. *Ans.* 436 gr.
10. Reduce 11000 grains to lb. *Ans.* 1 lb. 10 oz. 18 pwt. 8 gr.
11. In 25 lb. 17 pwt. how many grains?
12. In 87 lb. 5 oz. how many pennyweights?
13. In 8543 grains, how many pounds, &c.?
14. Reduce each of the following to pounds, and add the results: 40320 gr.; 960 oz.; 6960 pwt. *Ans.* 116 lb.
15. Reduce the following to grains, and add the results: 5 lb. 1 pwt. 18 gr.; 11 oz. 17 pwt.; 10 lb. 4 oz. 11 pwt. *Ans.* 94314 gr.
16. How many ounces in four lumps of gold, weighing 7 pwt., 13 pwt., 15 pwt., and 18 pwt.? *Ans.* 2 oz. 13 pwt.

---

234. From what does Troy Weight take its name?

17. What is the weight in pounds of a silver tea-pot weighing 200 pwt., and 24 table-spoons of 35 pwt. each? *Ans.* 4 lb. 4 oz.

18. How many pounds of gold will a miner dig in a year of 365 days, if he averages 6 pwt. daily? *Ans.* 9 lb. 1 oz. 10 pwt.

APOTHECARIES' WEIGHT.

235. Apothecaries' Weight is used by apothecaries in mixing medicines. They buy and sell their drugs, in quantities, by Avoirdupois Weight.

TABLE.

20 grains (gr.) make	1 scruple,	. . sc.	or ℈.
3 scruples,	1 dram,	. . dr.	or ʒ.
8 drams,	1 ounce,	. . oz.	or ℥.
12 ounces,	1 pound,	. . lb.	or ℔.

lb.		oz.		dr.		sc.		gr.
						1	=	20
				1	=	3	=	60
		1	=	8	=	24	=	480
1	=	12	=	96	=	288	=	5760

The only difference between Apothecaries' and Troy Weight lies in the division of the ounce. The grain, ounce, and pound, are the same in both.

EXAMPLES FOR PRACTICE.

1. Reduce 9247 gr. to pounds, &c. *Ans.* 1 ℔. 7 ℥ 2 ʒ 7 gr.

Which kind of Reduction does this example fall under? Recite the rule (§ 222). Name the numbers in order, by which we must divide. Prove the answer by reducing it back to grains.

2. Reduce 9 ℥ 6 ʒ 1 ℈ 7 gr. to grains. *Ans.* 4707 gr.

Which kind of Reduction does this example fall under? Recite the rule (§ 220). Name the numbers in order, by which we must multiply. Why do we not first multiply by 12? If drams had been the highest denomination given, by what would we have multiplied first? How can you prove the answer?

3. Reduce 15648 gr. to pounds, &c. *Ans.* 2 ℔. 8 ℥ 4 ʒ 2 ℈ 8 gr.

4. Reduce 1 ℔. 11 ℥ 2 ʒ 5 gr. to grains. *Ans.* 11165 gr.

5. Reduce 4763 ℈ to pounds, &c. *Ans.* 16 lb. 6 oz. 3 dr. 2 sc.

---

235. By whom is Apothecaries' Weight used? By what do they buy and sell their drugs in quantities? Recite the Table of Apothecaries' Weight. What is the only difference between Apothecaries' and Troy Weight?

6. Reduce 9 lb. 5 dr. to scruples. *Ans.* 2607 sc.

7. Reduce 843 dr. to pounds, &c. *Ans.* 8℔. 9℥ 8ʒ.

8. Reduce 14 lb. 8 oz. to drams.

9. Reduce 30019 gr. to pounds, &c.

10. In 2℔. 3℥ 4ʒ 1℈ 11 gr. how many grains?

11. Reduce the following to grains, and add the results: 1℔. 1 gr.; 10℥ 2℈; 5ʒ 1 gr.; 3℔. 1℈. *Ans.* 28202 gr.

12. Reduce the following to pounds, and add the results: 11520 gr.; 960 dr.; 864 sc.; 144 oz. *Ans.* 27 lb.

13. How many doses of 15 gr. each will 5 dr. of calomel make?

Reduce 5 dr. to grains. How many times are 15 grains contained therein?

14. How many grains in this mixture: benzoin, 2℥; cascarilla, 2ʒ; nitre, 1½ʒ; myrrh, 2℈; charcoal, 3℥? *Ans.* 2650 gr.

Reduce each item to grains; then add.

15. A druggist put up 24 powders of calomel, of 10 gr. each; if he had 1 oz. of calomel at first, how many grains will he have left?

AVOIRDUPOIS WEIGHT.

236. Avoirdupois Weight is used for weighing all articles not named under Troy and Apothecaries' Weight; such as groceries, meat, coal, cotton, all the metals except gold and silver, and drugs when sold in quantities.

TABLE.

16 drams (dr.) make	1 ounce, ...	oz.
16 ounces,	1 pound, . . .	lb.
25 pounds,	1 quarter, . . .	qr.
4 quarters,	1 hundred-weight,	cwt.
20 hundred-weight,	1 ton, ...	T.

T.		cwt.		qr.		lb.		oz.		dr.
								1	=	16
						1	=	16	=	256
				1	=	25	=	400	=	6400
		1	=	4	=	100	=	1600	=	25600
1	=	20	=	80	=	2000	=	32000	=	512000

236. For what is Avoirdupois Weight used? Recite the Table.

237. *Avoirdupois* is derived from the French words *avoir*, property, and *poids*, weight.—*Cwt.*, the abbreviation for hundred-weight, is formed of *c* for *centum*, one hundred, and *wt* for weight.

238. Formerly 28 pounds made a quarter, and 112 pounds a hundred-weight, in the United States, as they still do in Great Britain. But it is no longer customary to allow 112 pounds to the hundred-weight, except in the case of coal at the mines, iron and plaster bought in large quantities, and English goods passing through the Custom House.

Twenty hundred-weight of 112 pounds make a ton of 2240 pounds, which is distinguished as a Long or Gross Ton.

239. The Avoirdupois pound weighs 7000 grains Troy, and is therefore greater than the Troy pound, which contains 5760 grains. The Avoirdupois ounce weighs $437\frac{1}{2}$ grains, and is therefore less than the Troy ounce, which contains 480 grains.

1 lb. Avoir. = 7000 gr. = 1 lb. 2 oz. 11 pwt. 16 gr. Troy.
1 oz. Avoir. = $437\frac{1}{2}$ gr. = 18 pwt. $5\frac{1}{2}$ gr. Troy.

1 lb. Troy or Apoth. = 5760 gr. = $13\frac{29}{175}$ oz. Avoir.
1 oz. Troy or Apoth. = 480 gr. = $1\frac{17}{175}$ oz. Avoir.

EXAMPLES FOR PRACTICE.

1. Reduce 10 cwt. to drams.

Looking among the equivalents under the Table, we find 1 cwt. = 25600 dr. Then 10 cwt. = 10 × 25600 dr. *Ans.* 256000 dr.—When there are no intermediate denominations, the Table of equivalents can thus be used with advantage.

2. Reduce 4815 lb. to hundred-weight.

In the Table of equivalents we find 1 cwt. = 100 lb. Then in 4815 lb. there are as many cwt. as 100 lb. are contained times in 4815 lb. *Ans.* 48 cwt. 15 lb.

3. Reduce 3 T. 15 cwt. 16 lb. 5 oz. 5 dr. to drams.

Which kind of Reduction does this example fall under? Repeat the Rule (§ 220). What numbers must we multiply by? Prove the result. *Ans.* 1924181 dr.

4. Reduce 294400 oz. to tons, &c.

Which kind of Reduction does this example fall under? Repeat the Rule (§ 222). Mention the successive divisors. Prove the result.

5. Reduce 1 T. 15 lb. to ounces. *Ans.* 32240 oz.

6. Reduce 1792512 dr. to tons, &c. *Ans.* 3 T. 10 cwt. 2 lb.

7. How many pounds in two loads of $2\frac{1}{4}$ tons each?

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237. From what is the word *avoirdupois* derived? Of what is the abbreviation *cwt.* formed?—238. How many pounds formerly made a hundred-weight? In what alone is it now customary to allow 112 lb. to the hundred-weight? What is a Long Ton?—239. How many grains in the avoirdupois and the Troy pound respectively? In the avoirdupois and the Troy ounce? What is 1 lb. avoir. equivalent to in Troy weight? What is 1 lb. Troy equivalent to in avoirdupois weight?

8. How many pounds in four loads of 3⅜ tons each?

9. How many drams in 12⅞ tons?

10. Reduce 24 lb. 3 oz. 14 dr. to drams.

11. How many tons, &c., in 94500 oz.?

12. Reduce 2 T. 2 cwt. 2 qr. 2 lb. 2 oz. 2 dr. to drams.

13. How many drams in 27 long tons? *Ans.* 15482880 dr.

14. In 42½ long tons how many pounds? *Ans.* 95200 lb.

15. Reduce 5 T. 1 cwt. 13 lb. to drams. *Ans.* 2588928 dr.

16. Reduce the following to drams, and add the results: 7½ tons; 2¼ long tons; 11 cwt.; 4 lb. 4 oz. *Ans.* 5284928 dr.

17. Reduce the following to hundred-weight, and add the results: 6400 oz.; 1700 lb.; 281600 dr.; 28 qr. *Ans.* 39 cwt.

18. How many more pounds in 1 long ton than 1 common ton? In 25 long tons than 25 common tons?

19. If a coal-merchant buys a cargo of 200 long tons, and sells 200 common tons, how many pounds has he left? How many common tons? How many long tons? *Ans.* 21³⁄₇ long tons.

20. How many two-ounce weights can be made out of 50 pounds of brass?

How many oz. in 50 lb.? How many times are 2 oz. contained therein?

21. How many five-pound weights can be made out of 5½ cwt. of iron? Out of 6¼ cwt.?

22. How many more grains in 1 lb. avoirdupois than in 1 lb. Troy? (See § 239.) In 14 lb. avoir. than in 14 lb. Troy?

23. How many pounds Troy are 144 lb. avoir. equal to?

How many grains in 1 lb. avoir.? How many in 144 lb. avoir.? How many lb. Troy in these, if 1 lb. Troy contains 5760 grains?

24. Reduce 1225 lb. Troy to avoirdupois pounds.

25. Reduce 875 oz. apothecaries' weight to pounds avoir.

How many grains in 1 oz. apoth.? How many in 875 oz.? Reduce these grains to pounds avoirdupois.

26. Reduce 2880 oz. avoir. to Troy ounces. *Ans.* 2625 oz.

27. What cost 54½ cwt. of pork, at 11 c. a pound? *Ans.* $599.50.

28. What cost 26 cwt. of hams, at 6d. a lb.? *Ans.* £65.

29. What cost 9 T. of iron at 1¾d. a lb. *Ans.* £131 5s.

30. What cost 475 T. of iron, at $4.50 a cwt.?

31. What cost 100¾ cwt. of cheese, at 10 c. a pound?

240. Miscellaneous Table.

The pounds in this Table are avoirdupois.

14 pounds, . .	1 stone of iron or lead.
60 pounds, . .	1 bushel of wheat.
100 pounds, . .	1 quintal of dried fish.
100 pounds, . .	1 cask of raisins.
196 pounds, . .	1 barrel of flour.
200 pounds, . .	1 bar. of beef, pork, or fish.
280 pounds, . .	1 bar. of salt at the N. Y. State works.

EXAMPLES FOR PRACTICE.

1. How many ounces in 14 stone? *Ans.* 3136 oz.

2. How many stone are 7 cwt. equal to? *Ans.* 50 st.

Reduce 7 cwt. to pounds. Divide by the number of pounds in 1 stone.

3. How many barrels will 98 cwt. of flour make?

4. At 7 c. a pound, what cost 46 quintals of cod-fish?

5. How many bushels in 330 lb. of wheat?

6. If flour is $9.80 a barrel, how much is that a pound?

7. How many hundred-weight in 25 barrels of salt bought at the N. Y. State salt works?

8. How many seven-pound boxes can be filled from 21 casks of raisins?

9. If $\frac{1}{4}$ of a barrel of flour is sold, how many pounds remain in the barrel?

10. At 2d. a pound, what cost 125 quintals of dried fish?

LONG OR LINEAR MEASURE.

241. There are three dimensions: length, or distance from end to end; breadth, or distance from side to side; and thickness, or distance from top to bottom.

A line has length; a surface, length and breadth; a solid, length, breadth, and thickness.

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240. Recite the Miscellaneous Table. What kind of pounds are these?—241. How many dimensions are there? Name and define them. Which of these dimensions has a line? A surface? A solid?

242. Long or **Linear Measure** is used in measuring length and distance. It begins with the inch.

1 inch.

TABLE.

12	inches (in.)	make	1 foot, ...	ft.
3	feet,		1 yard, ...	yd.
$5\frac{1}{2}$	yards,		1 rod, ...	rd.
40	rods,		1 furlong, . . .	fur.
8	furlongs,		1 mile, ...	mi.

m.		fur.		rd.		yd.		ft.		in.
								1	=	12
						1	=	3	=	36
				1	=	$5\frac{1}{2}$	=	$16\frac{1}{2}$	=	198
		1	=	40	=	220	=	660	=	7920
1	=	8	=	320	=	1760	=	5280	=	63360

243. The following denominations also occur:—

The Line	= $\frac{1}{12}$ inch.	The Pace	= 3 feet.
The Hand	= 4 inches.	The Fathom	= 6 feet.
The Span	= 9 inches.	The Geographical Mile	= $1\frac{3}{20}$ + mi.
The Cubit	= 18 inches.	The League	= 3 miles.

The Hand is used in measuring the height of horses; the Fathom, in measuring depths at sea. The mile of the Table (5280 feet) is the land mile recognized by law in the United States and England, and is therefore distinguished as the Statute Mile. The land league consists of 3 statute miles; the nautical league, of 3 geographical or nautical miles.—A vessel is said to run as many *knots* as she sails geographical miles in an hour.—Rods are sometimes called *poles*, or *perches*.

244. CLOTH MEASURE.—In measuring drygoods, as cloth, muslin, &c., the yard of long measure is used, divided into halves, quarters, eighths, and sixteenths. The sixteenth of a yard, also called a Nail, contains $2\frac{1}{4}$ inches.

242. For what is Long or Linear Measure used? Recite the Table of Long Measure.—243. To what is the Line equal? The Hand? The Span? The Cubit? The Pace? The Fathom? The Geographical Mile? The League? What is the Hand used in measuring? The Fathom? How is the mile of the Table (5280 feet) distinguished? Of what does the land league consist? The nautical league? What is meant by a vessel's running a certain number of *knots*?—244. What is used in measuring drygoods? What is the sixteenth of a yard sometimes called? How many inches in a Nail?

The Ell, which in Flanders consists of 3 qr., in England of 5 qr., and in France of 6 qr., is not used in the United States.

EXAMPLES FOR PRACTICE.

1. Reduce 3534 inches to rods, &c.

```
12) 3534 in.
     3) 294 ft. 6 in.
         98 yd.
          2
   11) 196 half-yd.
         17 rd. 9 hf.-yd.
   17 rd. 4 yd. 1½ ft.
                  ½ ft.
Ans. 17 rd. 4 yd. 2 ft.
```

Divide by 12, to reduce to feet. Divide the quotient by 3, to reduce to yards. Divide the quotient by $5\frac{1}{2}$, or $\frac{11}{2}$, to reduce to rods. To divide by $\frac{11}{2}$, multiply by the fraction inverted $\frac{2}{11}$. Multiplying by 2 reduces the yards to half-yards, and on dividing by 11 we get 17 rd., and 9 half-yards remainder. But 9 hf.-yd $= 4\frac{1}{2}$ yd. $= 4$ yd. $1\frac{1}{2}$ ft. Adding to this the first remainder, 6 inches $= \frac{1}{2}$ ft., we get *Ans.* 17 rd. 4 yd. 2 ft.

After multiplying by $\frac{2}{11}$, therefore, to reduce yards to rods, *if there is a remainder, divide it by 2, to bring it to yards.*

2. Reduce 5 mi. 3 fur. 10 rd. to inches. *Ans.* 342540 in.

Multiply 5 mi. by 8, and add in 3. Multiply this result by 40, and add in 10. Multiply by $5\frac{1}{2}$, by 3, by 12.

3. In 7860 inches how many rods, &c.?
4. In $6\frac{1}{4}$ miles how many feet? $(5280 \times 6\frac{1}{4})$
5. Reduce 6 fur. 5 rd. 1 yd. 2 ft. to inches.
6. Reduce 12012 inches to rods, &c. *Ans.* 60 rd. 3 yd. 2 ft.
7. How many inches in $3\frac{3}{4}$ miles? *Ans.* 237600 in.
8. Reduce 54954 inches to furlongs, &c. *Ans.* 6 fur. 37 rd. 3 yd.
9. Reduce 134507 ft. to miles. Reduce 5000 rd. to miles.
10. How many leagues (§ 243) in 9600 rods? *Ans.* 10 leagues.
11. How many feet high is a horse whose height is 15 hands?
12. How many paces in 1 mile? In 10 rods?
13. Reduce 14640 ft. to mi. *Ans.* 2 mi. 6 fur. 7 rd. 1 yd. 1 ft. 6 in.
14. Reduce 87844 in. to higher denominations.
Ans. 1 mi. 3 fur. 3 rd. 3 yd. 1 ft. 10 in.

How many quarters in the Ell of Flanders or Flemish Ell? In the English Ell? In the French Ell? Is the ell used in the U. S.? Solve Example 1, explaining the steps. After multiplying by $\frac{2}{11}$, to reduce yards to rods, if there is a remainder, what must be done with it?

15. Reduce the following to miles, and add the results: 60720 ft.; 12 fur.; 126720 in.; 8800 yd. *Ans.* 20 miles.

16. Reduce the following to inches, and add the results: 39 rd. 2 ft.; 6 fur. 5 in.; 1 mi. 5 yd. 1 ft. *Ans.* 118823 in.

17. How many times will a wheel 6 ft. around, turn in going 5 miles?

How many feet in 5 miles? How many times are 6 ft. contained therein?

18. In 108 inches how many cubits? (See § 243.) How many spans? How many hands? How many lines?

19. How many feet deep is the water in a certain bay, if soundings show a depth of 140 fathoms?

20. About how many statute miles are 15 geographical miles equal to?

21. How long will it take a vessel running 10 knots to sail 12 nautical leagues? *Ans.* $3\frac{3}{5}$ hours.

22. Sound moves 1120 ft. in a second. How far off is a thunder-cloud, when the clap is heard 11 seconds after the flash is seen? *Ans.* $2\frac{1}{3}$ mi.

23. How many inches long is a piece of muslin containing 44 yd.? How many nails in the same piece? (16 nails = 1 yd.)

24. Bought three pieces of silk containing 37, 38, and 39 yards. How many pieces half a yard long can be cut from them?

25. How many sixteenths in $23\frac{1}{8}$ yards?

26. How many nails in $4\frac{1}{4}$ yards of cloth?

27. What cost 5 yd. 1 nail of cloth, at $6.40 a yard?

1 nail = $\frac{1}{16}$ yd. $6.40 × $5\frac{1}{16}$.

SURVEYORS' MEASURE.

245. A Surveyor is one who measures land. In measuring land, Gunter's Chain (so called after an eminent English mathematician, who invented it) is commonly used. Its length is 4 rods, or 22 yards, and it is divided into 100 links.

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245. What is a Surveyor? In measuring land, what is commonly used? How long is Gunter's Chain?

TABLE.

7.92	inches (in.)	make	1 link, ...	l.
100	links,		1 chain, . . .	ch.
80	chains,		1 mile, ...	mi.

mi.		ch.		l.		in.
				1	=	7.92
		1	=	100	=	792
1	=	80	=	8000	=	63360

Links may be written decimally, as hundredths of a chain. 4 ch. 32 l. = 4.32 ch.

246. 1 chain = 4 rods. Hence, *to reduce chains and links to rods, write the links as the decimal of a chain*, and multiply by 4. Multiply this result by $5\frac{1}{2}$ to reduce to yards, or by $16\frac{1}{2}$ to reduce to feet.

EXAMPLES FOR PRACTICE.

1. Reduce 40 ch. 25 l. to feet.

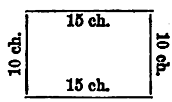
40.25 × 4 = 161 rods = $2656\frac{1}{2}$ ft. *Ans.*

2. Reduce 3 ch. 15 l. to inches. *Ans.* 2494.8 in.

3. A surveyor finds the distance between two bridges to be 340 chains; how many miles apart are they?

4. A farmer runs a fence on each side of a lane 20 chains in length. How many yards of fence does he put up? *Ans.* 880 yd.

5. An oblong field is 15 ch. in length and 10 ch. in width. How many feet long is the fence that encloses it? *Ans.* 3300 ft.



The field has four sides, two of them 15 ch. long, and two 10 ch. long. Find, by addition, the length of all four sides in chains; then reduce to feet.

6. How many rods long is a fence that surrounds an oblong field 12 chains long and 9 chains wide? *Ans.* 168 rd.

7. A man walks round a three-sided field, whose sides measure respectively 10, 8, and 4 chains; how many yards does he walk?

SQUARE MEASURE.

247. Square Measure is used in measuring surfaces; such as land, the walls of rooms, floors, &c.

Recite the Table of Surveyors' Measure. How may links be written?—246. Give the rule for reducing chains and links to rods.—247. In what is Square Measure used?

248. A **Square** is a figure that has four equal sides perpendicular one to another,—that is, leaning no more to one side than the other.

A Square Inch is a square whose sides are each an inch long. A Square Foot is a square whose sides are each a foot long.

A Square Inch.

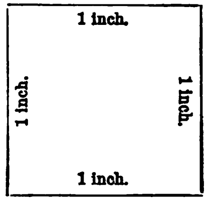


Table.

144	square inches (sq. in.),	1 square foot,	sq. ft.
9	square feet,	1 square yard,	sq. yd.
$30\frac{1}{4}$	square yards,	1 square rod,	sq. rd.
40	square rods,	1 rood, ...	R.
4	roods,	1 acre, ...	A.
640	acres,	1 square mile,	sq. mi.

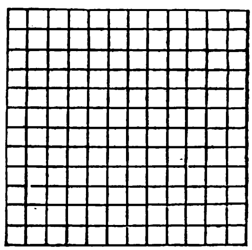
sq. mi.	A.	R.	sq. rd.	sq. yd.	sq. ft.	sq. in.
					1 =	144
				1 =	9 =	1296
			1 =	$30\frac{1}{4}$ =	$272\frac{1}{4}$ =	39204
		1 =	40 =	1210 =	10890 =	1568160
	1 =	4 =	160 =	4840 =	43560 =	6272640
1 =	640 =	2560 =	102400 =	3097600 =	27878400 =	4014489600

249. Stone-cutters often estimate their work by the square foot; plasterers and pavers, by the square yard.

250. 12 inches make a foot, but 144 *square* inches make a *square* foot. Why?—Look at the figure on the right. Suppose each of its sides to be 1 foot long; it will then represent a square foot. Each side is divided into 12 equal parts, representing inches. By drawing lines across the figure from the inch divisions, we form a number of small squares, each of which represents 1 square inch. It will be seen that the 1 sq. ft. contains 12 rows of 12 square inches each, making in all 144 sq. in.

1 foot=12 inches.

1 foot = 12 inches.



So, 1 yd. = 3 ft. Then 1 sq. yd. = 3×3 (9) sq. ft.
1 rd. = $5\frac{1}{2}$ yd. Then 1 sq. rd. = $5\frac{1}{2} \times 5\frac{1}{2}$ ($30\frac{1}{4}$) sq. yd.

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248. What is a Square? What is a Square Inch? A Square Foot? Recite the Table of Square Measure.—249. How do stone-cutters, plasterers, and pavers often estimate their work?—250. Show why it is that 144 square inches make 1 square foot.

251. Roods and acres have no corresponding denomination in linear measure; hence we do not say *square* roods or *square* acres.—A square rod is also called a *pole* or *perch* (P.); and a square mile of land, a *section*. A *township* is a subdivision of a county, containing 36 square miles or sections.

252. The space contained in a surface is called its Area, or Superficial Contents. To find the area of a four-sided figure whose sides are perpendicular one to another, *multiply the length by the breadth.*

The length and breadth must be in the same denomination, and the answer will be in the corresponding denomination of square measure.

Thus, in the figure, the length is 12 in., the breadth 12 in.; the area is 12×12 sq. in. A length of 12 in. and breadth of 2, give an area of 12×2 sq. in., as will be seen by counting the squares in the two uppermost rows of the figure. A length of 12 in. and breadth of 3, make an area of 12×3, or 36, sq. in., &c.

253. Surveyors, taking the dimensions of land in chains, on multiplying the length and breadth together, get the area in square chains, 10 of which make an acre. Hence, *to reduce square chains to acres, divide by* 10.

EXAMPLES FOR PRACTICE.

1. Reduce 10638 sq. ft. to square rods, &c.

Divide by 9, to reduce to sq. yds. Divide the quotient by $30\frac{1}{4}$, or $\frac{121}{4}$, to reduce to sq. rods. To divide by $\frac{121}{4}$, multiply by the fraction inverted $\frac{4}{121}$. Multiplying by 4 reduces the sq. yds. to quarters of a sq. yd., and on dividing by 121 we get 39 sq. rods, and 9 quarters of a sq. yd. remainder. Reduce the remainder to sq. yards by dividing by 4.

After multiplying by $\frac{4}{121}$, therefore, to reduce square yards to square rods, *if there is a remainder, divide it by 4, to bring it to square yards.*

```
     9)10638 sq. ft.
       1182 sq. yd.
          4
  121)4728(39 sq. rd.
      363
      1098
      1089
     4)9 quarter-sq.-yd.
       2¼ sq. yd.
```

Ans. 39 sq. rd. $2\frac{1}{4}$ sq. yd.

2. Reduce 1793664 sq. in. to roods. *Ans.* 1 R. 5 P. $22\frac{3}{4}$ sq. yd.

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251. Why do we not say *square roods* or *square acres*? What is a square rod also called? What is a Section? What is a Township?—252. What is meant by the Area or Superficial Contents of a surface? Give the rule for finding the area of a four-sided figure whose sides are perpendicular one to another. What will be the denomination of the answer? Apply this rule in the given example.—253. How many square chains make an acre? Give the rule for reducing square chains to acres.

3. Reduce 3 A. 27 sq. rd. to square inches. *Ans.* 19876428 sq. in.

4. Reduce 1118448 sq. in. to sq. rods. *Ans.* 28 sq. rd. 16 sq. yd.

5. In 3 sq. mi. how many perches?

6. How many acres in $1\frac{1}{2}$ sections (§ 251)?

7. How many acres in a township (§ 251)?

8. Reduce 262683 sq. ft. to acres, &c. *Ans.* 6 A. 4 P. 26 sq. yd.

9. Reduce 45 A. 3 R. 21 P. to poles (§ 251). *Ans.* 7341 P.

10. How many sq. yards in a garden 5 rd. long by 4 rd. wide?
See § 252. 5 rd. × 4 rd. = 20 sq. rd. Reduce 20 sq. rd. to sq. yards.

11. How many sq. yards in a court, 20 ft. long, 18 ft. wide?

12. A piece of land is 45 chains in length and 30 in breadth. How many acres does it contain (§ 253)? *Ans.* 135 A.

13. How many acres in a field, 40 rd. long, 24 rd. wide? *Ans.* 6.

14. How many square rods in a garden 100 feet by 90?

15. In a tract measuring 60 chains in length and 53.50 chains in width, how many acres? *Ans.* 321 A.

16. How many square yards of oil-cloth will be required to cover an office 18 feet by 14 feet?

17. How many yards of yard-wide carpeting will be needed to cover a room 27 feet by 16 feet? *Ans.* 48 yd.

18. At 35 cents a square yard, what will it cost to plaster a wall 15 feet high and 54 feet long? *Ans.* $31.50.

19. What will be the cost of a piece of land 80 rods square, at $45.50 an acre? *Ans.* $1820.

CUBIC MEASURE.

254. Cubic Measure is used in measuring bodies, which have length, breadth, and depth or thickness; as stone, timber, earth, boxes, &c.

255. A Cube is a body bounded by six equal squares.

A Cubic Inch is a cube, one inch long, one inch broad, and one inch thick. Each of its six sides, or *faces*, is a square inch.

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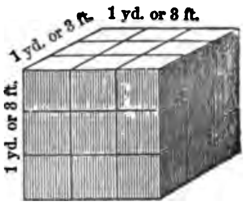
254. In what is Cubic Measure used?—255. What is a Cube? What is a Cubic Inch?



The engraving represents a Cubic Yard. It is 1 yard, or 3 feet, in length, breadth, and depth. It will be seen that each of its six faces is 1 square yard, or 9 (3×3) square feet.

The top of this cube contains 9 square feet. Hence, if it were only 1 foot deep, it would contain 9 cubic feet. As it is 3 feet deep, it contains 3 times 9, or 27, cubic feet. Hence 27 cubic feet make 1 cubic yard.

So, $12 \times 12 \times 12$, or 1728, cubic inches make 1 cubic foot.

Table.

1728	cubic inches (cu. in.),	1 cubic foot,	cu. ft.
27	cubic feet,	1 cubic yard,	cu. yd.
40	cu. ft. of round, or	1 ton or load,	T.
50	cu. ft. of hewn timber,		
16	cubic feet,	1 cord foot, . .	cd. ft.
8	cord feet,	1 cord, ...	Cd.

Cd.		cd. ft.		cu. yd.		cu. ft.		cu. in.
						1	=	1728
				1	=	27	=	46656
		1	=	—	=	16	=	27648
1	=	8	=	—	=	128	=	221184

256. The ton in this Table is a measured ton; the avoirdupois ton is a ton of weight. Round timber is wood in its natural state. A ton of round timber consists of as much as, when hewn, will make 40 cubic feet.

257. A cord of wood is a pile, 8 ft. long, 4 ft. wide, and 4 ft. high. Multiplying these dimensions together, we find 128 cubic feet in the cord. One foot in length of such a pile is called a cord foot.

258. Cubic Measure is used in estimating the amount of work in solid masonry, in digging cellars, making embankments, &c.

259. The space contained in a cube or other solid is called its Solidity, or Solid Contents. To find the solid contents of a body with six faces perpendicular one to another, *multiply its length, breadth, and depth together.*

What does the engraving represent? How does it show that 27 cubic feet make 1 cubic yard? Recite the Table of Cubic Measure.—256. How does the ton in this Table differ from the avoirdupois ton? What is meant by round timber?—257. What is meant by a cord of wood?—258. What is Cubic Measure often used in estimating? —259. What is meant by Solidity or Solid Contents? Give the rule for finding the solidity of a body with six faces perpendicular one to another.

The dimensions must be in the same denomination, and the answer will be in the corresponding denomination of cubic measure. Thus, let it be required to find the solid contents of a box, 6 ft. long, 4 ft. wide, and 36 inches deep.

36 in. = 3 ft. $6 \times 4 \times 3 = 72$ cu. ft. *Ans.*

EXAMPLES FOR PRACTICE.

1. How many cubic inches in $43\frac{1}{2}$ cu. yd.? *Ans.* 2029536 cu. in.
2. Reduce 264384 cu. in. to cu. yd. *Ans.* 5 cu. yd. 18 cu. ft.
3. How many cubic feet in 120 cords?
4. How many cords of wood in a pile, 25 feet long, 4 feet wide, and 8 feet high?

$25 \times 4 \times 8 = 800$ cu. ft. $800 \div 128 = 6\frac{1}{4}$ Cd. *Ans.*

5. How many cords in a pile of wood, 48 feet long, 4 feet wide, and 10 feet high? *Ans.* 15 Cd.
6. Reduce 56 cubic yards, 26 cubic feet, 943 cubic inches, to cubic inches.
7. What will it cost to dig a cellar, 30 ft. long, 20 ft. wide, and 9 ft. deep, at $62\frac{1}{2}$ cents a cubic yard? *Ans.* $125.

How many cubic feet in the cellar (§ 259)? How many cubic yards? Multiply by the price per cubic yard.

8. At 75 cents a cubic yard, what will it cost to dig a cellar, 36 ft. long, 18 ft. wide, and 10 ft. deep?
9. What will it cost to make an embankment containing 999999 cu. ft. of earth, at 70 cents a cubic yard?
10. At $3.50 a cord, what is the value of a pile of wood, 32 ft. long, 4 ft. wide, and 7 ft. high? *Ans.* $24.50.
11. At £1 5s. a cord, what is the value of a pile of wood, 48 ft. long, $10\frac{2}{3}$ ft. high, and 4 ft. wide? *Ans.* £20.
12. How many cubic inches in $8\frac{3}{4}$ cords of wood?

LIQUID MEASURE.

260. Liquid or **Wine Measure** is used in measuring liquids generally; as, liquors (beer sometimes excepted), water, oil, milk, &c.

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260. In what is Liquid or Wine Measure used?

TABLE.

4 gills (gi.) make	1 pint, ...	pt.
2 pints,	1 quart, ...	qt.
4 quarts,	1 gallon, ...	gal.
$31\frac{1}{2}$ gallons,	1 barrel, ...	bar.
2 barrels (63 gal.),	1 hogshead, . . .	hhd.
2 hogsheads,	1 pipe, ...	pi.
2 pipes,	1 tun, ...	tun.

tun.		pi.		hhd.		bar.		gal.		qt.		pt.		gl.
												1	=	4
										1	=	2	=	8
								1	=	4	=	8	=	32
						1	=	$31\frac{1}{2}$	=	126	=	252	=	1008
				1	=	2	=	63	=	252	=	504	=	2016
		1	=	2	=	4	=	126	=	504	=	1008	=	4032
1	=	2	=	4	=	8	=	252	=	1008	=	2016	=	8064

42 gallons make 1 tierce (tier.); 2 tierces, 1 puncheon (pun.).

261. Liquids are put up in casks of different sizes, called barrels, tierces, hogsheads, puncheons, pipes or butts, and tuns; but these casks seldom contain the exact number of gallons assigned them in the Table. The contents are found by gauging, or actual measurement.—When the barrel is used in connection with the capacity of cisterns, vats, &c., $31\frac{1}{2}$ gallons are meant; in Massachusetts, 32 gallons.

262. The wine gallon of the United States, which is the same as the Winchester wine gallon of England, contains 231 cubic inches. The Imperial gallon, established in Great Britain by act of Parliament in 1825, contains 277.274 cubic inches, or about 1.2 of our wine gallons.

EXAMPLES FOR PRACTICE.

1. Reduce 30 gal. 3 qt. 1 pt. to gills. *Ans.* 988 gi.

Multiply 30 gal. by 4, to reduce them to quarts, and add in 3 qt. Multiply the quarts thus obtained by 2, to reduce them to pints, and add in 1 pt. Multiply the pints thus obtained by 4, to reduce them to gills.

2. Reduce 72 gal. 1 pt. 3 gi. to gills. *Ans.* 2311 gi.
3. Reduce 180024 gi. to hhd., &c. *Ans.* 89 hhd. 18 gal. 3 qt.
4. How many pipes are needed, to hold 23184 pt. of wine?

Recite the Table. How many gallons in a tierce? In a puncheon?—261. Name the casks of different sizes in which liquids are put up. How are their contents found? When the barrel is used in connection with the capacity of cisterns, how many gallons are generally meant? How many in Massachusetts?—262. How many cubic inches does the wine gallon of the United States contain? The Winchester wine gallon of England? The Imperial gallon?

5. How many barrels in 2100 gal.?

As many as $31\frac{1}{2}$ gall. are contained times in 2100 gall. $31\frac{1}{2} = \frac{63}{2}$. Multiply by the divisor inverted, $\frac{2}{63}$. Multiplying by 2 reduces the gallons to half-gallons, and on dividing by 63 there is a remainder of 42 half-gallons, which we divide by 2, to reduce them to gallons.

$$2100 \times 2 = 4200$$
$$4200 \div 63 = 66, \text{ 42 rem.}$$
$$42 \div 2 = 21$$

Ans. 66 bar. 21 gal.

6. How many quarts in $3\frac{1}{2}$ hogsheads?

7. How many pints in 1 tierce, of 42 gallons?

8. How many gills in 1 hd. holding 61 gall. 3 qt. 1 pt.?

9. How many pints in 3 tuns?

10. What cost 15 gal. of kerosene, at 20 c. a qt.? *Ans.* \$12.

11. What cost 24 qt. of wine, at \$5.50 a gal.? *Ans.* \$33.

12. What cost 32 qt. of oil, at 9s. a gal.? *Ans.* £3 12s.

13. How many quart bottles can be filled from a puncheon of rum? *Ans.* 336 bottles.

14. How many gallons will a cistern hold that has a capacity of 10 barrels?

15. Reduce the following to gills, and add the results: 15 gal. 1 pt.; 19 gal. 3 qt.; $1\frac{3}{4}$ pt. *Ans.* 1123 gills.

16. Reduce the following to gallons, and add the results: 740 qt.; 608 gi.; 312 pt. *Ans.* 243 gal.

17. A milkman mixes a gill of water with every pint of milk. How many gallons will he thus make out of 48 quarts of pure milk? *Ans.* 15 gal.

BEER MEASURE.

263. Beer Measure was formerly employed in measuring beer and milk. It is now but little used, wine measure having for the most part taken its place.

TABLE.

2	pints (pt.)	make	1 quart, ...	qt.
4	quarts,		1 gallon, ...	gal.
36	gallons,		1 barrel, ...	bar.
$1\frac{1}{2}$	barrels (54 gal.),		1 hogshead, . . .	hhd.

263. In what was Beer Measure formerly employed? What is said of its use at the present day? Recite the Table of Beer Measure.

						qt.		pt.
				gal.		1	=	2
		bar.		1	=	4	=	8
hhd.		1	=	36	=	144	=	288
1	=	$1\frac{1}{2}$	=	54	=	216	=	432

The beer gallon contains 282 cubic inches. The gallon, quart, and pint of this measure, are therefore greater than those of Wine Measure. 1 gal. beer measure = $1\frac{17}{77}$ gal. wine measure.

EXAMPLES FOR PRACTICE.

1. Reduce $3\frac{1}{2}$ hhd., beer measure, to quarts.
2. How many quarts in 5 barrels, beer measure?
3. Reduce 9640 pt. to barrels, beer measure.
4. At 7 c. a quart, what cost 5 bar. of beer? *Ans.* $50.40.
5. What costs 1 hhd. of porter, at 12 c. a qt.? *Ans.* $25.92.
6. If a barrel of ale costs $11.52, what is the cost per pt.?
7. One third of a hhd. of porter has leaked out. How many quart bottles can be filled from what remains? *Ans.* 144.
8. If a man buys a barrel of beer for $8.75, and retails it at 9 c. a quart, how much does he make? *Ans.* $4.21.

DRY MEASURE.

264. Dry Measure is used in measuring grain, seeds, vegetables, roots, fruit, salt, coal, and other articles not liquid.

TABLE.

2 pints (pt.) make	1 quart, ...	qt.
8 quarts,	1 peck, ...	pk.
4 pecks,	1 bushel, ...	bu.
36 bushels,	1 chaldron, ...	chal.

						qt.		pt.
				pk.		1	=	2
		bu.		1	=	8	=	16
chal.		1	=	4	=	32	=	64
1	=	36	=	144	=	1152	=	2304

How many cubic inches does the beer gallon contain? How many wine gallons does 1 beer gallon equal?—264. In what is Dry Measure used? Recite the Table.

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265. The U. S. standard bushel is the Winchester bushel of Great Britain, which contains 2150.42 cubic inches.

1 qt. of Dry Measure = $1\frac{1}{6}$ qt. nearly of Wine Measure.—What is called the Small Measure contains 2 quarts.

266. Foreign coal is imported by the chaldron. American coal is bought and sold, in large quantities, by the ton; in small quantities, by the bushel.

EXAMPLES FOR PRACTICE.

1. Reduce 23 bu. 2 pk. 7 qt. to pints. *Ans.* 1518 pt.
2. Reduce 18564 pt. to bushels, &c. *Ans.* 290 bu. 2 qt.
3. How many pecks in 42 chaldrons?
4. Reduce 15 bu. 6 qt. to pints.
5. How many small measures in 25 bushels?
6. At 9 cents a quart, what will a bushel of peaches cost?
7. How much will a grocer make on 14 bushels of potatoes, if he buys them at 75 cents a bushel, and retails them at 12 cents a half peck? *Ans.* $2.94.
8. Reduce the following to pints, and add the results: 7 qt.; 5 bu. 3 pk.; 2 pk. 6 qt. *Ans.* 426 pt.
9. Reduce the following to pecks, and add the results: 14 chal.; 240 pt.; 19 bu.; 136 qt. *Ans.* 2124 pk.
10. How many barrels, holding $2\frac{1}{4}$ bushels each, will 40 chaldrons of coal fill?
11. Reduce 1879 bu. 3 pk. to quarts.

TIME.

267. The natural divisions of time are the year and the day. The year is the period in which the Earth makes one revolution round the Sun; the day, that in which it makes one revolution on its axis.

The year is divided into twelve calendar months; the day, into hours, minutes, and seconds.

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265. What is the standard bushel of the U. S.? How many cubic inches does it contain? How many wine quarts does a quart of dry measure equal? What is the Small Measure?—266. How is foreign coal imported? How is American coal bought and sold?—267. Name the natural divisions of time. What is the year? What is the day? Into what is the year divided? Into what is the day divided?

TABLE.

60 seconds (sec.) make	1 minute,	. . .	min.
60 minutes,	1 hour,	. . . .	h.
24 hours,	1 day,	. . . . .	da.
7 days,	1 week,	. . . .	wk.
365 days or 12 calendar months,	1 year,	. . . .	yr.
366 days,	1 leap year.		
100 years,	1 century,	. . .	cen.

yr.		wk.		da.		h.		min.		sec.
								1	=	60
						1	=	60	=	3600
				1	=	24	=	1440	=	86400
		1	=	7	=	168	=	10080	=	604800
1	=	$52\frac{1}{7}$	=	365	=	8760	=	525600	=	31536000

268. The twelve calendar months, with the number of days they contain, are as follows:—

				DAYS.					DAYS.
1st	mo.	January	(Jan.)	31.	7th mo.	July	(July)	31.
2d	mo.	February	(Feb.)	28.	8th mo.	August	(Aug.)	31.
3d	mo.	March	(Mar.)	31.	9th mo.	September	(Sept.)	30.
4th	mo.	April	(Apr.)	30.	10th mo.	October	(Oct.)	31.
5th	mo.	May	(May)	31.	11th mo.	November	(Nov.)	30.
6th	mo.	June	(June)	30.	12th mo.	December	(Dec.)	31.

269. The days in these months, added together, make 365 days in the year. But the solar year exceeds this by nearly six hours, its exact length being 365 days 5 h. 48 min. 49.7 sec. To cover this excess, every fourth year (except three in four centuries) is made a Leap Year of 366 days, the additional day being placed at the end of February, the shortest month, which then contains 29 days. Leap Year is also called Bissextile.

Every year that can be divided by 4 without remainder, as 1868, 1872, 1876, is a leap year, except the years that are multiples of 100 and are

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Recite the Table.—268. Name the twelve calendar months in order, with the number of days they contain.—269. How many days in these twelve months? What is the exact length of the solar year? What provision is made for covering the difference between the common and the solar year? What other name is applied to Leap Year? What years are leap years?

not exactly divisible by 400. The year 1900 will not be a leap year, but 2000 will be.

270. In business calculations, 30 days are generally allowed to the month. In common language, the term *month* is often applied to an interval of 4 weeks.

The following lines will help the pupil to remember the number of days in each calendar month :—

"Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Except February alone;
Which has but four and twenty-four,
And every leap year one day more."

271. The following Table will be found useful :—

TABLE,

SHOWING THE NUMBER OF DAYS FROM ANY DAY OF ONE MONTH TO THE SAME DAY OF ANY OTHER MONTH WITHIN A YEAR.

FROM ANY DAY OF	TO THE SAME DAY OF											
	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
JANUARY..	365	31	59	90	120	151	181	212	243	273	304	334
FEBRUARY.	334	365	28	59	89	120	150	181	212	242	273	303
MARCH ...	306	337	365	31	61	92	122	153	184	214	245	275
APRIL.....	275	306	334	365	30	61	91	122	153	183	214	244
MAY	245	276	304	335	365	31	61	92	123	153	184	214
JUNE	214	245	273	304	334	365	30	61	92	122	153	183
JULY	184	215	243	274	304	335	365	31	62	92	123	153
AUGUST...	153	184	212	243	273	304	334	365	31	61	92	122
SEPTEMBER.	122	153	181	212	242	273	303	334	365	30	61	91
OCTOBER...	92	123	151	182	212	243	273	304	335	365	31	61
NOVEMBER.	61	92	120	151	181	212	242	273	304	334	365	30
DECEMBER.	31	62	90	121	151	182	212	243	274	304	335	365

EXAMPLE.—How many days from Nov. 6, 1865, to the 15th of the following April?—Find November in the vertical column on the left, and April over the top. At the intersection of these two lines we find 151, which is the number of days from November 6, 1865, to April 6, 1866. To April 15 will be 9 more days; $151 + 9 = 160$, the number of days required.

One more day than is given in the above Table must be allowed for intervals embracing the end of February falling in a leap year.

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270. In business calculations, how many days are generally allowed to the month? To what is the term *month* often applied in common language?—271. What does the Table show? Give an example, to illustrate its use.

EXAMPLES FOR PRACTICE.

1. Reduce 9 yr. 3 da. 59 min. to seconds. *Ans.* 284086740 sec.

2. Reduce 63142980 sec. to years, &c. *Ans.* 2 yr. 19 h. 43 min.

3. How many seconds in a solar year (§ 269)? *Ans.* 31556929.7

4. How many leap years from the year 1800 to 1900?

5. How many days from Apr. 14, 1865, to Dec. 31, 1865? (See Table.) To October 9, 1865? To Aug. 29, 1865?

6. When $4\frac{1}{4}$ hours of a day have passed, how many seconds remain?

7. How much time will a person waste in a year, who wastes ten minutes every day? *Ans.* 2 da. 12 h. 50 min.

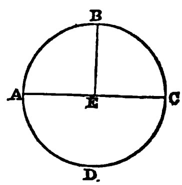
8. If a clock loses 3 sec. every hour, how many minutes too slow will it be at the end of a week? *Ans.* 8 min. 24 sec.

9. Find the length in days, &c., of the lunar month, which contains 2551443 seconds. *Ans.* 29 da. 12 h. 44 min. 3 sec.

10. If a person's income is 1 c. a minute, what will it amount to in the months of June, July, and August? *Ans.* $1324.80.

CIRCULAR MEASURE.

272. Circular Measure is used in connection with angles and parts of circles.



273. A Circle is a figure bounded by a curve, every point of which is equally distant from a point within, called the Centre.

The Circumference of a circle is the curve that bounds it. A Diameter is a straight line drawn through the centre, terminating at both ends in the circumference. A Radius (plural *radii*) is a straight line drawn from the centre to the circumference, and is equal to half the diameter.

The Figure represents a Circle: ABCD is the circumference; E, the centre; AC, the diameter; EA, EB, EC, are radii.

An Angle is the difference in direction of two straight lines that meet.

A Right Angle is an angle made by one straight line meeting another in such way as to make the two adjacent angles equal,—that is, so as to incline no more to one side than the other. In the above Figure, BEA and BEC are right angles.

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272. With what is Circular Measure used in connection?—273. What is a Circle? What is the Circumference of a Circle? What is a Diameter? What is a Radius? What is an Angle? What is a Right Angle?

274. Every circle may be divided into 360 equal parts, called Degrees. The actual length of the degree will of course depend on the size of the circle. A degree is divided into 60 equal parts, called Minutes; and a minute into 60 equal parts, called Seconds.

TABLE.

60 seconds (″)	make	1 minute, . . .	′
60 minutes,		1 degree, . . .	°
30 degrees,		1 sign, ...	S.
12 signs,		1 circle, ...	C.

						1′	=	60″
		S.		1°	=	60	=	3600
C.		1	=	30	=	1800	=	108000
1	=	12	=	360	=	21600	=	1296000

275. The Sign is used only in Astronomy.—1 minute of the circumference of the earth constitutes a geographical or nautical mile, which, as we have seen, is about $1\frac{3}{20}$ statute miles.

EXAMPLES FOR PRACTICE.

1. How many seconds in $\frac{1}{4}$ of a circle? *Ans.* 324000″.
2. Reduce 40° 41′ 42″ to seconds. *Ans.* 146502″.
3. Reduce 251989″ to degrees, &c. *Ans.* 69° 59′ 49″.
4. How many minutes in two signs?
5. How many geographical miles in 5° of latitude?

276. PAPER.

24 sheets	make	1 quire.
20 quires,		1 ream.
2 reams,		1 bundle.
5 bundles,		1 bale.

						quire.		sheets.
				ream.		1	=	24
		bundle.		1	=	20	=	480
bale.		1	=	2	=	40	=	960
1	=	5	=	10	=	200	=	4800

274. Into what may every circle be divided? How is a degree divided? How is a minute divided? Recite the Table of Circular Measure.—275. In what alone is the Sign used? What does 1 minute of the circumference of the earth constitute? —276. Recite the Table used in connection with paper.

277. COLLECTIONS OF UNITS.

12 units	make	1 dozen, doz.
12 dozen,		1 gross.
12 gross,		1 great gross.
20 units,		1 score.

great gross.		gross.		doz.		units.
				1	=	12
		1	=	12	=	144
1	=	12	=	144	=	1728

EXAMPLES FOR PRACTICE.

1. How many sheets in 10 bundles of paper?

2. If paper is \$6 a ream, what does it cost a quire?

3. A bookseller bought 10 reams of paper, at \$2¼ a ream; he retailed it at 1 cent a sheet. What was his gain? *Ans.* \$23.

4. How many reams of paper will be needed for 1000 books, if each book requires a dozen sheets? *Ans.* 25 reams.

5. If a score of boys have each 5 boxes of pens, containing a gross apiece, how many pens have they in all?

6. A tailor uses 13 dozen buttons out of a great gross; how many buttons has he left?

7. If a stationer manufactures 48 dozen copy-books a day, excluding Sundays, how many great gross will he make in fifty-two weeks? *Ans.* 104 great gross.

Reduction of Denominate Fractions, Common and Decimal.

278. A Common Fraction or Decimal is called **Denominate** when it is used in connection with a denomination; as, £½, .25 oz.

279. Denominate Fractions, whether common or decimal, are reduced, like integers, *to lower denominations by multiplication, to higher denominations by division.*

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277. Recite the Table relating to collections of units.—278. When is a common fraction or decimal called *denominate*?—279. How are denominate fractions reduced to lower denominations? To higher denominations?

280. CASE I.—*To reduce one denominate fraction to another of a lower denomination.*

EXAMPLE.—Reduce $\frac{1}{112}$ gall. to the fraction of a gill.

This is a case of Reduction Descending. Multiply the given fraction by 4 (since 4 qt. = 1 gal.); by 2 (2 pt. = 1 qt.); by 4 (4 gi. = 1 pt.). Cancel such factors as are common, and multiply together those that are left.

$$\frac{1}{\cancel{112}} \times \frac{\cancel{4}}{1} \times \frac{2}{1} \times \frac{\cancel{4}}{1} = \frac{2}{7}$$
$$\cancel{28}\ 7$$

Ans. $\frac{2}{7}$ gill.

RULE.—*Multiply the given fraction by the number or numbers that connect its denomination with that of the required fraction.*

EXAMPLES FOR PRACTICE.

1. Reduce $\frac{1}{80000}$ ton to the fraction of an oz. *Ans.* $\frac{2}{5}$ oz.
2. Reduce £$\frac{1}{1452}$ to the fraction of a penny. *Ans.* $\frac{20}{121}$d.
3. What fraction of a pint is $\frac{11}{720}$ of a bushel? *Ans.* $\frac{44}{45}$ pt.
4. What part of a sq. foot is $\frac{9}{968000}$ acre? *Ans.* $\frac{81}{200}$ sq. ft.
5. What part of an inch is $\frac{3}{200000}$ of a mile? *Ans.* $\frac{594}{625}$ in.
6. What part of a second is $\frac{1}{840000}$ of a week? *Ans.* $\frac{18}{25}$ sec.
7. What part of a quire is $\frac{12}{625}$ of a bundle of paper?
8. Reduce $\frac{5}{1944}$ of a pound to the fraction of a scruple.

281. CASE II.—*To reduce a denominate fraction to whole numbers of lower denominations.*

EXAMPLE.—Reduce $\frac{2}{3}$ of a bushel to pecks, &c.

To reduce bushels to pecks, multiply by 4. Multiplying the numerator of the fraction by 4 and dividing the product by its denominator, we get $2\frac{2}{3}$ pk. Reduce the fraction, $\frac{2}{3}$ pk., to quarts. Multiplying its numerator by 8 and dividing by its denominator, we get $5\frac{1}{3}$ qt. Reduce the fraction, $\frac{1}{3}$ qt., to pints. Multiplying its numerator by 2 and dividing by its denominator, we get $\frac{2}{3}$ pt. Collect the integers in the several quotients, and the last fraction, for the answer.

```
  2
  4
3)8
  2 pk. | 2 rem.
          8
        3)16
          5 qt. | 1 rem.
                  2
                3)2
                  0⅔ pt.
```

Ans. 2 pk. 5 qt. $\frac{2}{3}$ pt.

280. What is the first Case of the reduction of denominate fractions? Solve the given example. Recite the rule.—281. What is Case II.? Go through the given example.

RULE.—*Multiply the numerator of the given fraction by the number that will reduce it to the next lower denomination, and divide by its denominator. If there is a remainder, multiply and divide it in the same way; and proceed thus to the lowest denomination. Collect the integers and the last fraction, if any, for the answer.*

EXAMPLES FOR PRACTICE.

Reduce the following to integers of lower denominations:—

1. $\frac{3}{5}$ of a pound Troy. *Ans.* 7 oz. 4 pwt.
2. $\frac{3}{4}$ of a sign. *Ans.* 22° 30′.
3. $\frac{11}{15}$ of a cubic yard. *Ans.* 19 cu. ft. 1382$\frac{2}{5}$ cu. in.
4. $\frac{9}{10}$ of a bar. (beer measure). *Ans.* 32 gal. 1 qt. 1$\frac{1}{5}$ pt.
5. $\frac{4}{7}$ mile (surveyors' measure). *Ans.* 45 ch. 71 li. 3.39$\frac{3}{7}$ in.
6. $\frac{5}{8}$ of a great gross. *Ans.* 7 gross 6 dozen.
7. $\frac{1}{125}$ of a hundred-weight. *Ans.* 12 oz. 12$\frac{4}{5}$ dr.
8. $\frac{3}{25}$ of a long ton. *Ans.* 268 lb. 12 oz. 12$\frac{4}{5}$ dr.
9. $\frac{3}{5}$ of a furlong.
10. $\frac{5}{8}$ of a shilling.
11. How many acres, &c., in a piece of land $\frac{1}{2}$ mile long and $\frac{4}{9}$ of a mile wide? *Ans.* 142 A. 35$\frac{5}{9}$ sq. rd.

Area = $\frac{1}{2} \times \frac{4}{9} = \frac{2}{9}$ sq. mi. Reduce $\frac{2}{9}$ sq. mi. to acres, &c.

12. Required the solid contents of a block of stone, 2$\frac{1}{4}$ yd. long, 1$\frac{1}{5}$ yd. wide, $\frac{2}{3}$ yd. thick. *Ans.* 1 cu. yd. 21 cu. ft. 1036$\frac{4}{5}$ cu. in.

282. CASE III.—*To reduce one denominate fraction to another of a higher denomination.*

EXAMPLE.—Reduce $\frac{2}{7}$ of a gill to the fraction of a gallon.

This is a case of Reduction Ascending. Divide the fraction: that is, multiply its denominator by 4 (since 4 gi. = 1 pt.); by 2 (2 pt. = 1 qt.); by 4 (4 qt. = 1 gall.). Cancel 2; multiply the remaining factors.

$$\frac{\cancel{2}}{7 \times 4 \times \cancel{2} \times 4} = \frac{1}{112}$$

Ans. $\frac{1}{112}$ gall.

Under Case I. we reduced $\frac{1}{112}$ gall. to $\frac{2}{7}$ gill. Here we have reduced

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Recite the rule for reducing a denominate fraction to whole numbers of lower denominations.—282. What is Case III.? Solve the given example. How may it be proved?

$\frac{2}{7}$ gill to $\frac{1}{112}$ gall. Hence the operations in Case I. and Case III. prove each other.

RULE.—*Divide the given fraction by the number or numbers that connect its denomination with that of the required fraction.*

EXAMPLES FOR PRACTICE.

1. Reduce $\frac{2}{3}$ of a rod to the fraction of a league. *Ans.* $\frac{1}{1440}$ lea.
2. Reduce $\frac{7}{15}$ pt. to the fraction of a puncheon. *Ans.* $\frac{1}{1440}$ pun.
3. Reduce $\frac{1}{3}$ fathom to the fraction of a mile. *Ans.* $\frac{1}{2640}$ mi.
   $\frac{1}{3}$ fathom $= 2$ feet $\frac{2}{1} \times \frac{1}{3} \times \frac{2}{11} \times \frac{1}{40} \times \frac{1}{8} = \frac{1}{2640}$
4. What part of a guinea is $\frac{1}{5}$ of a crown? *Ans.* $\frac{1}{21}$ guin.
5. What part of an eagle is $\frac{1}{2}$ of a dime? *Ans.* $\frac{1}{200}$ E.
6. What part of a long ton is $\frac{3}{7}$ of a pound?
7. What part of a pound is $\frac{7}{20}$ of a scruple?
8. What part of a circle is $\frac{3}{8}$ of a second?
9. What part of a piece of 40 yards is a nail of cloth?
   1 nail $= \frac{1}{16}$ yd. $\frac{1}{16} \times 40 = \frac{1}{640}$ *Ans.*
10. What part of 20 gallons is $\frac{10}{11}$ of a pint? *Ans.* $\frac{1}{176}$.
11. What part of a five-acre lot is $\frac{5}{6}$ of a perch? *Ans.* $\frac{1}{960}$.
12. What part of the month of Aug. is $\frac{7}{13}$ min.? *Ans.* $\frac{7}{580320}$.

283. CASE IV.—*To reduce one denominate number to the fraction of another.*

EXAMPLE I.—Reduce 16s. 6d. 2 far. to the fraction of a pound.

Reduce 16s. 6d. 2 far. to farthings, the lowest denomination mentioned: 16s. 6d. 2 far. $=$ 794 far.
Reduce £1 to the same denomination: £1 $=$ 960 far.
794 far. $= \frac{794}{960}$ of 960 far.
Reduce this fraction to its lowest terms. £$\frac{794}{960}$ $=$ £$\frac{397}{480}$ *Ans.*

EXAMPLE II.—Reduce 20 rods $2\frac{1}{2}$ yards to the fraction of a mile.

If the lowest denomination given contains $\frac{1}{2}$, we must reduce both numbers to halves of that denomination; if it contains thirds, to thirds,

---

Give the rule for reducing a denominate fraction to a higher denomination.—283. What is Case IV.? Solve Example I. If the lowest denomination given contains $\frac{1}{2}$, what must we do? If it contains thirds, what must we do? Illustrate this with Example II.

&c. In this example, for instance, we must reduce both numbers to half-yards.

$$20 \text{ rd. } 2\tfrac{1}{2} \text{ yd.} = 225 \text{ half-yards.}$$
$$1 \text{ mile} = 3520 \text{ half-yards.}$$
$$\tfrac{225}{3520} = \tfrac{45}{704} \text{ mile } Ans.$$

RULE.—*Reduce the given numbers to the lowest denomination in either. Of the numbers thus reduced, take the one of which the fraction is required for the denominator, and the other for the numerator.*

EXAMPLES FOR PRACTICE.

Reduce the following; give the fraction in its lowest terms:—

1. 8 bu. 1 pk. to the fraction of a chaldron. *Ans.* $\frac{11}{48}$ chal.
2. 1 oz. 1 pwt. 1 gr. to the fraction of a lb. *Ans.* $\frac{101}{1152}$ lb.
3. $5\frac{2}{3}$ oz. to the fraction of a stone. *Ans.* $\frac{17}{672}$ stone.
4. $3\frac{1}{5}$ cu. ft. to the fraction of a cord. *Ans.* $\frac{1}{40}$ cord.
5. $\frac{1}{4}$ inch to the fraction of a hand. *Ans.* $\frac{1}{16}$ hand.
6. 29 gal. 1 pt. to the fraction of a barrel. *Ans.* $\frac{233}{252}$ bar.
7. 1 English ell to the fraction of 1 French ell. *Ans.* $\frac{5}{6}$ ell Fr.
   Reduce both to the common denomination, quarters.
8. What part of 1 ch. 50 l. is $4\frac{1}{4}$ inches? *Ans.* $\frac{17}{4752}$.
9. What part of 6s. $8\frac{3}{4}$d. is 3s. 5d.? *Ans.* $\frac{164}{323}$.
10. Reduce $5\frac{1}{4}$ hours to the fraction of a leap year.

284. CASE V.—*To reduce a denominate decimal to whole numbers of lower denominations.*

EXAMPLE.—Reduce .471875 lb., apothecaries' weight, to ounces, &c.

This is a case of Reduction Descending. Multiply by 12, to reduce to ounces, pointing off the product as in multiplication of decimals. Reserve the integer, and reduce the decimal to drams by multiplying by 8. Again reserve the integer, and reduce the decimal to scruples by multiplying by 3. There being no integer, multiply this product by 20 to reduce it to grains. Finally, collect the integers in the several products for the answer.

	.471875 lb.
	12
oz.	5 | .662500
	8
dr.	5 | .300000
	3
sc.	.900000
	20
gr.	18.000000

Ans. 5 oz. 5 dr. 18 gr.

Recite the rule for reducing one denominate number to the fraction of another.—284. What is Case V.? Go through the given example, explaining the steps.

RULE.—*Multiply the given decimal by the number that will reduce it to the next lower denomination. Treat the decimal part of the product in the same way, and proceed thus to the lowest denomination. Collect the integers in the several products, with the last decimal, if there is one, for the answer.*

EXAMPLES FOR PRACTICE.

1. Reduce .725 lb. Troy to ounces, &c. *Ans.* 8 oz. 14 pwt.
2. Reduce .4156 cwt. to qr., &c. *Ans.* 1 qr. 16 lb. 8 oz. 15.36 dr.
3. Reduce .75 bale of paper. *Ans.* 3 bundles 1 ream 10 qui.
4. Reduce .9 of a great gross to gross, &c.
5. Reduce .002 bar. of beer to gallons, &c. *Ans.* .576 pt.
6. A lot is 50.3 rd. long, 29.25 rd. wide. What is its area in acres, &c.? *Ans.* 9 A. 31 sq. rd. 8 sq. yd. 2 sq. ft. 125.1 sq. in.

Area = 50.3 × 29.25 = 1471.275 sq. rd. Reduce 1471 sq. rd. to roods and acres. Reduce .275 sq. rd. to square yards, &c. Combine the results.

7. A cistern is 3.25 ft. long and wide, and 10 ft. deep. What is its capacity? *Ans.* 3 cu. yd. 24 cu. ft. 1080 cu. in.
8. A piece of land measures 32.72 ch. by 41.36 ch. Required its area in acres, roods, and perches. *Ans.* 135 A. 1 R. 12 perches +.

Area = 32.72 × 41.36 = 1353.2992 sq. ch. Dividing 1353.2992 sq. ch. by 10 (since 10 sq. ch. = 1 acre), we get 135.32992 acres. Reduce .32992 A. to roods and perches.

9. What is the area of an oblong field, 8.5 chains in length and 5.5 chains in width? *Ans.* 4 A. 2 R. 28 sq. rd.
10. How many degrees, &c., in .01 of a circle?
11. How many days, &c., in .12 of a year?
12. How many roods, &c., in .575 of an acre?
13. How many shillings, &c., in .49 of a pound sterling?

285. CASE VI.—*To reduce a compound number to the decimal of a higher denomination.*

EXAMPLE.—Reduce 5 oz. 5 dr. 18 gr. to the decimal of a pound.

Recite the rule for reducing a denominate decimal to whole numbers of lower denominations.—285. What is Case VI.? Solve the given example.

Begin with the lowest denomination. Reduce 18 gr. to the decimal of a dram, which is the next higher denomination given, by dividing by 60 (since 60 gr. = 1 dr.), annexing as many decimal naughts as may be necessary. Annex the result, .3 dr., to the drams in the given number, and divide by 8, to reduce to the decimal of an ounce. Annex the result, .6625 oz., to the ounces in the given number, and divide by 12, to reduce to the decimal of a pound.

$$
\begin{array}{r}
60\underline{)\,18.0\text{ gr.}} \\
.3\text{ dr.} \\
8\underline{)\,5.3\text{ dr.}} \\
.6625\text{ oz.} \\
12\underline{)\,5.6625\text{ oz.}} \\
\textit{Ans.}\ \ .471875\text{ lb.}
\end{array}
$$

The processes in Case V. and Case VI. prove each other:—
By Case V. .471875 lb. = 5 oz. 5 dr. 18 gr.
By Case VI. 5 oz. 5 dr. 18 gr. = .471875 lb.

RULE.—*Divide the lowest denomination by the number that will reduce it to the next higher denomination in the given number, and annex the decimal quotient to that next higher. Treat this result in the same way, and proceed thus till the required denomination is reached.*

EXAMPLES FOR PRACTICE.

Reduce the following; prove the answers:—

1. 2 gal. 2 qt. 1 pt. to the decimal of a hhd. *Ans.* $.041\dot{6}$ hhd.
2. 3s. 4½d. to the decimal of a pound. *Ans.* £.16875.
3. $5.10 to the decimal of a double eagle. *Ans.* .255.
4. 2 da. 3 h. 4 min. 6 sec. to the decimal of a week.
5. 7 cd. ft. 7 cu. ft. to the decimal of a Cd. *Ans.* .9296875 Cd.
6. 4 yd. 9 in. to the decimal of a rod. *Ans.* $.\dot{7}7\dot{2}$ rd.
7. 9d. 2 far. to the decimal of a crown. *Ans.* $.158\dot{3}$ crown.
8. 2 cwt. 3 lb. to the decimal of a ton.
9. 1 pk. 7 qt. 1 pt. to the decimal of a bushel.
10. 10 ℥ 2 ℈ to the decimal of a pound.
11. 7 dr. 18 gr. to the decimal of an ounce.
12. 8 pwt. 3 gr. to the decimal of an ounce.
13. 16 rd. to the decimal of a mile. *Ans.* .05 mi.
14. 8 in. to the decimal of a fathom. *Ans.* $.\dot{1}$ fathom.
15. 1 lb. 12 oz. to the decimal of a stone. *Ans.* .125 stone.
16. 24 lb. to the decimal of a long ton. *Ans.* .010714 long T.+.

Recite the rule for reducing a compound number to the decimal of a higher denomination.

Miscellaneous Questions.—In what denominations do American merchants keep their accounts? British merchants? What American coin is nearest in value to the British shilling? To the British sovereign? Why is Federal Money so called? Sterling Money?

Recite the three Tables used in connection with weight. For what is Avoirdupois Weight used? Apothecaries'? Troy? In which of these is the pound the greatest? In which is the ounce the greatest? Is the avoirdupois dram greater or less than the dram of apothecaries' weight? Is the grain of apothecaries' weight greater or less than the Troy grain? How many pennyweights is the dram of apothecaries' weight equal to?

What measure is used in reckoning distances? In surveying land? In expressing superficial contents? In expressing solid contents? In estimating the amount of work in solid masonry? In estimating surfaces to be plastered or paved? In measuring drygoods? What are the dimensions of a cord of wood?

What measure is now generally used for liquids? How are the contents of casks ascertained? How many cubic inches in the wine gallon? In the beer gallon? Which is greater, the beer or the wine quart? What is used in measuring grain and fruit? Which is greater, the quart of dry or that of liquid measure? In what two Tables do the second and minute occur? How do the second and minute of Circular Measure differ from those of Time Measure?

286. Miscellaneous Examples.

1. How many ducats, worth 9s. 3d. apiece, are equal in value to £74? *Ans.* 160 ducats.

2. If a cannon-ball could move with uniform velocity 1000 feet a second, how many miles, &c., would it go in a quarter of a minute?

3. How long would this ball be in reaching the sun, which is 95000000 miles from the earth? *Ans.* 5805 da. 13 h. 20 min.

4. A cubic foot of water weighs 1000 oz. What weight of water will a cistern 3 ft. by 4 ft. across, and 10 ft. deep, contain? *Ans.* 75 cwt.

5. Required the area in acres, &c., of an oblong piece of land, .5 miles long and .3 miles broad.

6. If three presses, each capable of striking off 1800 coins an hour, work, the first at quarter-dollars, the second at half-eagles,

and the third at dimes, what will be the whole amount coined in eight hours? *Ans.* $77040.

7. A silversmith, having on hand 20 lb. of silver, uses 4 oz. 18 gr. of it. What decimal is this of the amount he originally had? *Ans.* .0168229+.

Find what decimal it is of 1 lb., § 285; it will be $\frac{1}{20}$ as much of 20 lb.

8. What were the solid contents of the Ark, which was 300 cubits in length, 50 in breadth, and 30 in height—the sacred cubit being 22 inches? *Ans.* 102700 cu. yd. 16 cu. ft. 1152 cu. in.

9. In two dozen bottles, each holding 1.1 qt., how many gallons, &c.? *Ans.* 6 gal. 2 qt. 3.2 gi.

$1.1 \times 24 = 26.4$ qt. Reduce 26 qt. to gallons, and .4 qt. to lower denominations according to § 284.

10. An oblong piece of land measures 14 ch. 5 l. in width, and 36 ch. 24 l. in length. How many acres, roods, and perches, does it contain? *Ans.* 50 A. 3 R. 26.752 P.

11. What part of an acre is an oblong lot 75 feet wide and 150 feet in length? *Ans.* $\frac{125}{484}$ A.

12. What are the solid contents of a block of wood, $\frac{3}{5}$ yd. long, $\frac{3}{7}$ yd. wide, $\frac{2}{3}$ yd. thick? *Ans.* 4 cu. ft. $1086\frac{6}{35}$ cu. in.

13. How many acres, &c., are there in an oblong farm, $\frac{1}{8}$ mi. long, $\frac{3}{7}$ mi. wide?

14. If $\frac{1}{8}$ of a chaldron of coal is consumed daily, how many bushels will be used in a week?

15. If a thread 18 rods long can be spun from an ounce of silk, how many pounds of silk will be required for a thread 90 miles long? *Ans.* 100 lb.

16. Reduce $\frac{3}{4}$ qt. to the decimal of a bushel.

$\frac{3}{4} = .75$ qt. $.75 \div 8 = .09375$ pk. $.09375 \div 4 = .0234375$ bu.

17. Reduce $\frac{1}{8}$ qt. to the fraction of a hhd. *Ans.* $\frac{1}{2016}$ hhd.
To the fraction of a pint. *Ans.* $\frac{1}{4}$ pt.
To lower denominations. *Ans.* 1 gi.
To the decimal of a gallon. *Ans.* .03125 gal.

18. Reduce 1 ʒ $1\frac{4}{5}$ ℈ to the fraction of a lb. *Ans.* $\frac{1}{60}$ lb.
To the decimal of an ounce. *Ans.* .2 oz.

19. Reduce $\frac{1}{40}$ sq. rd. to lower denominations.

CHAPTER XIV.

COMPOUND ADDITION.

287. Compound Addition is the process of uniting two or more compound numbers in one, called their Sum. It combines addition and reduction ascending.

EXAMPLE.—Add 1 lb. 3 oz. 19 pwt. 23 gr.; 2 oz. 15 gr.; 3 lb. 17 pwt.; and 2 lb. 1 oz. 8 pwt. 10 gr.

That we may unite things of the same kind, we write pounds under pounds, ounces under ounces, &c., marking the denominations above.

	lb.	oz.	pwt.	gr.
	1	3	19	23
		2	0	15
	3	0	17	0
	2	1	8	10
Ans.	6	8	6	0

Beginning to add at the right, we find the sum of the grains to be 48. But 48 gr. = 2 pwt. Hence we carry 2 to the pennyweights, and write 0 under the grains.

The sum of the pennyweights, including the 2 carried, is 46. But 46 pwt. = 2 oz. 6 pwt. Write 6 under the pennyweights, and carry 2 to the column of ounces. The sum of the ounces is 8, which, not being reducible to pounds, we write under the ounces. The sum of the pounds is 6, which, not being reducible to any higher denomination, we write under the pounds added. *Ans.* 6 lb. 8 oz. 6 pwt.

288.—Observe that in Simple Addition there is a reduction similar to the above, when we carry. As the orders increase in value tenfold as we go to the left, to reduce to a higher order, we divide the sum of each column by 10. That is, we cut off the right-hand figure, and place it as a *remainder* under the column added; while the left-hand figure or figures, being the *quotient*, we carry to the next column.

289.—RULE.—1. *Write numbers of the same denomination in the same column.*

2. *Beginning at the right, add as in simple numbers. Write each sum under the numbers added, unless it can be reduced to a higher denomination; in which case, divide by the number that it takes to make one of that denomination. Write the remainder under the numbers added, and carry the quotient.*

3. *Prove by adding in the opposite direction.*

---

287. What is Compound Addition? What processes does it combine? Go through the given example, explaining the steps.—288. Show how in Simple Addition there is a similar reduction.—289. Recite the rule for Compound Addition.

290.—If a fraction occurs in the answer, it must be reduced to lower denominations, if there are any, and the result added to the previous sum with the fraction omitted. Thus, in dividing yards by $5\frac{1}{2}$, to reduce them to rods, a remainder containing $\frac{1}{2}$ yd. may occur, as in Example 2. But $\frac{1}{2}$ yd. = 1 ft. 6 in. We therefore add 1 ft. 6 in. to the integers of the answer first obtained.

EXAMPLE 2.

	rd.	yd.	ft.	
Add	5	2	1	
	6	3	2	
	7	1	1	
	19	1|$\frac{1}{2}$|	1	
$\frac{1}{2}$ yd. =			1	6 in.
Ans.	19	1	2	6

291. EXAMPLES FOR PRACTICE.

Add the following compound numbers:—

(1)

£	s.	d.	far.
3	7	8	3
	8	9	1
1	9	11	0
	7	10	2
2	6	10	2
8	1	2	0

(2)

℔.	℥	ʒ	℈	gr.
1	2	7	2	18
	6	5	1	19
	3	1	0	16
4	7	3	1	15
2	4	6	2	11
9	1	1	0	19

(3)

rd.	yd.	ft.	in.
19	5	2	4
2	3	0	9
	5	2	2
4	1	1	7
11	3	1	5
39	3	0	9

(4)

ch.	l.	in.
9	41	$6\frac{1}{10}$
14	9	5
8	57	3
22	16	1
35	82	$4\frac{3}{10}$
90	7	3.56

(5)

sq. rd.	sq. yd.	sq. ft.	sq. in.
6	29	2	93
8	30	7	86
5	18	0	101
3	27	6	79
14	14	8	128
39	30	1	91

(6)

Cd.	cd. ft.
4	2
8	6
32	3
15	7
29	5
90	7

(7)

gal.	qt.	pt.	gi.
1	3	1	3
	2	1	2
5	1	1	1
2	3	1	2

(8)

T.	cwt.	qr.	lb.	oz.
20	17	1	15	9
41	16	0	4	5
16	12	3	13	12
38	18	2	11	13

(9)

stone	lb.	oz.
5	12	4
7	11	7
1	13	2
12	10	3

10. A jeweller buys the following quantities of silver: 8 lb. 6 pwt.; 10 oz. 4 pwt. 21 gr.; 8 oz. 20 gr.; 3 lb. 6 oz. 8 pwt. 7 gr. How much does he buy in all?

---

290. If a fraction occurs in the answer, what must be done?

(11)

cu. yd.	cu. ft.	cu. in.
23	19	1698½
48	22	842⅚
79	8	1257⅓
52	18	208¾
87	14	1265⅕
65	16	1084 23/60
357	19	1173

(12)

mi.	fur.	rd.	yd.	ft.	in.
47	1	29	4	0	6.6
26	5	18	3	2	3.1
59	3	7	5	1	4.9
86	7	26	5	0	11.25
34	6	33	4	1	9.5
92	4	35	4	2	10.75
347	5	33	0	2	4.10

(13)

bu.	pk.	qt.	pt.
14	2	7	0.5
9	3	5	1.1
7	0	3	1.6
6	2	4	0.2
18	1	1	1.5

(14)

wk.	da.	h.	min.	sec.
2	2	22	42	36
	6	19	31	24
3	1	18	28	29
	5	13	19	59
8	4	14	57	57

(15)

°	′	″
8	36	24
4	8	14
6	9	36
3	25	58
2	51	42

16. What are the contents of four hogsheads, the first of which contains 63 gal. 2 qt. 1½ pt.; the second, 60 gal. 3 qt. 1.75 pt.; the third, 62 gal. 1 pt. 3 gi.; the fourth, 61 gal. 2 qt. 2 gi.?

Ans. 248 gal. 1 qt. 1 pt. 2 gi.

17. How much wood in three piles, the first of which contains 10 Cd. 6 cd. ft. 4 cu. ft.; the second, 12 Cd. 12 cu. ft.; the third, 17 Cd. 1 cd. ft.?

18. A surveyor measures four distances; the first he finds to be 40 ch. 59 l. 3 in., the second 28 ch. 43 l. 5 in., the third 16.27 ch., the fourth 12 ch. 7 in. What is the whole distance measured, expressed first in chains, &c., then in the denominations of linear measure?

Ans. 97 ch. 30 l. 7.08 in.: 1 mi. 1 fur. 29 rd. 1 yd. 10.68 in.

19. How many yards in 3 pieces of cloth, containing respectively 24 Ells French 3 qr. 1 in., 28 Ells English 3 qr. 2 nails, 40 Ells Flemish 1 qr. 1 nail 1¼ in.?

2¼ inches make 1 nail; 4 nails, 1 qr. of a yd.; 3 qr., 1 Ell Flemish; 5 qr., 1 Ell English; 6 qr., 1 Ell French. Reduce the ells to the common denomination, quarters; add the whole, and reduce the quarters to yards. *Ans.* 108 yd.

20. Find the sum total in pounds, &c., of the following items: £20 10s., £1 6s. 8d., 5 guineas 10s. 6d., 15 guineas, and £1 15s. 3¼d. *Ans.* £45 2s. 5¼d.

21. A person owning a section of land (§ 251) buys three additional tracts, containing 347 A. 2 R. 27 sq. rd., 201 A. 19 sq. rd., and 417 A. 3 R. 14 sq. rd. How much does he then own in all?

Ans. 2 sq. mi. 326 A. 2 R. 20 sq. rd.

22. How much coke in three carts, the first of which contains 1 chal. 5 bu. 2 pk., the second 1 chal. 5½ bu., and the third 35 bu. 3 pk.?

23. How much beer in four hogsheads, containing respectively 53 gal. 2 qt., 54 gal. 1 qt. 1 pt., 52 gal. 3 qt. 1 pt., and 51 gal. 3 qt. 1 pt.?

24. Add together 6 da. 37 min., 43 da. 5 h. 29 sec., 94 da. 19 h. 18 sec., 126 da. 7 h. 9 min. 8 sec., and 94 da. 16 h. 13 min. 5 sec. How many years in the sum?

25. How many yards in four pieces of cloth, containing respectively 30 yd. 1 qr., 20 Ells Fr. 1 na., 24 Ells En. 1¼ in., 32 Ells Fl. 2 qr. 2 na. ¾ in. *Ans.* 115 yd.

CHAPTER XV.

COMPOUND SUBTRACTION.

292. Compound Subtraction is the process of finding the difference between two numbers, when one or both are compound.

EXAMPLE 1.—From 20 lb. 5 oz. 3 dr. take 18 lb. 7 oz. 1 dr. Write the subtrahend under the minuend, pounds under pounds, &c., marking the denominations above. Begin to subtract at the right. 1 dr. from 3 dr. leaves 2 dr., which we write in the column of drams.

	lb.	oz.	dr.
	20	5	3
	18	7	1
Ans.	1	14	2

7 oz. can not be taken from 5 oz. We therefore take one of the next higher denomination (1 lb.), reduce it to ounces, and add it to the 5 oz.; 16 + 5 = 21. Then subtracting 7 from 21, we get 14, which we write under the ounces.—To balance the 16 oz. added to the minuend, we now

292. What is Compound Subtraction? Go through the given example explaining the steps.

add 1 lb. to the subtrahend, or carry 1 to the next column; 19 lb. from 20 lb., 1 lb. *Ans.* 1 lb. 14 oz. 2 dr.

This process involves the same principle as carrying in Simple Subtraction. In the latter, as the orders uniformly increase in value tenfold, we add 10 to the figure of the minuend when it is necessary, and to balance it carry 1 to the figure of the next higher order in the subtrahend.

293. RULE.—1. *Write the subtrahend under the minuend, placing numbers of the same denomination in the same column. Beginning at the right, subtract as in simple numbers.*

2. *If, in any denomination, the subtrahend exceeds the minuend, add to the latter as many as make one of the next higher denomination. Subtract, and carry* 1 *to the subtrahend in the next higher denomination.*

3. *Prove by adding remainder and subtrahend.*

We may have to carry several times in succession.—If fractions occur, proceed as in subtraction of fractions. Thus, in Example 2, $5\frac{2}{3}$ yd. can not be taken from 4 yd. Add, therefore, to the minuend $5\frac{1}{2}$ yd., which equal 1 rd. $4 + 5\frac{1}{2} = 9\frac{1}{2}$. Subtracting $5\frac{2}{3}$ from the sum, we get $3\frac{5}{6}$ yd., and carrying 1 successively to the columns of rods, furlongs, and miles, we find the remainder to be 0 in each case. *Ans.* $3\frac{5}{6}$ yd.

EXAMPLE 2.

	mi.	fur.	rd.	yd.
From	1	0	0	4
Take		7	39	$5\frac{2}{3}$
Rem.	0	0	0	$3\frac{5}{6}$

If a fraction occurs in any denomination of the remainder, except the lowest, it should be reduced and added, as in Addition, § 290.

294. To find the interval between different dates since the Christian era, *Write the earlier date under the later, representing the month in each by its number* (January, 1; February, 2, &c.). *Subtract, allowing* 30 *days to the month and* 12 *months to the year.*

EXAMPLE 3.—Washington was born Feb. 22, 1732. How old was he July 4, 1776?

Represent July, the seventh month, by 7—and February, the second month, by 2. Thirty days being allowed to the month, we subtract 22 from 30 + 4, and carry 1.

	yr.	mo.	da.
	1776	7	4
	1732	2	22
Ans.	44	4	12

Show how the same principle is involved in carrying in Simple Subtraction.—293. Recite the rule. If fractions occur, how are we to proceed? Illustrate this with the given example.—294. Give the rule for finding the interval between different dates since the Christian era. Apply this rule to Example 3.

EXAMPLES FOR PRACTICE.

(1)

	gal.	qt.	pt.	gi.
From	25	1	1	3
Take	6	0	1	2
Ans.	19	1	0	1

(2)

°	′	″
8	9	11
3	6	34
5	2	37

(3)

bu.	pk.	qt.	pt.
7	3	1	0
4	3	6	1
2	3	2	1

(4)

mi.	fur.	rd.	yd.	ft.	in.	
8	3	25	0	1	8	
5	5	26	1	0	0	
2	5	38	4|½|	1	8	
				1	6	= ½ yd.
2	5	38	5	0	2	*Ans.*

(5)

A.	R.	P.	sq. yd.	sq. ft.	sq. in.	
9	0	30	1	7	26	
3	2	37	30	2	97	
5	1	32	1|¼|	4	73	
				2	36	= ¼ sq. yd.
5	1	32	1	6	109	*Ans.*

(6)

ch.	l.	in.
20	8	3
16	17	$4\frac{1}{2}$
3	90	$6\frac{67}{100}$

(7)

£	s.	d.
6	5	$10\frac{1}{4}$
5	12	$8\frac{3}{4}$
	13	$1\frac{1}{2}$

(8)

hhd.	bar.	gal.	qt.	pt.
4	1	0	1	0
2	1	31*	1	1
1	0	31	3	1

Find the value of the following. Prove each example.

9. 25° 15′ 31″ — 18° 52′ 49″.
10. 20 guineas — 19s. 11d. 2 far. *Ans.* 19 guin. 1s. 2 far.
11. 1 mi. — 47 ch. 94 l. 6½ in. *Ans.* 32 ch. 6 l. 1.42 in.
12. 15 Cd. 4 cd. ft. — 10 Cd. 13⅜ cu. ft.
13. 30 gal. 3 qt. — 24 gal. 1 pt. 2 gi.
14. 200 da. 13 h. 15 sec. — 195 da. 21 h. 49 min.
15. 9 lb. 3 oz. 1 sc. 19 gr. — 8 lb. 2 dr. 2 sc. 6 gr.
16. 2 T. 9 cwt. 3 lb. 4 dr. — 3 qr. 24 lb. 15 oz. 13 dr.
17. 3 wk. 10 h. 11 min. — 1 wk. 6 da. 49 min. 57 sec.
18. 6 lb. 3 oz. 15 pwt. 15 gr. — 4 lb. 10 oz. 18 pwt. 22 gr.
19. 9 sq. mi. 3 sq. rd. 8 sq. ft. — 1 R. 29 sq. yd. 100 sq. in.
20. 11 cu. yd. 111 cu. in. — 8 cu. yd. 20 cu. ft. 1000½ cu. in.

* Wine Measure. Carrying 1, we get 32 gal., which can not be taken from *one* barrel reduced to gallons (31½ gal.). Hence we take *two* barrels, reduce to gallons, subtract, and carry *two*. 31½ × 2 = 63. 63 − 32 = 31. Remember, in such a case, to carry 2.

21. Henry Clay died June 29, 1852, aged 75 yr. 2 mo. 17 days. When was he born? *Ans.* April 12, 1777.

22. Andrew Jackson was born March 15, 1767; he became president March 4, 1829. What was his age at that time?

23. Shakespeare was born April 23, 1564. How long from that time to the first day of the present year?

24. How old was Shakespeare at the time of Milton's birth, December 9, 1608? *Ans.* 44 yr. 7 mo. 16 da.

25. A note dated Dec. 30, 1862, was paid Nov. 3, 1865. How long had it run? *Ans.* 2 yr. 10 mo. 3 da.

26. San Francisco is in 122° 23′ west longitude, Baltimore in 76° 37′ west; what is their difference of longitude? *Ans.* 45° 46′.

27. Longitude of Boston, 71° 3′ 58″ W.; of Rome, 12° 28′ 40″ E. What is their difference of longitude? *Ans.* 83° 32′ 38″.

The one being in *west* long., the other in *east*, to find the diff. of long., add.

28. New York is in 40° 42′ 43″ north latitude; New Orleans, in 29° 58′ N.; Charleston, in 32° 46′ 33″ N. What is the difference of latitude between New York and New Orleans? Between New York and Charleston? Between Charleston and N. O.?

29. Latitude of St. Louis 38° 27′ 28″ N.; of Cape Horn, 55° 58′ 40″ S. What is their difference of latitude? *Ans.* 94° 26′ 8″.

The one being in *north* lat., the other in *south*, to find the diff. of lat., add.

30. A grocer, having on hand 17 cwt. 3 qr. 5 lb. of sugar, buys 5 cwt. 20 lb. more, and then sells 12 cwt. 1 qr. 5 lb. 8 oz. How much has he remaining? *Ans.* 10 cwt. 2 qr. 19 lb. 8 oz.

31. From a piece of cloth containing 37 yd. 1 nail, are cut 6 yd. 3 qr. 2 nails, and afterwards 8 yd. 3 qr. 2 na. 1 in. How much is then left? *Ans.* 21 yd. 1 qr. 1¼ in.

32. Napoleon was born Aug. 15, 1769; Wellington, May 1, 1769. Which was the older, and how much?

33. How old was Napoleon when the battle of Waterloo took place, June 18, 1815? How old was Wellington?

34. A druggist, having bought 1 lb. 8 ℥ of salts, put 4 ℥ 3 ʒ 1 ℈ in one bottle, and 8 ℥ 2 ℈ 19 gr. in another. How much did what was left weigh? *Ans.* 7 ℥ 3 ʒ 2 ℈ 1 gr.

35. From 1 lb. Troy take 10 oz. 17 pwt. 18 gr.

CHAPTER XVI.

COMPOUND MULTIPLICATION.

295. Compound Multiplication is the process of taking a compound number a certain number of times. It combines multiplication and reduction ascending.

EXAMPLE.—Multiply 4 gal. 2 qt. 1 pt. 3 gi. by 36.

	gal.	qt.	pt.	gi.
	4	2	1	3
				36
Ans.	169	3	1	0

Write the multiplier under the lowest denomination of the multiplicand. Begin to multiply at the right.

3 gi. × 36 = 108 gi. = 27 pt. Write 0 in the column of gills, and carry 27. 1 pt. × 36 = 36 pt., and 27 carried makes 63 pt. = 31 qt. 1 pt. Write 1 in the column of pints, and carry 31. 2 qt. × 36 = 72 qt., and 31 carried makes 103 qt. = 25 gal. 3 qt. Write 3 under the quarts, and carry 25. 4 gal. × 36 = 144 gal., and 25 carried makes 169; write it under the gallons. *Ans.* 169 gal. 3 qt. 1 pt.

In stead of multiplying by 36 at once, we may multiply in turn by any factors that will produce 36; as, 4 and 9, or 6 and 6. The product will be the same.

gal.	qt.	pt.	gi.
4	2	1	3
			4
18	3	1	0
			9
169	3	1	0

gal.	qt.	pt.	gi.
4	2	1	3
			6
28	1	0	2
			6
169	3	1	0

296. RULE.—1. *Write the multiplier under the lowest denomination of the multiplicand.*

2. *Beginning at the right, multiply each denomination in turn, and write the product under the number multiplied, unless it can be reduced to a higher denomination. In that case, divide it by the number that it takes to make one of that denomination; write the remainder under the number multiplied, and carry the quotient to the next product.*

297. When Compound Division has been learned, Compound Multiplication is best proved by dividing the product by the multiplier, and seeing whether the multiplicand results.

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295. What is Compound Multiplication? What processes does it combine? Go through the given example, explaining the steps.—296. Recite the rule.—297. How is Compound Multiplication best proved?

298. Multiply by 12 or less in one line. If the multiplier exceeds 12 and is a composite number, it may be best to multiply by its factors. In this case, to prove the result, multiply by the factors in reverse order.

299. If a fraction occurs in the product, it must be reduced to lower denominations, if there are any, and the result added in. See Examples 1 and 2 below.

EXAMPLES FOR PRACTICE.

(1)

	rd.	yd.	ft.	in.
Multiply	3	1	1	8
By				5
	16	1|½|	2	4
½ yd. =			1	6
Ans.	16	2	0	10

(2)

	A.	R.	sq. rd.	sq. yd.	sq. ft.	sq. in.
		3	2	4	6	
					7	
	5	1	15	1|¾|	6	
sq. yd. =					6	108
	5	1	15	2	3	108

(3)

gal.	qt.	pt.	gi.
2	2	1	3
			10

(4)

cwt.	qr.	lb.	oz.	dr.
16	1	23	14	15
				11

(5)

da.	h.	min.	sec.
2	19	47	58
			12

6. Multiply 2° 13′ 12″ by 45.
7. Multiply 12 ch. 15 l. 5.7 in. by 55.
8. Multiply £14 17s. 8d. 3 far. by 56.
9. Multiply 3 sq. mi. 2 R. 15½ P. by 60.
10. Multiply 5 lb. 8 oz. 13 pwt. 19 gr. by 63.
11. Multiply 5 Cd. 3 cd. ft. 3 cu. ft. by 72.
12. Multiply 22 gal. 1 qt. 1 pt. 2½ gi. by 77.
13. Multiply 5 T. 14 cwt. 20 lb. 6 oz. 15 dr. by 99.
14. Multiply 12 lb. 6 dr. 2 sc. 18 gr. by 108 (9 × 12).
15. Multiply 1 mi. 37 rd. 4 yd. 2 ft. 9 in. by 132 (11 × 12).

16. A has 3 packages of silver, each weighing 1 lb. 11 oz. 14 pwt. 9 gr. B has 7 packages containing 8 oz. 23 gr. each. Which has the most, and how much? *Ans.* A 1 lb. 2 oz. 16 pwt. 10 gr.

17. From a pipe of wine holding 127 gal. 1 pt. were filled 25

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298. How should we multiply by 12 or less? If the multiplier exceeds 12 and is a composite number, how may it be best to proceed? In this case, how can we prove the result?—299. If a fraction occurs in the product, what must be done with it?

demijohns, each containing 5 gal. 1½ gi. How much wine remained in the pipe? *Ans.* 3 qt. 1 pt. 2½ gi.

18. D, having given C a note dated Aug. 2, 1864, and paid it Jan. 1, 1865, borrowed from C the same amount for a period three times as long. How long was that? *Ans.* 1 yr. 2 mo. 27 da.

19. If 5 suits, each requiring 6 yd. 1 qr. 1 na., are cut from a piece of cloth containing 40 yd. 3 na., how much will remain?

20. Henry Smith bought of Walter Rowe, of Liverpool, 2 barrels of flour, at £2 4s. 6d. per bar.; 27 lb. coffee, at 11½d. per lb.; 14 boxes sardines, at 3s. 6d. a box; 210 lb. citron, at 1s. 8½d. Paid on account £3 17s. 6¾d. Make out Smith's bill, showing the balance due. *Ans.* £22 8½d.

21. A printer, having on hand 4 bundles of paper, printed three pamphlets, each requiring 1 ream 6 quires 12 sheets. How much paper had he then left? *Ans.* 2 bundles 12 sheets.

22. If to a pile containing 20 Cd. 8 cd. ft. of wood, 13 loads of 1 Cd. 15 cu. ft. each, are carted, how much wood will there then be in the pile? *Ans.* 35 Cd. 4 cd. ft. 3 cu. ft.

23. A lady, having subscribed 100 guineas for the poor, pays four instalments of £15 8s. 6½d. each. How much has she yet to pay? *Ans.* £43 5s. 11d.

24. P and Q start from two points 175 miles apart, and walk towards each other. P averages 15 mi. 20 rd. 4 yd. a day, and Q 12 mi. 1 fur. 2 yd. 2 ft. How far apart are they at the end of five days? *Ans.* 39 mi. 13 rd. 5 yd. 6 in.

25. If six farms, each containing 40 A. 2 R. 15 P., are taken from a section of land, how much remains? *Ans.* 396 A. 1 R. 30 P.

26. Multiply 5 bu. 3 pk. 6 qt. 1 pt. by 7; 13; 23; 17; and add the products. *Ans.* 357 bu. 6 qt.

27. Multiply 10 cu. yd. 19 cu. ft. 1123 cu. in. by 11; 19; 29; 41; and add the products. *Ans.* 1072 cu. yd. 20 cu. ft. 1708 cu. in.

28. Multiply the sum of 40 ch. 99 l. 3.92 in. and 39 ch. 4 in. by 50.

29. From a heap of potatoes containing 243 bu. 2 pk. were filled 150 baskets, each holding 3 pk. 2 qt. How many bushels, &c., of potatoes remained in the heap?

300. Difference of Time and Longitude.—All places have not the same time. When it is noon here, it is sunset at some place east of us, and sunrise at some place west.

This is because the earth turns on its axis from west to east. Places east of a given point are, therefore, brought within sight of the sun before that point is, and have the sun in their meridian sooner.

301. The difference of time between any two places being known, their difference of longitude can be found. The earth turns on its axis once in 24 hours. A given point on its surface, therefore, completes a circle of 360° in 24 hours, moving 15° in 1 hour, 15′ in 1 minute, 15″ in 1 second. Hence,

To find the difference of longitude in degrees, minutes, and seconds, multiply the difference of time, expressed in hours, minutes, and seconds, by 15.

Navigators thus determine their longitude at sea. Taking with them a chronometer (an accurate watch) set to mark the time at a given place (as, Greenwich or Washington), they ascertain by an astronomical observation the time at the spot they are in, reduce the difference of time to difference of longitude by the above rule, and thus find that they are so many degrees east or west of the meridian of the place for which their chronometer is set.

Ex.—When it is noon at San Francisco, it is 4 min. 52 sec. after 3 P. M. at Philadelphia. What is their difference of longitude?

	h.	min.	sec.
Difference of time,	3	4	52
			15
Difference of longitude,	46°	13′	0″

29. The difference of time between Washington and Dublin is 4 h. 42 min. 51 sec. What is their difference of longitude?

Ans. 70° 42′ 45″.

30. When it is midnight at Detroit, it is 41 min. 13 sec. after 5 A. M. at Paris; what is the difference of long.? *Ans.* 85° 18′ 15″.

300. What is said of the difference of time at different places? Why is this?—301. Give the rule for finding the difference of longitude, when the difference of time is known. Why do we have to multiply by 15? How do navigators determine their longitude at sea?

CHAPTER XVII.

COMPOUND DIVISION.

302. Compound Division is the process of dividing a compound by an abstract number, or finding how many times one compound number is contained in another. It combines division and reduction descending.

Ex. 1.—Find $\frac{1}{29}$ of 32 rd. 4 yd. 3 ft.

The divisor being greater than 12, we must use Long Division. Write the divisor at the left of the dividend, and begin to divide at the left.

Divide 32 rd. by 29: quotient, 1 rd.; remainder, 3 rd. To continue the division, reduce the remainder to yards, and add in the 4 yd. in the dividend. $3 \times 5\frac{1}{2} = 16\frac{1}{2}$. $16\frac{1}{2} + 4 = 20\frac{1}{2}$.

29 is not contained in $20\frac{1}{2}$; hence we have 0 yd. for the quotient. Reduce $20\frac{1}{2}$ yd. to feet, and add in the 3 ft. of the dividend. $20\frac{1}{2} \times 3 = 61\frac{1}{2}$. $61\frac{1}{2} + 3 = 64\frac{1}{2}$.

Divide $64\frac{1}{2}$ ft. by 29: quotient, 2 ft.; remainder, $6\frac{1}{2}$ ft. Reduce the remainder to inches, and again divide. $6\frac{1}{2} \times 12 = 78$. $78 \div 29 = 2\frac{20}{29}$ in. Collect the several quotients for the answer.

```
       rd. yd. ft.
   29) 32   4   3 (1 rd.
       29
        3 rd.
        5½
   29) 20½ yd. (0 yd.
        3
   29) 64½ ft. (2 ft.
       58
        6½ ft.
       12
   29) 78 in. (2 20/29 in.
       58
       20
```

Ans. 1 rd. 2 ft. $2\frac{20}{29}$ in.

Ex. 2.—How many powders weighing 1℈ 5 gr. each can be put up from a mixture containing 1℥ 4ʒ $1\frac{1}{2}$℈?

As many as 1℈ 5 gr. is contained times in 1℥ 4ʒ $1\frac{1}{2}$℈. Reduce both divisor and dividend to grains, that being the lowest denomination in either, and then divide.

1℈ 5 gr. = 25 gr.
1℥ 4ʒ $1\frac{1}{2}$℈ = 750 gr.
750 gr. ÷ 25 gr. = 30 *Ans.*

303. Rule.—1. *To divide a compound by an abstract number, beginning at the left, divide each denomination in turn. When there is a remainder, reduce it to the next*

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302. What is Compound Division? What processes does it combine? Go through Examples 1 and 2, explaining the steps.—303. Recite the rule.

lower denomination, add in the number of that denomination in the dividend, if any, and continue the division. Collect the several quotients, each of the same denomination as its dividend, for the entire quotient.

2. *To divide one compound number by another, reduce both to the lowest denomination in either, and divide as in simple numbers.*

3. *Prove by finding whether the product of divisor and quotient equals dividend.*

Divide by 12 or less in one line. If the divisor exceeds 12 and is a composite number, its factors may be used in dividing.

EXAMPLES FOR PRACTICE.

(1)

	T.	cwt.	qr.	lb.	oz.	dr.
10)	4	5	3	21	9	8
Ans.		8	2	9	10	$8\frac{4}{5}$

(2)

	sq. mi.	A.	R.	sq. rd.	sq. yd.
12)	115	11	1	26	8
Ans.	9	374	1	5	$15\frac{19}{24}$

3. Divide 22 sq. yd. 6 sq. ft. 85 sq. in. by 11. } *Same ans.*
4. Divide 47 sq. yd. 4 sq. ft. $112\frac{3}{11}$ sq. in. by 23. } *for both.*
5. Divide 20 yd. 1 qr. 1 na. by 9. *Ans.* 2 yd. 1 qr. $\frac{1}{9}$ na.
6. Divide 228 ch. 39 l. 4.62 in. by 57. *Ans.* 4 ch. 5.5 in.
7. Divide 3 cu. yd. 20 cu. ft. 709 cu. in. by 401. *Ans.* 437 cu. in.
8. Divide 1 yr. 27 da. 22 h. 30 min. 30 sec. by 65.
9. Divide 127 lb. 10 oz. 18 pwt. 19 gr. by 164.
10. Divide 43 Cd. 4 cd. ft. 11 cu. ft. by 19.
11. Divide 101 chal. 34 bu. $6\frac{1}{2}$ pk. by 83.
12. Divide 6 mi. 31 rd. 4 yd. 2 ft. 2 in. by 42 (7 × 6).
13. Divide 12 ℔ 11 ℥ 7 ʒ 2 ℈ 19 gr. by 121 (11 × 1 1).
14. Divide £147 17s. 4d. 2 far. by 3; by 18; by 29; and add the quotients. *Ans.* £62 12s. 0d. $3\frac{26}{87}$ far.
15. Divide 57 cwt. 15 lb. 5 oz. by 8; by 13; by 58; and add the quotients. *Ans.* 12 cwt. 2 qr. 2 lb. 9 oz. $7\frac{339}{377}$ dr.
16. Divide 3 fur. 4 yd. 6 in. by 34; by 14; find the difference between the quotients. *Ans.* 5 rd. 1 ft. $2\frac{74}{119}$ in.
17. Find $\frac{1}{20}$ of 13 T. 14 cwt. 3 qr. 15 lb.

18. Find $\frac{5}{9}$ of 20 bu. 3 pk. 7 qt. 1 pt. *Ans.* 11 bu. 2 pk. 5 qt. $\frac{1}{9}$ pt.

Multiply the compound number by the numerator of the fraction, and divide the product by its denominator.

19. Find $\frac{7}{17}$ of 3 ℔ 18 gr. *Ans.* 1 ℔ 2 ℥ 6 ʒ 2 ℈ $2\frac{12}{17}$ gr.

20. Find $\frac{13}{14}$ of 7 guin. 10s. 6d. *Ans.* 6 guin. 20s. 3d.

21. How many times is 2 cu. ft. 34 cu. in. contained in 1 cu. yd. 5 cu. ft.? (See Example 2, p. 179.) *Ans.* $15\frac{1473}{1745}$ times.

22. How many spoons, weighing 1 oz. 9 pwt. 13 gr. apiece, can be made out of 1 lb. 18 pwt. of silver? *Ans.* $8\frac{520}{709}$.

23. D, having 431 A. 3 R. 21 P. of land, bought 126 A. 31 P. more, and then divided the whole equally among his 4 sons and 3 daughters. How much did each receive? *Ans.* 79 A. 2 R. 36 P.

24. From a puncheon of rum, containing 80 gal. 1 qt. $1\frac{1}{2}$ pt., 2 qt. leaked out, and what remained was put up in bottles holding 1 pt. 1 gi. apiece. How many bottles were filled? *Ans.* $511\frac{3}{5}$.

25. A lady went out with £20, and spent £8 6s. 3d. How many books, at 3s. $8\frac{1}{2}$d., could she buy with what remained?

26. What is the average speed per minute of a train that runs 30 mi. 30 rd. 5 yd. in one hour, and 27 mi. 4 fur. 30 rd. 1 yd. the next? *Ans.* 3 fur. 33 rd. 4 yd. 1 ft. $10\frac{4}{5}$ in.

How far did the train go in two hours? How many minutes in 2 h.? The average rate per minute will be $\frac{1}{120}$ of the distance travelled in 2 h.

27. B has $\frac{1}{4}$ as much silver plate as his father, who has 18 lb. 4 oz. At 3c. an ounce, what tax has B to pay on this silver, 40 oz. being exempt from taxation?

28. How many times longer is a field 13 ch. 23 l. 1.4 in. in length, than one that measures 1 ch. 1 l. $6\frac{1}{5}$ in.? *Ans.* 13 times.

29. Two fields, of $1\frac{3}{4}$ A. each, produce respectively 36 bu. 3 pk. and 34 bu. 2 pk. 1 qt. of wheat. What is the average yield per square rod? *Ans.* $4\frac{41}{560}$ qt.

30. During February, 1864, a grocer sold 15 cwt. 20 lb. 8 oz. of sugar; what was his average daily sale? *Ans.* 52 lb. $6\frac{26}{29}$ oz.

31. A druggist, having 3 ℔ 8 ℥ 2 ℈ of soda, put up from it 3 dozen powders of $1\frac{1}{3}$ ʒ each, and divided the rest into 6 equal parts; what did each of these weigh? *Ans.* 6 ℥ 2 ʒ 2 ℈ $6\frac{2}{3}$ gr.

32. A person owning a section of land sold 50 A. 1 R. 22 sq.

rd., and gave away 20 A. 39 sq. rd. 30 sq. yd. What remained, he divided equally among his five sons. What was each son's share?

Ans. 113 A. 3 R. 19 sq. rd. $18\frac{1}{5}$ sq. yd.

33. Twenty-four men agree to construct 7 mi. 1 fur. 24 rd. of road; after completing $\frac{1}{8}$ of it, they employ 8 more men. What distance does each man construct before and after the 8 men were employed? *Ans.* 16 rd. before; 1 fur. 20 rd. after.

304. DIFFERENCE OF LONGITUDE AND TIME.—1 hour being the difference of time for 15° of longitude (§ 301), 1 minute for 15′, 1 second for 15″,

To find the difference of time between two places, in hours, minutes, and seconds, divide their difference of longitude, in degrees, minutes, and seconds, by 15.

When the time at a given place is known, add the difference to find the time of any place east of it, subtract for any place west.

EXAMPLE.—St. Petersburg is in 30° 19′ E., New York in 74° 3″ W. longitude. When it is 3 P. M. at N. Y., what o'clock is it at St. Petersburg?

Difference of longitude, 104° 19′ 3″

104° 19′ 3″ ÷ 15 = 6 h. 57 min. 16 sec. + *Diff. of time.*

St. Petersburg being east of N. Y., add: 3 h. + 6 h. 57 min. 16 sec.

Ans. 57 min. 16 sec. past 9 P. M.

34. When it is noon at Buffalo, what is the time at Naples, the former being in 78° 55′ West longitude, the latter in 14° 15′ East? *Ans.* 12 min. 40 sec. past 6 P. M.

35. When it is 6 o'clock A. M. at Portland, what is the time at San Francisco, the former being in 70° 15′ W. long., and the latter in 122° 23′ W.? *Ans.* 31 min. 28 sec. past 2 A. M.

36. Required the difference of time between Buffalo and San Francisco. *Ans.* 2 h. 53 min. 52 sec.

37. Between Portland and Naples. *Ans.* 5 h. 38 min.

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304. How may the difference of time between two places be found, when their difference of longitude is known? In what case must the difference of time be added, and in what case subtracted?

305. MISCELLANEOUS EXAMPLES IN COMPOUND NUMBERS.

1. If a man wastes 4 minutes a day, how much time will he waste in the years 1867, 1868? *Ans.* 2 da. 44 min.

2. A druggist bought 3 lb. $1\frac{1}{4}$ oz. Av. of magnesia; he sold 5 packages of 1 dr. 1 sc. each; how many pounds, &c., Troy, has he remaining? (See § 239.) *Ans.* 3 lb. 8 oz. 5 pwt. $16\frac{1}{4}$ gr.

3. What is the difference of cost between 3 tons of hewn timber, at $1 a cu. ft., and $2\frac{1}{2}$ tons round timber, at 88c. a cu. ft.?

4. What fraction of 1 mile is 6 fathoms?

5. A grocer's quart measure was too small by half a gill. How much did he thus dishonestly make in selling four barrels of cider, averaging 34 gal. 2 qt. 1 pt. each, if the cider was worth 24 cents a gallon? *Ans.* $2.0775.

6. A lady, for ten successive years, went into the country on the 20th of May, and returned the 17th of the following October. At 90c. a day, what did her board cost her for the whole time? (See Table, § 271.) *Ans.* $1350.

7. A farmer owns a horse 15 hands high and a lamb $1\frac{2}{3}$ ft. high. What common fraction, and what decimal, is the lamb's height of the horse's, and how much higher is the horse than the lamb? *Ans.* $\frac{1}{3}$; $.\dot{3}$; 3 ft. 4 in.

8. Which is the greater, .65 lb. Troy or $\frac{108}{175}$ lb. Avoir.?

9. Washington was born Feb. 22, 1732; died Dec. 14, 1799. Franklin was born Jan. 17, 1706; died April 17, 1790. How much did Franklin's age exceed Washington's? *Ans.* 16 yr. 5 mo. 8 da.

10. From a hogshead containing 63 gal. of wine, 1 pt. leaked out; what fraction of the original quantity was thus lost? *Ans.* $\frac{1}{504}$.

11. From 1 qt. 1 pt. of grain was raised 1 bu. What decimal was the seed of the crop? *Ans.* .046875.

12. How many angles of 3° 45′ will fill the same space as 1 right angle, of 90°?

13. How many square yards of carpeting will be required for a room 26 feet by 32 feet?

14. What part of 1 perch is $\frac{2}{111}$ of an acre? *Ans.* $\frac{320}{111}$ P.

306. ***To add or subtract denominate fractions, common or decimal, of different denominations.***

Ex. 1.—Add £$\frac{3}{8}$, $\frac{9}{16}$s., and $\frac{1}{2}$d.

1. Reduce each fraction to integers of lower denominations (§ 281), and then add.

	s.	d.	far.
£$\frac{3}{8}$ =	7	6	0
$\frac{9}{16}$s. =		6	3
$\frac{1}{2}$d. =			2
Ans.	8	1	1

£$\frac{3}{8}$ = $\frac{60}{8}$s. = $7\frac{1}{2}$s.
$7\frac{1}{2} + \frac{9}{16} = 8\frac{1}{16}$s.
$\frac{1}{16}$s. = $\frac{3}{4}$d.
$\frac{3}{4}$d. + $\frac{1}{2}$d. = $1\frac{1}{4}$d.
Ans. 8s. 1d. 1 far.

2. Or, reduce £$\frac{3}{8}$ to shillings, and add in $\frac{9}{16}$s. Reserving 8, the integer, reduce the fraction $\frac{1}{16}$s. to pence, and add in $\frac{1}{2}$d. Reserving 1, the integer, reduce $\frac{1}{4}$d. to farthings. Finally, collect the integers for the answer.

Ex. 2.—From .825 T. subtract .62 cwt.

Proceed by either method shown under Example 1.

	cwt.	qr.	lb.
.825 T. =	16	2	
.62 cwt. =		2	12
Ans.	15	3	13

.825 T. = 16.5 cwt.
16.5 − .62 = 15.88 cwt. =
15 cwt. 3 qr. 13 lb. *Ans.*

Ex. 3.—Add .875 ℔, .7 ℥, and .4 ʒ.

In adding decimals of different denominations, the second method is generally preferable.

Reduce .875 ℔ to ounces, and add in .7 ℥. Reserving 11, the integer, reduce .2 ℥ to drams, and add in .4 ʒ. Collect the integers for the answer. *Ans.* 11 ℥ 2 ʒ.

```
  .875 ℔
    12
 10.500 ℥
    .7
 11|.2 ℥
     8
   1.6 ʒ
    .4
   2|.0 ʒ
```

Rule.—1. *Reduce the given fractions to integers of lower denominations; then add or subtract, as required.*

2. *Or, reduce the fraction of the highest denomination to integers of lower denominations, taking care, as each is reached, to add or subtract, as may be required, any given fractional term belonging to that denomination.*

15. Add $\frac{1}{2}$ cwt., $\frac{1}{3}$ qr., and $\frac{1}{4}$ lb. *Ans.* 2 qr. 8 lb. 9 oz. $5\frac{1}{3}$ dr.

16. Add $\frac{1}{10}$ bu., $\frac{1}{6}$ pk., and $\frac{1}{2}$ qt. *Ans.* $5\frac{1}{30}$ qt.

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306. In how many ways may we add or subtract fractions of different denominations? Illustrate these two modes with the given examples. Recite the rule.

17. From $\frac{2}{5}$ oz. take $\frac{3}{8}$ pwt. *Ans.* 7 pwt. 15 gr.

18. From .375 da. take .2 min. *Ans.* 8 h. 59 min. 48 sec.

19. From .22 ch. take .43 l. *Ans.* 21 l. 4.5144 in.

20. Add £.75, .8s., .36d., .9 far. *Ans.* 15s. 10d. 0.74 far.

21. Add $\frac{2}{3}$ wk., $\frac{1}{5}$ da., $\frac{1}{3}$ h. *Ans.* 4 da. 21 h. 8 min.

22. Add $\frac{3}{5}$ mi., $\frac{5}{6}$ fur., $\frac{1}{3}$ rd., $\frac{2}{3}$ yd., $\frac{3}{4}$ ft., $2\frac{67}{70}$ in. } *Same ans.*

23. Add $\frac{5}{9}$ mi., $\frac{7}{8}$ fur., $12\frac{3}{5}$ rd., $\frac{4}{7}$ yd., $\frac{2}{5}$ ft., $\frac{3}{10}$ in. } *for both.*

24. Add .6 cu. yd., .875 cu. ft., .4 cu. in. *Ans.* 17 c. ft. 130 c. in.

25. Add .375 sq. mi., .54 A., .6 R. *Ans.* 240 A. 2 R. 30.4 sq. rd.

26. From $\frac{5}{7}$ A. take $\frac{1}{4}$ ot 3 roods. *Ans.* 2 R. $4\frac{2}{7}$ P.

27. From $\frac{1}{9}$ hhd. take $\frac{2}{3}$ qt. *Ans.* 6 gal. 3 qt. $\frac{2}{3}$ pt.

28. From .32 ℔ take .9 ℥. *Ans.* 2 oz. 7 dr. 1 sc. 11.2 gr.

29. From $\frac{1}{2}$ of $\frac{3}{4}$ of a day take $\frac{1}{3}$ of $1\frac{1}{2}$ hours.

30. Add $\frac{1}{5}$ lb. Troy, $\frac{1}{4}$ oz., and $\frac{1}{7}$ pwt. *Ans.* 2 oz. 13 pwt. $3\frac{3}{7}$ gr.

31. A man had to plough 3 A., $\frac{1}{3}$ R., $\frac{1}{2}$ P. When $\frac{3}{5}$ A. $\frac{2}{3}$ R. $\frac{3}{4}$ P. was ploughed, how much had he to do? *Ans.* 2 A. 1 R. $10\frac{5}{12}$ P.

32. How many cu. in. in 3 gal. 2 qt. 1 pt., Wine? *Ans.* $837\frac{3}{8}$.

33. In $\frac{2}{3}$ of a gallon + $\frac{1}{2}$ of a quart, Beer? *Ans.* 223.25 cu. in.

34. In 1 bushel 3 pecks? *Ans.* 3763.235 cu. in.

35. How many feet in $\frac{3}{4}$ of a chain + $\frac{3}{10}$ fur.? *Ans.* $247\frac{1}{2}$ ft.

36. From a piece of cloth containing 20 yd. 2 qr. 2 nails, 3 suits, each requiring $4\frac{1}{2}$ yd., were cut. One third of the remainder was sold for \$10.68$\frac{3}{4}$; what did it bring per yard? *Ans.* \$4.50.

$\frac{1}{3}$ remainder = 2 yd. 1 qr. 2 na. = 2.875 yd. \$10.6875 ÷ 2.875 = \$4.50.

37. What cost 4 bu. 3 pk. 6 qt. of potatoes, at 75c. a bushel?

As the price is given by the *bushel*, reduce, by § 285, 4 bu. 3 pk. 6 qt. to *bushels* and the decimal of a bushel (4.9375 bu.), and multiply by the price. *Ans.* \$3.70.

38. What cost 4 bu. 3 pk. 6 qt. of potatoes, at 18c. a pk.?

As the price is given by the *peck*, reduce 4 bu. 3 pk. 6 qt. to *pecks* and the decimal of a peck, and multiply by the price. 4 bu. 3 pk. = 19 pk. 6 qt. = .75 pk. 19.75 × .18 = \$3.555. *Ans.*

39. What cost 57 A. 2 R. 20 P., at \$20 an acre? *Ans.* \$1152.50.
At \$3.75 a rood? *Ans.* \$864.375.

40. What cost 7 gal. 3 qt. 1 pt. of wine, at \$8 a gal.? *Ans.* \$63.
At \$1.50 a quart? *Ans.* \$47.25.

41. What cost 5 T. 17 cwt. 20 lb. of hay, at \$30.50 a ton?
At \$1.60 a hundred-weight? *Ans.* \$187.52.

42. What cost 3 lb. 6 oz. 1 dr. 2 sc. of quinine, at $2.75 per oz.?

43. Find the cost of a gold ornament, weighing 4 oz. 18 pwt. 20 gr., at £4 9s. per ounce. *Ans.* £21 19s. 9d. 2.8 far.

44. What is the cost of a block of marble, 9 ft. long, 4 ft. 4 in. wide, and 3 ft. 6 in. thick, at $5 a cubic foot? *Ans.* $682.50.

45. What cost 3 bundles 8 quires of paper, at $6 a ream?

46. What cost a field 4 ch. 8 l. square, at $6.25 a rood?

47. What cost $4\frac{1}{2}$ A. $3\frac{1}{4}$ R. $\frac{1}{2}$ P. of land, at $4.25 a rood?

48. What cost $\frac{1}{6}$ T. $\frac{1}{5}$ cwt. 25 lb., at $3 per cwt.? *Ans.* $11.35.

49. 18 Cd. 8 cu. ft. of wood, at $8 a cord? *Ans.* $144.50.

50. 7 T. 14 cwt. 3 qr. 10 lb., at $75 a ton? *Ans.* $580.687 +.

51. 3℔. 6 ʒ 15 gr. of calomel, at $1.50 an ounce?

Practice.

307. Practice is a short method of operating with compound numbers, by means of aliquot parts. It was applied to Federal Money on p. 123, and may be extended to compound numbers generally. The aliquot parts most frequently used are as follows:—

Table of Aliquot Parts.

Sterling Money.					Avoir. Wt.		Time.				
s.	d.	£	d.	s.	lb.	cwt.	mo.	da.	yr.	da.	mo.
10		= $\frac{1}{2}$	6	= $\frac{1}{2}$	50	= $\frac{1}{2}$	6		= $\frac{1}{2}$	15	= $\frac{1}{2}$
6	8	= $\frac{1}{3}$	4	= $\frac{1}{3}$	$33\frac{1}{3}$	= $\frac{1}{3}$	4		= $\frac{1}{3}$	10	= $\frac{1}{3}$
5		= $\frac{1}{4}$	3	= $\frac{1}{4}$	25	= $\frac{1}{4}$	3		= $\frac{1}{4}$	6	= $\frac{1}{5}$
4		= $\frac{1}{5}$	2	= $\frac{1}{6}$	20	= $\frac{1}{5}$	2	12	= $\frac{1}{5}$	5	= $\frac{1}{6}$
3	4	= $\frac{1}{6}$	$1\frac{1}{2}$	= $\frac{1}{8}$	$16\frac{2}{3}$	= $\frac{1}{6}$	2		= $\frac{1}{6}$	3	= $\frac{1}{10}$
2	6	= $\frac{1}{8}$	1	= $\frac{1}{12}$	$12\frac{1}{2}$	= $\frac{1}{8}$	1	15	= $\frac{1}{8}$	1	= $\frac{1}{30}$
2		= $\frac{1}{10}$			10	= $\frac{1}{10}$	1	6	= $\frac{1}{10}$		
1	8	= $\frac{1}{12}$			$8\frac{1}{3}$	= $\frac{1}{12}$	1		= $\frac{1}{12}$		
1		= $\frac{1}{20}$			5	= $\frac{1}{20}$	30 da. allowed to 1 mo.				

307. What is Practice? Give the aliquot parts of £1. Of a shilling. Of a hundred-weight. Of a year. Of a month.

Ex. 52.—What cost 960 Grammars, at 1s. 8d. each?

At £1 each, 960 Grammars would cost £960. But 1s. 8d. = $\frac{1}{12}$ of £1; therefore, at 1s. 8d., they will cost $\frac{1}{12}$ of £960, or £80.

12)	£960
Ans.	£80

Ex. 53.—If it costs $17.50 to insure a house 1 year, what will it cost to insure it for 3 yr. 1 mo. 15 da.?

1 mo. 15 da. = $\frac{1}{8}$	$17.50
	3
	$52.50
	2.1875
Ans.	$54.6875

For 3 years take 3 times the cost for 1 yr. For 1 mo. 15 da., which is $\frac{1}{8}$ of 1 yr., take $\frac{1}{8}$ of the cost for 1 yr. Find the whole by adding these two parts.

Ex. 54.—How much seed will be needed for 10 A. 1 R. 30 P., allowing 1 bu. 2 pk. 4 qt. to an acre?

For 10 A. take 10 times the quantity required for 1 A. For 1 R., which is $\frac{1}{4}$ of 1 A., take $\frac{1}{4}$ the quantity required for 1 A. 30 P. not being an aliquot part of 1 rood, take first for 20 P., which is $\frac{1}{2}$ of 1 R.; then for 10 P., which is $\frac{1}{2}$ of 20 P. Find the whole by adding these parts.

	bu.	pk.	qt.
1 R. = $\frac{1}{4}$	1	2	4
			10
	16	1	0
	0	1	5
20 P. = $\frac{1}{2}$	0	0	6.5
10 P. = $\frac{1}{2}$	0	0	3.25
Ans.	16	3	6.75

55. What cost 14 dozen Readers, at 3s. 4d. apiece? *Ans.* £28.

56. 14 cwt. 12$\frac{1}{2}$ lb. of cheese, at $16 a cwt.? *Ans.* $226.

57. 1 gross of knives, at 2s. 6d. apiece? *Ans.* £18.

58. 5 yd. 1 qr. 1 na. of cloth, at $6.25 a yd.? *Ans.* $33.20+.

59. 26 gal. 1 qt. 1 pt. 1 gi. of wine, at $7 a gal.? *Ans.* $184.84+.

60. What will it cost to travel 1200 miles, at 1$\frac{1}{2}$d. a mile?

61. At $25 a month, what will be a man's wages for 1 year 7 months 12 days?

62. What will be the yield of 16 A. 25 P. of land, at the rate of 24 bu. 3 pk. 1 qt. per acre? *Ans.* 400 bu. 1 pk. 3.90625 qt.

63. What cost a plate of glass, measuring 7 ft. by 5 ft. 6 in., at 4s. 6d. a square foot? *Ans.* £8 13s. 3d.

64. What cost 10 panes of glass, each 4 ft. by 2 ft. 9 in., at 1s. 3d. a square foot? *Ans.* £6 17s. 6d.

65. Find the rent of a farm of 250 A. 2 R. 15 P., at £1 12s. 4d. an acre. *Ans.* £405 2s. 6d. 1.5 far.

CHAPTER XVIII.

DUODECIMALS.

308. Duodecimals are a system of compound numbers, sometimes used as measures of length, surface, and solidity.

The foot, whether linear, square, or cubic, is the unit; and the other denominations arise from successive divisions by 12. Hence the term *duodecimals*, *duodecim* being the Latin for *twelve.*

1 of any denomination in this system makes 12 of the next lower; and, conversely, 12 of any denomination make 1 of the next higher.

TABLE.

1 foot (ft.) = 12 primes, marked $'$.
1 prime = 12 seconds, marked $''$.
1 second = 12 thirds, marked $'''$.
1 third = 12 fourths, marked $''''$, &c.

$$\begin{array}{ccccccccc} & & & & & & 1''' & = & 12'''' \\ & & & & 1'' & = & 12''' & = & 144'''' \\ & & 1' & = & 12'' & = & 144''' & = & 1728'''' \\ 1\text{ ft.} & = & 12' & = & 144'' & = & 1728''' & = & 20736'''' \end{array}$$

The marks used to distinguish the denominations ($'$ $''$ $'''$ $''''$) are called In'dices (*singular*, Index).

309. When used in connection with one dimension simply, as length or breadth, the prime, being $\frac{1}{12}$ of a foot, is equivalent to 1 inch.

When applied to surfaces, the prime, being $\frac{1}{12}$ of a foot, equals 12 square inches. The second, being $\frac{1}{12}$ of $\frac{1}{12}$ of a foot, equals 1 sq. in.

When applied to solid contents, the prime, being $\frac{1}{12}$ of a foot, equals 144 cubic inches; the second = 12 cubic inches; the third = 1 cubic inch.

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308. What are Duodecimals? What is the unit? How do the other denominations arise? Whence is the term *duodecimals* derived? Recite the Table. What are the marks used to distinguish the denominations called?—309. What is the prime equivalent to, when used in connection with one dimension simply? When applied to surfaces? When applied to solid contents?—310. How are duodecimals added, subtracted, multiplied, and divided?

310. Duodecimals may be added, subtracted, multiplied, and divided, like other compound numbers.

EXAMPLES FOR PRACTICE.

(1)

Add	1 ft.	11'	6"	10'''	7''''
	4 ft.	9'	7"	11'''	5''''
		11'	8"	9'''	3''''
	2 ft.	0'	6"	3'''	9''''
Ans.	9 ft.	9'	5"	11'''	0''''

(2)

From	8 ft.	1'	0"	6'''	10''''
Take	5 ft.	0'	4"	9'''	11''''
Ans.	3 ft.	0'	7"	8'''	11''''

(3)

Multiply		6'	8"	9'''
By				12
Ans.	6 ft.	8'	9"	0'''

(4)

Divide 9 ft. 1" 4''' by 32.

4)	9 ft.	0'	1"	4'''	
8)	2 ft.	3'	0"	4'''	
Ans.	0 ft.	3'	4"	6'''	6''''

5. Find the sum of 3 ft. 1" 5'''', 16 ft. 6' 7''', 19 ft. 8' 9" 11''' 11'''', 10' 5" 8'''', and 5 ft. 7" 11'''. *Ans.* 45 ft. 2' 7'''.

6. From 25 ft. 1" take 16 ft. 3' 9''' 8''''. *Ans.* 8 ft. 9' 2''' 4''''.

7. Multiply 3 ft. 6' 5" 7''' by 12. *Ans.* 42 ft. 5' 7".

8. Divide 6 ft. 4''' 10'''' by 31. *Ans.* 2' 3" 10''' 7'''' +.

9. From 100 ft. subtract 7 times 8' 9'''. *Ans.* 95 ft. 3' 6" 9'''.

10. From 59 ft. take $\frac{1}{16}$ of 6 ft. 6". *Ans.* 58 ft. 7' 5" 7''' 6''''.

11. Add 365 ft. 1' 7" 9''' 8'''', 521 ft. 10' 10" 11''', 605 ft. 8' 8" 1''', and 731 ft. 3' 8''' 4''''. *Ans.* 2224 ft. 3" 6'''.

12. What is the sum of 14 ft. 5' 6''' 9'''' and 11' 11" 10''' 10''''? What is their difference?

13. What is the sum, and what the difference, of 47 ft. 1' 1" 1''' and 13 ft. 11' 11" 11'''? Of 10' 10" 10''' and 10" 10''' 10''''?

14. From the sum of 8' 9" 8''' and 10' 10" 10''' take the sum of 4' 8" 9'''' and 11" 8''' 3''''. *Ans.* 1 ft. 2' 9'''.

15. Multiply by 36 the sum of 8" 3''', 4' 9", and 2 ft. 3' 4" 7'''. Divide the product by 7. *Ans.* 14 ft. 9" 5''' 1'''' +.

16. What is the sum of 100 ft. 8' 8", 135 ft. 1" 9''', 65 ft. 9' 2" 7''', 45 ft. 3' 3", and 200 ft. 6' 6" 8'''?

17. Which is greater, $\frac{1}{3}$ of 10 ft. 6" 7''' or $\frac{1}{8}$ of 30 ft. 1' 1", and how much?

311. Multiplication of Duodecimals by Duodecimals.—1 ft. is the unit. Hence, multiplying by 1 ft. is simply multiplying by 1, and the denomination of the product will be the same as that of the multiplicand. $3' \times 1 \text{ ft.} = 3'$.

$1' = \frac{1}{12}$ ft. Hence, multiplying by $1'$ is multiplying by $\frac{1}{12}$, and the denomination of the product will be *one* degree lower than that of the multiplicand. $3' \times 1' = 3''$.

$1'' = \frac{1}{12}$ of $\frac{1}{12}$ ft. Hence, multiplying by $1''$ is multiplying by $\frac{1}{12}$ of $\frac{1}{12}$, and the denomination of the product will be *two* degrees lower than that of the multiplicand. $3' \times 1'' = 3'''$.

$1''' = \frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of a ft. Hence, multiplying by $1'''$ is multiplying by $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$, and the denomination of the product will be *three* degrees lower than that of the multiplicand. $3' \times 1''' = 3''''$.

From the above it will be seen that *The index of a product equals the sum of the indices of its factors.*

Thus $6' \times 3' = 18''$; $6' \times 3'' = 18'''$; $6'' \times 3' = 18'''$; $6'' \times 8'' = 18''''$.

Example.—Multiply 14 ft. 7′ 8″ by 2 ft. 6′.

	14 ft.	7′	8″
		2 ft.	6′
7 ft.	3′	10″	0‴
29 ft.	3′	4″	
36 ft.	7′	2″	*Ans.*

Set the multiplier under the multiplicand, with their right-hand terms in line. Begin to multiply at the right, reducing and carrying as in compound multiplication.

$8'' \times 6' = 48''' = 4''$; carry 4 to the next product. $7' \times 6' = 42''$, and $4''$ carried makes $46'' = 3'\ 10''$; write down $10''$, and carry $3'$ to the next product. $14 \text{ ft.} \times 6' = 84'$, and $3'$ carried makes $87' = 7 \text{ ft. } 3'$.

Next multiply by 2 ft., remembering that, when we multiply by feet, the product is of the same denomination as the multiplicand. Set the terms of this product under like denominations in the former one. Finally, add the partial products.

312. Rule.—1. *Write the multiplier under the multiplicand, with their right-hand terms in line.*

311. How does the denomination of the product compare with that of the multiplicand, when we multiply by 1 ft.? When we multiply by 1′? When we multiply by 1″? When we multiply by 1‴? What rule is hence deduced, for the index of a product? Solve and explain the given example.—312. Recite the rule for the multiplication of duodecimals.

2. *Beginning at the right, multiply by each term of the multiplier, giving each product an index equal to the sum of the indices of its factors, and reducing and carrying as in compound multiplication. Write terms of the same denomination in the partial products in the same column, and finally add the partial products.*

EXAMPLES FOR PRACTICE.

1. Multiply 3 ft. 7′ 2″ by 7 ft. 6′ 3″. *Ans.* 27 ft. 7″ 9‴ 6⁗.
2. Multiply 7 ft. 8′ 9″ by 6 ft. 4′ 3″; by 12 ft. 5′; by 9 ft. 8″.
3. Multiply 6 ft. 9′ 7″ by 4 ft. 2′. *Ans.* 28 ft. 3′ 11″ 2‴.
4. What is the area of a slab 7 ft. 3′ long and 2 ft. 11′ broad?
5. What is the area of a hall 37 ft. 3′ long by 10 ft. 7′ wide?
6. How many square ft., &c., in a garden 100 ft. 6′ by 39 ft. 7′?
7. How many square ft. in 12 boards, each 12 ft. 8′ by 1 ft. 9′?

Solve this and the next two examples by the above rule. Then prove the result by expressing the primes as fractions of a foot, multiplying, and reducing the fraction of a foot in the product, if there is any, to primes, &c. Thus, in Ex. 7:—
12 ft. 8′ = $12\frac{2}{3}$ ft. 1 ft. 9′ = $1\frac{3}{4}$ ft.

$12\frac{2}{3} \times 1\frac{3}{4} \times 12 =$ sq. ft. *Ans.*

8. How many cubic feet, primes, &c., in a wall, 80 ft. 9′ long, 1 ft. 8′ wide, and 3 ft. 4′ high?

9. How many cubic feet in a pile of wood, 156 ft. long, 4 ft. 8′ high, 6 ft. 4′ wide? How many cords? *Ans.* $36\frac{1}{48}$ Cd.

10. A room is 18 ft. long, 14 ft. 6′ wide, 9 ft. 8′ high. It contains four windows, each 5 ft. 6′ by 3 ft.; and two doors, each 6 ft. 9′ by 2 ft. 10′. What will be the cost of plastering said room, at 25c. per square yard? *Ans.* $21.81.

The four walls and ceiling are to be plastered.
Two of the walls have an area of 18 ft. × 9 ft. 8′ each.
The two other walls have an area of 14 ft. 6′ × 9 ft. 8′ each.
The ceiling has an area of 18 ft. × 14 ft. 6′.
Deduct from the sum of these areas, the areas of the windows and doors, which are not to be plastered: 4 windows, each 5 ft. 6′ × 3 ft.: 2 doors, each 6 ft. 9′ × 2 ft. 10′. Reduce the sq. feet to sq. yards, and multiply by the price.

11. What will it cost to paint a house 42 ft. 6′ deep, 28 ft. 6′ wide, and 19 ft. 6′ high, at 24c. per square yard, no allowance being made for windows? *Ans.* $73.84.

313. Division of Duodecimals by Duodecimals.

In multiplying duodecimals, we assign to a product an index equal to the sum of the indices of its factors. Hence, in dividing, *To find the index of the quotient, we subtract the index of the divisor from that of the dividend.*

Thus, $18'' \div 6' = 3'$; $18''' \div 6' = 3''$; $18''' \div 6'' = 3'$.

314. *If the index of the divisor exceeds that of the dividend, reduce the dividend to the same denomination as the divisor, and the quotient will be feet.*

Example.—Divide 18 square feet by $6''$.

18 ft. $= 2592''$ $2592'' \div 6'' = 432$ ft. *Ans.*

Example.—Divide 27 sq. ft. $7''\ 9'''\ 6''''$ by 3 ft. $7'\ 2''$.

Write the divisor at the left of the dividend, as in other cases of compound division. Begin to divide at the left.

3 ft. $7'\ 2''$)	27 ft.	$0'$	$7''$	$9'''$	$6''''$	(7 ft. $6'\ 3''$ *Ans.*
	25 ft.	$2'$	$2''$			
	1 ft.	$10'$	$5''$	$9'''$	$6''''$	
	1 ft.	$9'$	$7''$	$0'''$		
			$10''$	$9'''$	$6''''$	
			$10''$	$9'''$	$6''''$	

27 sq. ft. $\div$ 3 ft. $=$ 9 ft. But, making allowance for the primes in the divisor, which amount to more than half a foot, we write 7 ft. as the first term in the quotient. We then multiply the whole divisor by 7 ft., and subtract the product from the dividend.

3 ft. is not contained in 1 ft., the first term of the new dividend; hence we reduce 1 ft. to primes, and add in $10'$. Dividing $22'$ by 3 ft. (making allowance, as above), we get $6'$. Write $6'$ in the quotient, multiply the divisor by it, and subtract.

Dividing $10''$ by 3 ft., we get $3''$, which we write as the third term in the quotient. Multiplying the divisor by this term and subtracting the product, we find there is no remainder.

If, on multiplying the divisor by any term of the quotient, the product is greater than the partial dividend, the quotient term must be diminished.

315. Rule.—1. *Divide the highest term of the dividend by that of the divisor, making it divisible, if neces-*

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313. In dividing duodecimals, how do we find the index of the quotient?—314. If the index of the divisor exceeds that of the dividend, how must we proceed? Solve and explain the given example. In what case must the term placed in the quotient be diminished?—315. Recite the rule for the division of duodecimals.

sary, by reducing it to a lower denomination, and adding in the given number of that denomination. Write the result in the quotient, multiply the whole divisor by it, and subtract the product from the dividend.

2. *Divide the highest term of the new dividend as before. Write the result in the quotient, multiply the divisor by it, and subtract. Proceed thus till the division terminates, or a quotient sufficiently exact is obtained.*

EXAMPLES FOR PRACTICE.

1. Divide 32 ft. 9′ 9″ by 7 ft. 3′ 6″. *Ans.* 4 ft. 6′.
2. Divide 18 ft. 4′ 6″ by 3′ 6″ (§ 314). *Ans.* 63 ft.
3. Divide 32 ft. 9′ 9″ by 29 ft. 2′. *Ans.* 1 ft. 1′ 6″.
4. Divide 42 ft. 10′ 10″ 4‴ by 6 ft. 1′ 4″. *Ans.* 7 ft. 3″.
5. Divide 9′ 11″ 8‴ 6⁗ by 4″ 3‴. *Ans.* 28 ft. 2′.
6. What is the breadth of a marble slab, whose area is 21 ft. 1′ 9″, and its length 7 ft. 3′? *Ans.* 2 ft. 11′.
7. A carpenter bought 920 sq. ft. of boards. If their united length was 480 ft., what was their average breadth?
8. A board fence 6 ft. 4′ high contains 510 ft. 10′ 8″ of surface. How long is the fence? *Ans.* 80 ft. 8′.
9. In digging a cellar 42 ft. 10′ long and 12 ft. 6′ wide, 4283 cu. ft. 4′ of earth was thrown out. What was its depth?

Divide the solid contents, represented by the amount of earth thrown out, by the product of the two given dimensions. *Ans.* 8 ft.

MISCELLANEOUS QUESTIONS.—When do we add, to find the difference of latitude between two places? To find the difference of longitude? How is the difference of time found from the difference of longitude? How is the difference of longitude found from the difference of time?

What is the unit of duodecimals? What is meant by the indices of duodecimals? What is the index of the foot? Of the prime? How many inches are equal to a prime, when used in connection with length or width only? When used in connection with surface? With solid contents? Recite the rule for multiplying duodecimals by duodecimals. How may the operation be proved? Recite the rule for dividing duodecimals by duodecimals. How may the operation be proved?

CHAPTER XIX.

PERCENTAGE.

316. Per cent., from the Latin words *per centum*, means *by* or *on the hundred.* One per cent. means *one on every hundred*, or *one hundredth;* it is written briefly 1%, and is equivalent to $\frac{1}{100}$ or .01. Two per cent., 2 on 100, or two hundredths, is written 2%, and equals $\frac{2}{100}$ or .02.

317. Any per cent. or number of hundredths may thus be written either as a common fraction or a decimal; but the decimal form is preferred, as easier to operate with.

Any integral per cent. less than 100 is expressed by two decimal figures. 1% = .01. 10% = .10.

100%, being $\frac{100}{100}$, is written 1.; 150% = 1.50; 200% = 2., &c.

Any part of 1% may be expressed by taking the like part of .01:

$\frac{1}{2}\% = \frac{1}{2}$ of .01 = .005. $\frac{3}{8}\% = \frac{3}{8}$ of .01 = .00375.

Any part of 1% that can not be exactly expressed as a decimal may be written as a common fraction after the place of hundredths. Thus, $\frac{1}{3}\% = .00\frac{1}{3}$. $\frac{1}{9}\% = .00\frac{1}{9}$.

318. The following examples will show how to express different rates per cent. decimally:—

7%	=	.07	525%	=	5.25	$\frac{1}{8}\%$	=	.00125
12%	=	.12	$\frac{1}{2}\%$	=	.005	$4\frac{1}{10}\%$	=	.041
40%	=	.40	$\frac{1}{3}\%$	=	$.00\frac{1}{3}$	$15\frac{3}{4}\%$	=	.1575
100%	=	1.00	$\frac{1}{5}\%$	=	.002	$23\frac{2}{5}\%$	=	.234
300%	=	3.00	$\frac{1}{6}\%$	=	$.00\frac{1}{6}$	$30\frac{7}{10}\%$	=	.307

In the case of an integral per cent., the decimal point must not be prefixed when the sign % or the words *per cent.* are used. 25% is very different from .25%; the former being equivalent to $\frac{25}{100}$ or $\frac{1}{4}$,—the latter to $\frac{25}{100}$ of $\frac{1}{100}$, or $\frac{1}{400}$.

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316. What is the expression *per cent.* derived from? What does it mean? What does *one per cent.* mean? How is it written? To what is it equivalent? Two per cent.?—317. How may any per cent. be written? Which form is preferred, and why? How many decimal figures are required to express any integral per cent. less than 100? How is 100 per cent. written decimally? 150 per cent.? 200 per cent.? How may any part of 1 per cent. be expressed? How may any part of 1 per cent. that can not be exactly expressed as a decimal be written?—318. Give examples of the mode of expressing different rates. What caution is given in the case of an integral per cent.?

319. Exercise.

1. Write the following rates per cent. as decimals: 6%; $4\frac{1}{5}$%; 25%; 101%; $\frac{1}{4}$%; 13%; $\frac{5}{6}$%; 200%; $\frac{5}{8}$%; $3\frac{3}{10}$%; $8\frac{2}{5}$%; 80%; $50\frac{3}{4}$%; 425%; 42%; $5\frac{2}{3}$%; 9%; $\frac{7}{12}$%; $81\frac{1}{7}$%; $15\frac{4}{9}$%; $98\frac{1}{9}$%; 312%; $53\frac{1}{4}$%; $\frac{1}{100}$%; $100\frac{5}{8}$%; 1000%.

2. Read the following as so many per cent.: .0825 (*eight and a quarter per cent.*); .04; 2.00 (*two hundred* %); .17; .105; .20; 4.00; .1175; $.33\frac{1}{3}$; $3.33\frac{1}{3}$; $.03\frac{1}{3}$; $.05\frac{2}{3}$; .052; .074 ($7\frac{2}{5}$%); .094; 1.15; .008; 8.00; $.00\frac{1}{3}$; .0003 (*three hundredths of* 1%); .0007.

3. What per cent. is each of the following common fractions equivalent to? $\frac{1}{2}$ (*Annex two naughts to the numerator, and divide by the denominator:* $1.00 \div 2 = .50 = 50\%$); $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{6}$; $\frac{1}{7}$; $\frac{1}{8}$; $\frac{1}{9}$; $\frac{1}{10}$; $\frac{1}{12}$; $\frac{3}{4}$; $\frac{2}{3}$; $\frac{3}{5}$; $\frac{5}{6}$; $\frac{3}{8}$; $\frac{7}{8}$; $\frac{1}{20}$; $\frac{1}{16}$; $\frac{1}{50}$; $\frac{1}{75}$; $\frac{3}{20}$.

4. What common fraction is each of the following equivalent to? 25% ($= \frac{25}{100} = \frac{1}{4}$); $\frac{1}{2}$% ($= \frac{1}{2}$ of $\frac{1}{100} = \frac{1}{200}$); 7%; 6%; 14%; 20%; 10%; 40%; 50%; 12%; 4%; 8%; $\frac{1}{3}$%; $\frac{1}{5}$%; 9%; $16\frac{2}{3}$%; 15%; 60%; 18%; $\frac{3}{8}$%; $\frac{1}{9}$%; 28%; 200%; $\frac{1}{4}$%.

320. In connection with the subject of Percentage, three things are to be considered:—

1. The **Rate**, or number of hundredths taken.

2. The **Base**, or number of which the hundredths are taken.

3. The **Percentage**, or number obtained by taking certain hundredths of the base.

Two of these being known, the third can be found; for the Percentage is the product of the Base and Rate.

Example 1.—How much is 7% of $16.85?

Here the base and rate are given, and the percentage is required. 7% is $\frac{7}{100}$. Taking $\frac{7}{100}$ is equivalent to multiplying by $\frac{7}{100}$ (§ 161). Hence we multiply the base, $16.85, by .07 (the rate expressed decimally), and point off the product as in multiplication of decimals.

$$\begin{array}{r} \$16.85 \\ .07 \\ \hline \textit{Ans.}\ \$1.1795 \end{array}$$

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320. How many things are to be considered in connection with the subject of Percentage? Name them, and define each. What relation subsists between the Percentage, Base, and Rate? Show this from Example 1.

It will be seen from this example that the percentage is the product of the base and rate.

EXAMPLE 2.—What per cent. of \$16.85 is \$1.1795?

Here the percentage (the product) and the base (one of its factors) are given, and the rate (the other factor) is required. Divide the product by the given factor, and the quotient will be the required factor (§ 89).

$$16.85)\ 1.1795\ (.07$$
$$\underline{11795}$$

Ans. 7%.

EXAMPLE 3.—\$1.1795 is 7% of what number?

$$.07)\ \$1.1795$$
Ans. \$16.85

Here again the product and one factor are given, and the other factor is required. Divide the product, \$1.1795, by the given factor, 7%, expressed decimally; and point off the quotient as in division of decimals.

321. RULES.—I. *To find the percentage, multiply the base by the rate expressed decimally.*

II. *To find the rate, divide the percentage by the base; the figures of the quotient to the hundredths' place inclusive will denote the rate %, and the remaining figures, if any, the decimal of 1%.*

III. *To find the base, divide the percentage by the rate expressed decimally.* Hence these formulas:—

$$\text{PERCENTAGE} = \text{BASE} \times \text{RATE}$$

$$\text{RATE} = \frac{\text{PERCENTAGE}}{\text{BASE}} \qquad \text{BASE} = \frac{\text{PERCENTAGE}}{\text{RATE}}$$

PROOF.—These rules may be used to prove one another. Thus:—

If the percentage has been found by Rule I., divide it by the rate, according to Rule III., and see whether the given base results.

If the rate has been found by Rule II., multiply the base by it, according to Rule I., and see whether the given percentage results.

If the base has been found by Rule III., multiply it by the rate, according to Rule I., and see whether the given percentage results.

Be very careful to place the decimal point correctly.

322. EXAMPLES FOR PRACTICE.

1. How much is 15% of £10 4s. 6d.?

By § 285, £10 4s. 6d. = £10.225. £10.225 × .15 = £1.53375.
By § 284, £1.53375 = £1 10s. 8d. .4 far. *Ans.*

Explain Ex. 2. Explain Ex. 3.—321. Recite the rule for finding the percentage. For finding the rate. For finding the base. Express these rules briefly in formulas. Show how each operation may be proved.

2. How much is 50 % of £64 18s. 8d.? *Ans.* £32 9s. 4d.

50 % being ½, the shortest way is to take ½ at once.

So, for 33⅓ % take ⅓.	For 12½ % take ⅛.
For 25 % " ¼.	For 10 % " $\frac{1}{10}$.
For 20 % " ⅕.	For 8⅓ % " $\frac{1}{12}$.
For 16⅔ % " ⅙.	For 5 % " $\frac{1}{20}$.

3. Find 6 % of $1000. *Ans.* $60.
4. 8 % of $28.98. *Ans.* $2.318.
5. ¼ % of £120. *Ans.* 6s.
6. 4⅖ % of 75 gal. *Ans.* 3.3 gal.
7. $11\frac{1}{10}$ % of 3 yd. *Ans.* 11.988 in.
8. 37½ % of $60.005. *Ans.* $22.50+.
9. 20 % of £10 5d. *Ans.* £2 1d.
10. ⅓ % of 9171 acres. *Ans.* 30.57 A.
11. 2½ % of 50 guin. *Ans.* 1 g. 5s. 3d.
12. Find 9 % of $995.
13. 25 % of 78 bu. 2 pk.
14. 6¼ % of $75.
15. 24 % of £10 10s.
16. 8⅜ % of 33 cwt. 8 lb.
17. 30¾ % of $122.50.
18. 12½ % of £8 1s. 4d.
19. 18⅜ % of $240.505.
20. 100 % of 16 lb. 5 oz. 1 dr.

21. Find 38 % of 4. 97 % of 16. 500 % of 7. 840 % of 28½. 365 % of ¼. 92 % of ⅜. *Sum of answers*, 292.6975.

22. Find the percentage on $987634.37 at each of the following rates: ½ %; 2½ %; ¼ %; 3¼ %; ⅜ %; 6⅜ %; 25 %; 412 %; 900 %; 43 %. *Sum of answers*, $13760215.86 +.

23. A farmer, raising 1097 bu. of wheat, gives 10 % of it for thrashing, and sells 10 % of the remainder. How much is left?

24. A merchant, who had $6480 invested in business, lost 75 % of it. How much did he save? *Ans.* $1620.

25. A and B invested $100 each in speculations. A lost 100 % of his investment, and B made 200 % on his. How much better off was B than A on these speculations?

26. If 36 % of the contents leak out of a hhd. of molasses, how many gallons will be left? *Ans.* 40.32 gal.

27. A coal-dealer bought 17180 tons of coal; he sold 62 % of it at $6.75 a ton, and the rest at $7. How much did the whole bring? *Ans.* $117597.10.

28. What is the sum of ½ % of $40 and .3 % of $30?

29. A California miner, having obtained 15½ lb. of gold dust, finds that it loses 5 % in refining, and pays 6 % for coining. What weight should he receive after it is coined?

30. What % is 1 of 20? (Ex. 2, p. 196.)
31. .048 of 240? *Ans.* .02 %.
32. \$117 of \$900? *Ans.* 13 %.
33. 200 bu. of 50 bu.? *Ans.* 400 %.
34. 3s. of £100?* *Ans.* .15 %.
35. \$2.25 of \$112.50? *Ans.* 2 %.
36. 2s. 9d. of £2 15s.? *Ans.* 5 %.
37. 8 ℨ of 25 lb? *Ans.* $\frac{1}{8}$ %.
38. 1.6 pt. of 40 gal.? *Ans.* $\frac{1}{2}$ %.
39. \$8.736 of \$1248? *Ans.* .007 %.
40. What % of $\frac{4}{5}$ is $\frac{1}{2}$?
41. What % of $\frac{5}{6}$ is $\frac{1}{8}$?
42. What % of $\frac{3}{8}$ is $\frac{1}{4}$?
43. What % of 1600 is 1000?
44. What % is \$6 of \$18?
45. \$36.10 of \$902.50?
46. 76 trees of 400 trees?
47. 1 of 10000?
48. 5 mills of 1 dime?
49. 2 cwt. of 100 tons?

50. What % of a long ton is a common ton?* *Ans.* 89$\frac{2}{7}$ %.

51. What % is a pound Troy of a pound avoir.?* *Ans.* 82$\frac{2}{7}$ %.

52. What % is an ounce Troy of an ounce avoir.? *Ans.* 109$\frac{5}{7}$ %.

53. What % is the wine gal. of the beer gal.? *Ans.* 81$\frac{41}{47}$ %.

54. A lady divides \$300 among her three sons, giving the first \$100, the second \$25, and the third the rest. What per cent. of the whole does each receive?

55. A person owns a house and lot worth \$5500. The lot is worth \$1000; what % is that of the value of the house?

56. 25 is 4 % of what? *Ans.* 625.
57. \$10 is 12 % of what? \$83$\frac{1}{3}$.
58. 9s. is $\frac{1}{2}$ % of what? £90.
59. 17 qt. is 8$\frac{1}{2}$ % of what? 50 gal.
60. 20c. is .01 % of what? \$2000.
61. 1.25 lb. is $\frac{1}{4}$ % of what? 5 cwt.
62. 1s. is 3$\frac{1}{3}$ % of what? £1 10s.
63. \$40 is 150 % of what? \$26$\frac{2}{3}$.
64. \$56 is 14 % of what?
65. \$81.50 is 100 % of what?
66. 21c. is .3 % of what?
67. 1200 is 40 % of what?
68. 28.8 bu. is 2$\frac{2}{3}$ % of what?
69. \$456.75 is 105 % of what?
70. 1 % of 240 is 80 % of what?
71. 10 % of 300 is 5 % of what?

72. A farmer keeps 25 % of his sheep in one field, 15 % in another, and the rest, numbering 48, in a third. How many sheep has he? *Ans.* 80 sheep.

73. A collector, who gets 3 % for his services, makes \$33.33 by collecting a certain bill. How large is the bill, and how much must he pay over to his employer? *Last ans.* \$1077.67.

74. Of what number is $\frac{1}{2}$ % of 90 three hundred %? *Ans.* .15.

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* Before dividing, reduce dividend and divisor to the same denomination.

75. The deaths in a certain county average 320 a month, and the number of deaths each year is 3 % of the population. What is the population? *Ans.* 128000.

76. A person worth $40000 gave 30 % of it to his son, and this amount was 75 % of what his son had before. How much had the son after receiving the father's gift? *Ans.* $28000.

323. *To find the base, the rate and the sum or difference of the percentage and base being given.*

EXAMPLE 1.—A farmer, having a certain number of sheep, bought $33\frac{1}{3}$ % of that number more, and then had 256. How large was his flock at first?

As he increased his flock by $33\frac{1}{3}$ % of itself, he must then have had $133\frac{1}{3}$ % of the original number or base. As 256 is $133\frac{1}{3}$ % of the base, to find the base, divide 256 by $1.33\frac{1}{3}$, or its equivalent $\frac{4}{3}$ (Rule III. § 321).

$$100 + 33\tfrac{1}{3} = 133\tfrac{1}{3}\% = 1.33\tfrac{1}{3} = 1\tfrac{1}{3} = \tfrac{4}{3}$$
$$256 \div \tfrac{4}{3} = 192$$
Ans. 192 sheep.

EXAMPLE 2.—A farmer, having a certain number of sheep, sold $33\frac{1}{3}$ % of them, and then had 128 left. How large was his flock at first?

$$100 - 33\tfrac{1}{3} = 66\tfrac{2}{3}\%$$
$$66\tfrac{2}{3}\% = .66\tfrac{2}{3} = \tfrac{2}{3}$$
$$128 \div \tfrac{2}{3} = 192$$
Ans. 192 sheep.

As he sold $33\frac{1}{3}$ % of his flock, he must have had left $66\frac{2}{3}$ % of the original number or base. As 128 is $66\frac{2}{3}$ % of the base, to find the base, divide 128 by $.66\frac{2}{3}$ or its equivalent $\frac{2}{3}$ (Rule III. § 321).

RULE.—*Divide the given number by* 1 *increased or diminished by the rate expressed decimally, according as the sum or difference of the percentage and base is given.*

77. When I add to a certain number 25 % of itself, I get 540; what is the number? *Ans.* 432.

78. What number is that which diminished by $\frac{1}{2}$ % of itself is 778.09? *Ans.* 782.

79. A gentleman, having bought a house, spent 10 % of the purchase price in repairs, and then found that the whole cost was $8800? What was the purchase price?

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323. In stead of the percentage, what may be given, with the rate, to find the base? Explain Examples 1 and 2. Recite the rule for finding the base, the rate and the sum or difference of the percentage and base being given.

80. A merchant, having lost 7 % of his capital, has $23250 left; what was his capital?

81. A farmer set out some apple-trees; 5 % of them died the next summer, and 3 % the following winter; 138 lived. How many trees did he set out? *Ans.* 150 trees.

82. A lady spent 75 % of her money for a cloak, and 5 % for gloves; she then had $16 left. How much did she have at first?

324. Applications of Percentage.—The rules of Percentage are applied in many of the most common mercantile transactions. They form the basis of computations in Profit and Loss, Interest, Discount, Commission, Bankruptcy, Insurance, Assessment of Taxes, &c.

Profit and Loss.

325. Profit (or gain) and Loss are generally reckoned at a certain per cent. of the cost.

The *cost* is the *base.*

The *per cent.* of profit or loss is the *rate.*

The *amount* of profit or loss is the *percentage.*

326. Hence, applying the Rules of Percentage (§ 321),

$$\text{Profit or Loss} = \text{Cost} \times \text{Rate}$$

$$\text{Rate} = \frac{\text{Profit or Loss}}{\text{Cost}} \qquad \text{Cost} = \frac{\text{Profit or Loss}}{\text{Rate}}$$

327. *When the cost and selling price are given, their difference will be the profit or loss,—profit if the selling price is the greater, loss if the cost is the greater.*

328. *To find the selling price,—when there is profit, add it to the cost; when there is loss, subtract it from the cost.*

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324. What is said of the application of the rules of Percentage? In what do they form the basis of computations?—325. How are Profit and Loss generally reckoned? What corresponds to the *base*? What, to the *rate*? What, to the *percentage*?—326. Give the formulas that apply.—327. When the cost and selling price are given, what will their difference be?—328. How do you find the selling price, when there is profit? When there is loss?

EXAMPLES FOR PRACTICE.

Find the PROFIT or LOSS,

1. On goods that cost \$145, sold at 3 % advance. *Ans.* \$4.35 pr.
2. On goods costing £2500, sold at $4\frac{1}{2}$ % loss. *Ans.* £112 10s.
3. On furniture bought for \$850.75, sold at 7 % below cost.
4. On paper costing \$1485.50, and sold at a profit of 15 %.
5. On coal bought for \$9020, and sold at a loss of $6\frac{1}{4}$ %.
6. On tea sold at $\frac{1}{8}$ % below cost, which was \$666.66.

Find the SELLING PRICE of goods,

7. Bought at \$88.65, sold at $3\frac{1}{3}$ % below cost. *Ans.* \$85.695.
8. Bought at £120, and sold at 8 % advance. *Ans.* £129 12s.
9. Sold at 20 % below cost, bought for \$18000.
10. Sold at $10\frac{3}{4}$ % above their cost, which was \$5050.

Find the RATE % of profit or loss on goods,

11. Bought for \$13000, sold at a profit of \$292.50. *Ans.* $2\frac{1}{4}$ %.
12. Bought for \$80, sold for \$60. *Ans.* 25 %.
13. Bought for \$113.25, sold so as to gain \$113.25.
14. Bought for \$5601.30, sold so as to lose \$2800.65.
15. Bought for £250, sold for £200 (§ 327). *Ans.* 20 % loss.
16. Bought for \$1250, sold for \$1375. *Ans.* 10 % prof.
17. Sold for \$1090, bought for \$1000. *Ans.* 9 % prof.
18. Sold for \$245.18, bought for \$235.75.
19. Bought for \$800, and sold for \$894.40.
20. Bought for \$740, and sold for \$627.15.
21. Bought for \$815, and sold for \$220.05.
22. Bought for \$350.50, and sold for \$701.
23. Sold for \$540, at a profit of \$40. *Ans.* 8 %.
24. Sold for \$600.35½, at a loss of \$26.64½. *Ans.* $4\frac{1}{4}$ %.
25. Sold for \$200, at a loss of \$100.

Find the COST of goods,

26. Sold at a profit of \$40, being 20 % on the cost. *Ans.* \$200.
27. Sold at 7 % below cost, at a loss of \$350. *Ans.* \$5000.
28. Sold at $12\frac{1}{2}$ % above cost, at a profit of \$240. *Ans.* \$1920.
29. Sold so as to lose \$53, and $\frac{1}{2}$ % of cost.
30. Sold so as to gain \$10.50, and $\frac{1}{2}$ % of cost.

329. *To find the cost, when the selling price and rate of profit or loss are given.*

EXAMPLE 1.—A sold a horse for \$175, and by so doing gained 40 %. What did the horse cost?

This question is analogous to Example 1, § 323, under Percentage. As he gained 40 % of the cost, the selling price must have been 100 + 40, or 140, % of the cost. The question then becomes, \$175 is 140 % of what number? (Rule III., § 321.)

$100 + 40 = 140\%$
$175 \div 1.40 = 125$
Ans. \$125.

EXAMPLE 2.—A sold a horse for \$175, and by so doing lost 40 %. What did the horse cost?

$100 - 40 = 60\%$
$175 \div .60 = 291\frac{2}{3}$
Ans. \$291.66⅔

This is analogous to Ex. 2, § 323. As he lost 40 % of the cost, the selling price must have been 100 − 40, or 60, % of the cost. The question then becomes, \$175 is 60 % of what number?

RULE.—*Divide the selling price by* 1 *increased by the rate of profit, or diminished by the rate of loss, expressed decimally.*

31. By selling a house and lot for \$5790, the owner lost 3½ %. What was their cost? *Ans.* \$6000.

32. Sold 517 barrels of flour for \$8.10 a barrel, at a profit of 8 %. What was the whole cost? *Ans.* \$3877.50.

33. Sold 1100 tons of coal for £1361 5s., thereby losing 1 %. What was the cost per ton? *Ans.* £1 5s.

34. Some linen was sold for 61¾c. a yd., at a loss of 5 %. What was the cost of 7 pieces of this linen, averaging 13 yd. to the piece? *Ans.* \$59.15.

35. Sold a book-case for £15, and some books for £33 2s. 6d., and thereby gained 20¼ %. What was the cost of case and books? *Ans.* £40 4d. 3 far. +

36. D bought 5000 bu. of corn, but lost 10 % of it by fire; he sold what was left for \$3408.75, and by so doing gained 1 % on its cost. What did he give for the 5000 bu.? *Ans.* \$3750.

37. Selling price, \$4773.75; gain, ⅛ %; required, the cost.

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329. Explain Examples 1 and 2. Recite the rule for finding the cost, when the selling price and the rate of profit or loss are given.

330. *To find the rate of profit or loss at a proposed selling price, when the actual selling price and rate of profit or loss are given.*

Ex.—If, by selling a cow for \$60, I gain 20%, what % would I have gained or lost by selling her for \$25?

First find the cost, § 329:	\$60 ÷ 1.20 = \$50, cost.
Then find the gain or loss at the proposed selling price:	\$50 − \$25 = \$25, loss.
Find the rate, by dividing the loss by the cost:	25 ÷ 50 = 50% loss. *Ans.*

RULE.—*From the selling price obtain the cost* (§ 329); *then find the gain or loss at the proposed selling price by subtraction, and divide it by the cost.*

38. A profit of 4% is realized by selling some cloths for \$228.80; had they been sold for \$215.60, what % would have been gained or lost? *Ans.* 2% lost.

39. Some grain is sold for \$1835, at a loss of 11%; what amount would have been gained or lost, and what %, if it had been sold for \$3000? *Last ans.* 100% gd.

40. By selling some goods at \$1537.90, a profit of 12⅘% was realized; what per cent. would have been gained or lost, if they had sold for \$1651.65? *Ans.* 21% gd.

41. 2½% was lost by selling a farm for \$13650; what % would have been gained or lost by selling it for \$13986?

42. If by selling some wood for \$850 I made 100%, what % would I have gained by selling it for \$1275?

43. Sold a house for \$1000, thereby making \$200; what would I have had to sell it for, to gain 50%?

44. By selling some goods for \$4759.79, ⅐ of 1% was gained. What would these goods have had to be sold for, to realize a profit of 7%? *Ans.* \$5085.71.

45. A merchant bought 320 barrels of flour at \$7.50 a barrel, and sold them at a loss of 10%. How much did he lose?

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330. Explain the given example. Recite the rule for finding the rate of profit or loss at a proposed selling price, when the actual selling price and rate of profit or loss are given.

46. Bought 300 yd. merino at \$2.25 a yd., and sold the same at \$2.50 a yd. How much was gained, and what %?

47. Twenty-five cords of wood were bought at \$4.50 a cord, and sold at an advance of 25%. 40% of the bill was paid in cash; how much remained to be paid? *Ans.* \$84.375.

48. Bought a lot for \$600, fenced it for \$50, and built a house on it for \$1550. Sold the whole at a profit of $8\frac{1}{4}$%; what did it bring? *Ans.* \$2381.50.

49. A buys \$1000 worth of goods, which he sells to B at a gain of 5%. B sells them to C at a profit of 5%, and C sells them to D at a like profit. What did they cost D? *Ans.* \$1157.625.

50. S sells T some goods that cost him \$1480, at a loss of $3\frac{1}{2}$%. A few days afterwards, T sells them back again to S at a gain of $3\frac{1}{2}$%. How much less does S pay for them the second time than the first? *Ans.* \$1.81 +.

51. P buys an article for £50 13s. 6d., and sells it to Q at a profit of 10%. Q in turn sells it to R at a loss of 10%. What % of the original cost does R pay? *Ans.* 99%.

52. If a person buys 600 barrels of flour at \$9.25 a barrel, and sells $33\frac{1}{3}$% of the same at a profit of 10%, and the rest at a profit of $12\frac{1}{2}$%, how much will he receive in all, and what % will he gain on the whole? *Last ans.* $11\frac{2}{3}$%.

53. Bought 3000 bu. of wheat at \$1.60 a bushel. Sold 10 per cent. of it at a loss of 3%, 50 per cent. of it at a gain of 10%, and the rest at a gain of 2%. How much was made on the whole, and what per cent.? *Ans.* \$48, and 1%.

54. Sold some muslin for \$199.50, at a loss of $\frac{1}{4}$%; some linen for \$148.50, at a loss of 1%; some cloth for \$520, at a profit of 4%. What did the muslin, linen, and cloth cost? *Ans.* \$850.

55. Sold a horse for \$198, at a loss of 10%. Bought 3 cows for \$135. What must I sell the cows for apiece, to make up the loss on the horse and \$44 besides? *Ans.* \$67.

56. The difference between 50% and 71% of a certain number is 525. What is the number?

57. A house that cost \$5000, was repaired at an expense of \$1000. It was then sold for \$7500; what was the gain or loss %?

CHAPTER XX.

INTEREST.

331. Interest is what is paid for the use of money.

The **Principal** is the money used, for which interest is paid.

The **Rate** is the number of dollars paid for the use of $100 for a certain time, usually for a year (*per annum*). It is written and operated with as so many per cent. When no time is mentioned with the rate, a year is meant.

The **Amount** is the sum of the principal and interest.

I borrow $100 for a year, and pay $6 for its use; the Principal is $100, the Interest $6, the Rate 6 %, the Amount $106.

332. Interest is distinguished as Simple and Compound. It is called Simple, when reckoned on the principal only; Compound, when allowed on interest as well as principal. When the word *interest* is used alone, Simple Interest is meant.

333. There is a rate of interest fixed by law, called the **Legal Rate**, for cases in which no other rate is specified. Parties may always agree on a lower rate than the Legal Rate, and in some of the states on a higher one; but there is generally a limit fixed, beyond which the taking of interest is forbidden under certain penalties—the offence being called Usury.

The legal rate in England and France is 5 %; in Canada, Nova Scotia, and Ireland, 6 %. In all of the United States it is 6 %, except the following: Louisiana, 5 %; New York, Michigan, Wisconsin, Minnesota, South Carolina,

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331. What is Interest? What is the Principal? What is the Rate? How is the Rate written and operated with? What is the Amount? Illustrate these definitions.—332. How is interest distinguished? When is it called Simple? When, Compound?—333. What is meant by the Legal Rate? May the parties agree on a lower rate than the legal one? On a higher one? What is Usury? In what countries is the legal rate 5 per cent.? In what, 6 per cent.? In which of the United States is it 5 per cent.? In which, 7 per cent.? In which, 8 per cent.? In which,

and Georgia, 7%; Alabama, Florida, Mississippi, and Texas, 8%; California and Kansas, 10%; Oregon, 12½%.

334. Interest is an application of Percentage, the additional element of *time* being introduced. The principal is the base; the interest is the percentage, reckoned at a certain rate, *for a certain time.*

To find the Interest.

335. Case I.—*To find the interest for any number of years, when the principal and rate are given.*

Ex. 1.—What is the interest of $124.50, for 1 year, at 6%?

That is, what is 6%, or $\frac{6}{100}$, of $124.50? Taking $\frac{6}{100}$ is equivalent to multiplying by $\frac{6}{100}$. Hence, multiply the principal by .06.

$124.50	Prin.
.06	Rate.
$7.4700	Int.

Ex. 2.—Find the amount of $124.50, at 6%, for 5 years.

$124.50	Prin.
.06	Rate.
7.4700	Int. 1 yr.
5	
37.3500	Int. 5 yr.
124.50	Prin.
$161.85	Amt. 5 yr.

Find the interest for 1 yr. as above, $7.47. For 5 yr. it will be 5 times $7.47; and, as the amount is required, add the principal to the last product.

In stead of multiplying by the rate and years separately, it sometimes saves work to multiply by their product. Thus, in Example 2, it would be shorter to multiply by .30 than by .06 and 5.

Rule.—*Multiply the principal by the rate per annum expressed decimally, and that product by the number of years.*

Aliquot parts of a year may be expressed fractionally. Thus, 5 yr. 6 mo. = 5½ yr. See Table, page 186.

336. So, when the rate is given by the *month*, the interest may be found for any number of months, *by multiplying the principal by the rate per month expressed decimally, and that product by the number of months.*

337. *For the Amount, add the principal to the interest.*

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10 per cent.? In which, 12½ per cent.? In the rest?—334. Of what is Interest an application? What additional element is introduced?—335. What is Case I.? Explain Example 1. Go through Example 2. Recite the rule. How may aliquot parts of a year be expressed?—336. When the rate is given by the month, how may the interest be found for any number of months?—337. How is the amount found?

EXAMPLES FOR PRACTICE.

1. Find the interest of \$1, at $3\frac{1}{3}$%, for 3 years. *Ans.* 10c.
2. Find the amount of \$540, at 7%, for 9 yr. *Ans.* \$880.20.
3. Find the interest of \$90, at $4\frac{1}{2}$%, for 6 yr. *Ans.* 24.30.
4. Find the amount of £1400, at 8%, for $2\frac{1}{2}$ yr. *Ans.* £1680.
5. Find the interest of \$825, at $6\frac{1}{4}$%, for 4 yr. *Ans.* \$206.25.
6. What is the interest of \$33120.01, for 5 yr., at 6%?
7. What is the interest of \$987.41, for 13 yr., at 7%?
8. Find the interest of \$69582.57, at 5%, for $2\frac{1}{4}$ years.
9. Find the amount of \$9812.17, at $4\frac{3}{4}$%, for 4 years.
10. Find the amount of \$700, at 6%, for 2 yr. 6 mo. ($2\frac{1}{2}$ yr.).
11. Find the amount of \$820, at 3%, for 4 yr. 4 mo. ($4\frac{1}{3}$ yr.).
12. Find the amount of \$660, at 5%, for 3 yr. 3 mo.
13. What is the interest of \$60.50, for 3 months, at 1% a month? (See § 336.) *Ans.* \$1.815.
14. What is the amount of \$12198.75, for 2 months, at $\frac{3}{4}$% a month? *Ans.* \$12381.73.
15. What is the interest of £600, at $\frac{1}{2}$% a month, from Jan. 1 to April 1 of the same year? *Ans.* £9.
16. What is the amount of \$8250, from April 3, 1861, to April 3, 1866, at $5\frac{3}{4}$% per annum? *Ans.* \$10621.875.
17. Borrowed, Jan. 1, 1865, in California, \$900 (no rate specified). What amount must be repaid, Jan. 1, 1866? *Ans.* \$909.
18. A owes B interest on \$450, from Feb. 2 to Oct. 2; B owes A interest on \$575, from April 2 to Oct. 2. What is the balance of interest, and to whom is it due, the rate being $\frac{1}{2}$% a month? *Ans.* 75c., to B.
19. What is the interest on \$68.40, at $4\frac{1}{2}$%, for 4 yr. 2 mo. ($4\frac{1}{6}$ yr.)? On \$712, for 6 yr. 3 mo., at 5%? On \$2688.88, at $6\frac{1}{2}$%, for 1 yr. 6 mo.? On \$1263.25, for 5 mo., at 1% a month? *Sum of answers*, \$560.1783.
20. Loaned, New York, Feb. 1, 1864, \$1050. What amount should I receive for loan and interest, March 1, 1866?
21. C, living in Canada, owes \$500 with interest for 3 yr. 2 mo. He pays \$550 on account; how much remains due?

338. Case II.—*To find the interest, at 6 per cent., for years, months, and days.*

1. The interest or amount of any principal for a given time and rate, equals the interest or amount of $1 for the same time and rate, multiplied by the given principal.

Thus, the interest of $50, for 5 mo., at 6 %, is 50 times the interest of $1, for 5 mo., at 6 %. The amount of $60, at 7 %, for 30 days, is 60 times the amount of $1, at 7 %, for 30 days.

2. The interest of $1, at 6 %, is 6 cents for 1 year. Hence it is 1 cent for every two months, and 1 mill for ($\frac{1}{10}$ of 2 months, or) 6 days.

6 days are $\frac{1}{10}$ of 2 months, if 30 days are allowed to the month, according to general usage in the United States. Each day's interest is thus made $\frac{1}{360}$, in stead of $\frac{1}{365}$, of 1 year's interest; it thus exceeds the exact interest by $\frac{5}{365}$, or $\frac{1}{73}$, of itself. To find the exact interest for any number of days, see § 342.

Example 1.—What is the interest of $1200, at 6 %, for 3 yr. 7 mo. 18 da.?

First Method.—First find the interest of $1, at 6 %, for the given time.

As the interest is 1 cent for every 2 months, for 3 years 7 months, or 43 months, it will be ½ of 43 cents, or $.215. As the interest is 1 mill for 6 days, for 18 days it will be ⅙ of 18 mills, or $.003. Adding $.215 and $.003, we find the interest of $1 for the given time to be $.218. For $1200 it will be 1200 times as much, or $261.60.

$.215	
.003	
$.218	
1200	
$261.600	*Ans.*

Second Method.—First find the interest of $1200 for 1 yr., then for 3 yr., as in Case I. For the months and days apply the principles of Practice, § 307.

7 mo. are not an aliquot part of 1 yr., but 6 mo. = ½ yr.; therefore, for 6 mo. take ½ of 1 year's interest, and for 1 mo., which remains, take ⅙ of the interest for 6 mo. 18 days are not an aliquot part of 1 mo., but 15 da. = ½ mo.; therefore, for 15 days take ½ of 1 month's interest, and for 3 days, which remain, take ⅕ of the interest for 15 days. Finally, add the several items of interest.

	$1200
	.06
6 mo. = ½	72.00
	3
	216.00
	36.00
1 mo. = ⅙	6.00
15 da. = ½	3.00
3 da. = ⅕	.60
Ans.	$261.60

338. What is Case II.? To what is the interest or amount of any principal for a given time and rate equal? Give examples. For how long a time will the interest of $1, at 6 per cent., be 1 cent? 1 mill? How does the interest for 1 day thus

339. RULE.—1. *Take $\frac{1}{2}$ the number of months as cents, $\frac{1}{6}$ the number of days as mills, for the interest of \$1 for the given time. Multiply this sum and the given principal together.*

2. *Or, find the interest first for the given number of years, as in § 335, then for the months and days by taking the necessary parts, and add the results.*

The first method is generally shorter and easier.

When the *amount* is required, and the first method is used, multiply the *amount* of \$1 and the given principal together.

EXAMPLE 2.—What is the amount of \$66.60, at 6%, for 1 year 11 mo. 11 da.?

1 yr. 11 mo. = 23 mo. Taking $\frac{1}{2}$ the number of months as cents, we have \$.115. Taking $\frac{1}{6}$ the number of days as mills, we have \$.001$\frac{5}{6}$. As the amount is required, add in \$1. \$.115 + .001$\frac{5}{6}$ + 1 = 1.116$\frac{5}{6}$. Multiply the principal by this sum.

```
        66.60
        1.116⅚
        ------
         5550
        39960
        6660
       6660
      6660
      --------
Ans. $74.38110
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EXAMPLES FOR PRACTICE.

At 6 per cent., required the

1. Interest of \$49.37, for 1 yr. 1 mo. 15 da. *Ans.* \$3.33 +.
2. Amount of \$341.13, for 7 yr. 9 da. *Ans.* \$484.916 +.
3. Amount of \$591.03, for 4 yr. 3 mo. 7 da. *Ans.* \$742.43 +.
4. Interest of \$0.134, for 4 months 3 days. *Ans.* \$ .0027 +.
5. Amount of \$7.50, for 7 months. *Ans.* \$7.76 +.
6. Interest of \$371.01, for 4 years 15 days. *Ans.* \$89.969 +.
7. Interest of \$57.92, for 3 yr. 7 mo. 9 da. *Ans.* \$12.53968.
8. Amount of \$329, for 5 years 13 days. *Ans.* \$428.41 +.
9. Amount of \$47.39, for 1 year 7 months.
10. Interest of \$2250, for 2 yr. 2 mo. 24 da.
11. Interest of \$5762, for 6 yr. 4 mo. 19 da.
12. Amount of \$840.75, for 11 months 21 days.
13. Interest of 98.76, for 3 yr. 5 mo. 22 da. *Ans.* \$20.60792.

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computed compare with the true interest? Go through Example 1 according to each method.—339. Recite the Rule. Which method is preferred? When the amount is required, what must be done? Explain Example 2.

14. Interest on \$718, from April 19 to Aug. 3 following. From Oct. 29, 1865, to Feb. 11, 1866. *Sum of answers*, \$24.65 +.

15. Interest of £500, for 2 yr. 4 mo. 12 da. *Ans.* £71.

Compute the interest on pounds as on dollars. A decimal in the answer must be reduced to shillings, &c.

16. Amount of £2500, for 1 year 9 months 18 days.

17. Interest of £480, for 1 yr. 3 mo. 20 da. *Ans.* £37 12s.

18. Amount of £60, for 8 yr. 6 mo. 2 da. *Ans.* £90 12s. 4d. +.

19. P owes Q \$975, with interest for 1 yr. 10 mo. 10 da.; Q owes P \$720, with interest for 2 yr. 25 da. The rate being 6%, what is the balance, and to whom is it due? *Ans.* \$274.475, to Q.

20. A merchant collects the interest on \$400, at 7%, for 1 yr. 6 mo.; on \$220, at 6%, for 8 mo. 8 da.; on \$694.10, for 2 yr. 2 da., at 6%; and on \$1180.50, for 26 days, at 6%. How much does he collect in all? *Ans.* \$139.732 +.

340. Merchants often have to cast interest, at 6%, for 30, 60, and 90, also for 33, 63, and 93 days. The following short methods can be used mentally:—

For 60 *days, simply move the decimal point in the principal two places to the left,*—for this will be multiplying it by .01, the interest of \$1 for 60 days.

For 30 *days, take* $\frac{1}{2}$ *of this result.*

For 3 *days, take* $\frac{1}{10}$ *of the interest for* 30 *days,*—that is, move the decimal point one place to the left.

Combine these results as may be required.

Example.—Required the interest of \$560, at 6%, for 30, 60, 90, 33, 63, and 93 days.

Int. 60 days, \$5.60
Int. 30 days, \$2.80
Int. 3 days, \$ .28

Then
Int. 90 days, \$5.60 + \$2.80 = \$8.40
Int. 33 days, \$2.80 + \$0.28 = \$3.08
Int. 63 days, \$5.60 + \$0.28 = \$5.88
Int. 93 days, \$8.40 + \$0.28 = \$8.68

At 6 per cent., what is the interest of

21. \$700 for 60 days? For 33 days? For 90 days?

22. \$1200 for 30 days? For 60 days? For 90 days?

23. £1000 for 63 days? For 90 days? For 93 days?

24. $74.75 for 60 days? For 63 days? For 93 days?
25. $180.90 for 33 days? For 63 days? For 93 days?
26. $2000.50 for 30 days? For 90 days? For 33 days?

341. CASE III.—*To find the interest, at any rate, for years, months, and days.*

Ex.—Find the interest of $120, at 7 %, for 2 yr. 5 mo. 6 da.

First find the int. at 6 %: $\left\{\begin{array}{l}\frac{1}{2} \text{ number of mo.,}\\ \frac{1}{6} \text{ number of days,}\end{array}\right.$	$ .145 .001
Interest of $1, for the given time, at 6 %,	$ .146
Multiply by the given principal,	120
Interest of $120, for the given time, at 6 %,	$17.520
For 7 %, add to the interest at 6 %, $\frac{1}{6}$ of itself,	2.920
(for $7 = 6 + \frac{1}{6}$ of 6).	$20.440 *Ans.*

	$120	Princ.
	.07	Rate.
4 mo. = $\frac{1}{3}$	$8.40	Int. 1 yr.
	2	
	$16.80	Int. 2 yr.
	2.80	Int. 4 mo.
1 mo. = $\frac{1}{4}$	.70	Int. 1 mo.
6 da. = $\frac{1}{5}$	.14	Int. 6 da.
Ans.	$20.44	Int. 2 y. 5 m. 6 d.

Or, we may find the interest at once at 7 %, for 1 yr.; then for 2 yr., by multiplying by 2; then for the months and days, by taking parts. For 4 mo., take $\frac{1}{3}$ of 1 year's interest; for 1 mo., take $\frac{1}{4}$ of 4 months' interest; for 6 days, take $\frac{1}{5}$ of 1 month's interest. For the whole, add these parts.

RULE.—1. *Find the interest at 6 %, and add thereto, or subtract therefrom, such a part of itself as must be added to or subtracted from 6 to produce the given rate.*

For 7 %, add $\frac{1}{6}$ ($7 = 6 + \frac{1}{6}$ of 6).	For 5 %, subt. $\frac{1}{6}$ ($5 = 6 - \frac{1}{6}$ of 6).
For 8 %, add $\frac{1}{3}$ ($8 = 6 + \frac{1}{3}$ of 6).	For $4\frac{1}{2}$ %, subt. $\frac{1}{4}$ ($4\frac{1}{2} = 6 - \frac{1}{4}$ of 6).
For 9 %, add $\frac{1}{2}$ ($9 = 6 + \frac{1}{2}$ of 6).	For 4 %, subt. $\frac{1}{3}$ ($4 = 6 - \frac{1}{3}$ of 6).
For 10 %, add $\frac{2}{3}$ ($10 = 6 + \frac{2}{3}$ of 6).	For 3 %, take $\frac{1}{2}$ the interest at 6 %.

2. *Or, find the interest at the given rate, for the given number of years, as in* § 335; *then for the months and days, by taking the necessary parts; and add the results.*

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341. What is Case III? Give both solutions of the Example. Recite the rule. For 7 per cent., what must we do, and why? For 4 per cent.? For 10 per cent.? For 8 per cent.? For 9 per cent.? For $4\frac{1}{2}$ per cent.? For 8 per cent.? For 5 per cent.?

EXAMPLES FOR PRACTICE.

What is the interest (by either or both of the methods given in the preceding Rule) of

1. \$5.37, for 4 years 12 days, at 8%? *Ans.* \$1.73 +.
2. \$40.17, for 3 months 18 days, at 3%? *Ans.* 36c. +
3. \$37.13, for 5 months 12 days, at $4\frac{1}{2}$%? *Ans.* 75c. +
4. \$194.10, for 1 yr. 7 mo. 13 da., at 7%? *Ans.* \$22. +
5. \$321.21, for 5 yr. 9 mo. 21 da., at 9%? *Ans.* \$167.91 +.
6. \$9872.86, for 1 yr. 5 mo. 11 da., at 7%? *Ans.* \$1000.175 +.
7. \$999.99, for 11 months 29 days, at 5%? *Ans.* \$49.86 +.
8. \$27541.03, for 2 yr. 10 mo. 22 da., at 7%? *Ans.* \$5580.11 +.
9. \$137.50, for 6 mo. 10 da., at $6\frac{1}{2}$%? (Add $\frac{1}{12}$.) *Ans.* \$4.717 +.
10. \$4650, for 3 yr. 4 mo. 12 da., at 7%? *Ans.* \$1095.85.
11. \$2000, for 33 days, at 10%? For 63 days?
12. \$11500, for 60 days, at 4%? For 90 days?
13. \$8260, for 3 yr. 29 da., at $5\frac{1}{2}$%? (Subtract $\frac{1}{12}$.)
14. \$428.07, for 1 yr. 1 mo. 1 da., at 7%?
15. \$ .75, for 10 yr. 10 mo. 10 da., at 5%?
16. A, living in New York, owes B \$625, with interest from Jan. 1 to Sept. 15, no rate specified. He pays on account \$540.25; how much remains due? *Ans.* \$115.618.
17. What is the amount of \$469.10, for 3 yr. 2 mo., at 7%? For 1 yr. 20 days, at 4%? For 11 mo. 19 da., at 5%? For 6 mo. 6 da., at 8%? *Sum of answers*, \$2042.318 +.
18. A merchant living in Mississippi collects \$100, with interest for 3 yr. (no rate specified); \$427.50, with interest for 8 mo. 9 da., at 7%; \$1100, with interest for 1 yr. 18 da., at 6%. How much does he collect in all? *Ans.* \$1741.50.

342. CASE IV.—*To find the exact interest for days.*

The exact number of days between two dates within a year of each other can be found by the Table on page 156.

Each day being $\frac{1}{365}$ of 1 year, the exact interest for

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342. What is Case IV.? How may the exact number of days between two dates within a year of each other be found? What fraction of 1 year's interest will the exact interest for any number of days be? Recite the rule. Solve the Example.

any number of days will be as many 365ths of 1 year's interest as there are days. Hence the following

RULE.—*Multiply the interest for* 1 *year at the given* % *by the number of days, and divide the product by* 365.

EXAMPLE.—What is the exact interest of \$37.37, from May 3, 1865, to Dec. 27, 1865, at 7%?

1 year's interest = \$37.37 × .07 = \$2.6159.
By Table, p. 156, we find the number of days to be 238.
We must therefore take $\frac{238}{365}$ of \$2.6159.
\$2.6159 × 238 = \$622.5842. \$622.5842 ÷ 365 = \$1.705. *Ans.*

1. What is the exact interest of \$100, at 6%, from Jan. 13 to Nov. 15, it being leap year? *Ans.* \$5.047.
2. What is the exact interest of £1000, from June 20 to Aug. 13, at 7%? *Ans.* £10.356 = £10 7s. 1d. 1 far. +
3. What is the exact interest of \$730, from July 4 to Dec. 25, at 6%? *Ans.* \$20.88.
4. What is the exact interest of \$2160, from March 10 to Dec. 1, at 5%? What is the amount? *Ans.* Amt., \$2238.71.
5. What is the exact interest of \$21450, at 8%, for 20 days?
6. What is the exact interest of £4500, at 4½%, for 25 days?

343. CASE V.—*To find the interest or amount of pounds, shillings, pence, and farthings, at any rate, for any time.*

EXAMPLE.—What is the interest of £84 10s. 6d., at 4%, for 1 yr. 3 mo.?

For convenience of multiplying and dividing, we reduce 10s. 6d. to the decimal of a pound, § 285. The principal thus becomes £84.525. Now, proceeding as in Federal Money, we find the interest to be £4.22625,—or, reducing the decimal to shillings, &c., § 284, £4 4s. 6d. 1.2 far.

$$
\begin{array}{rr}
 & £84.525 \\
 & .04 \\ \hline
3 = \frac{1}{4} & 3.38100 \\
 & .84525 \\ \hline
 & £4.22625 \\
 & 20 \\ \hline
 & \text{s. } 4.52500 \\
 & 12 \\ \hline
 & \text{d. } 6.30000 \\
 & 4 \\ \hline
 & \text{far. } 1.20000
\end{array}
$$

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343. What is Case V.? Go through and explain the given Example. Recite the rule for finding the interest or amount of pounds, shillings, &c., at any rate, for any time.

RULE.—*Reduce the shillings, &c., of the principal to the decimal of a pound, find the interest or amount as in Federal Money, and reduce the decimal in the result to lower denominations.*

344. If the rate is 5% and the time 1 year, the interest is readily found by taking 1s. for every pound of the given principal, 3d. for every 5s., and 1 far. for every 5d. For, 5% is $\frac{1}{20}$; and 1s. is $\frac{1}{20}$ of £1, 3d. is $\frac{1}{20}$ of 5s., 1 far. is $\frac{1}{20}$ of 5d.

EXAMPLE.—Required the interest of £86 17s. 6d., at 5%, for 1 year.

1s. on £1 of the given principal gives		86s. =	£4 6s.
3d. for every 5s.	17s. 6d. ÷ 5s. = $3\frac{1}{2}$ $3\frac{1}{2} \times 3$	=	$10\frac{1}{2}$d.
		Ans.	£4 6s. $10\frac{1}{2}$d.

EXAMPLES FOR PRACTICE.

Find the interest of

1. £760 5s. 6d., at 5%, for $2\frac{1}{4}$ yr. *Ans.* £88 13s. 11d. 2.8 far.
2. £1 7s. 6d., at $4\frac{1}{2}$%, for 2 yr. 6 mo. *Ans.* 3s. 1d. $\frac{1}{2}$ far.
3. £8260 18s., at $3\frac{1}{2}$%, for 21 da. (§ 342). *Ans.* £16 12s. 8d. +
4. £275 10d., at 4%, for 5 yr. 10 mo. *Ans.* £64 3s. 6d. +
5. Find the amount of £7 15s., for 1 yr., at 5% (§ 344).
6. Find the amount of £42 2s. 6d., for 1 yr., at 5%.
7. Find the amount of £88 7s. 6d., for 1 yr., at 5%.
8. Find the amount of £68 12s. 6d., for 1 yr., at 5%.
9. Find the amount of £100 15s. 3d., at $4\frac{3}{4}$%, for 2 yr. 7 mo.

345. Observe that whenever the product of the rate per annum and the number of years is 100 (or 1, if the rate is expressed decimally), the interest equals the principal. The interest of \$75, at 5%, for 20 years, is \$75; since $5 \times 20 = 100$ (or $.05 \times 20 = 1$).

346. In stead of computing interest by any of the methods that have been given, many use Interest Tables. These being constructed for different principals, rates, and periods of time, the required interest is found in some cases by a simple reference, in others by an addition of items.

344. If the rate is 5 per cent. and the time 1 year, what short method may be used? Explain the principle on which this method is based.—345. Under what circumstances does the principal equal the interest? Give an example.—346. In stead of computing interest, what do many use?

347. To find the Rate,

the principal, interest or amount, and time, being given.

Ex. 1.—At what rate will \$400 yield \$55 interest, in 2 yr. 6 mo.?

$400 × .01 = \$4, int. 1 yr.
\$4 × 2½ = \$10, int. 2½ yr.
\$10) \$55
5½ *Ans.*

The interest of \$400, at *one per cent.* for 2 yr. 6 mo., is \$10. To produce \$55 interest, the rate must be as many times 1% as \$10 is contained times in \$55, or 5½. *Ans.* 5½%.

RULE.—*Divide the given interest by the interest of the principal, for the given time, at 1%.*

348. If, in stead of the interest, the amount is given, subtract the principal from it, to find the interest, and proceed as above.

349. PROVE by trying whether, at the rate found, the principal will produce the given interest in the given time.

Ex. 2.—At what rate will \$630 amount to \$665.28, in 9 mo. 18 da.?

Find the interest of \$630, for 9 mo. 18 da., at 1%.

First find it at 6%, according to § 339: { ½ number of months, .045; ⅙ number of days, .003 }

At 1% it will be ⅙ as much as at 6%: 6) .048

Interest of \$1 for the given time, at 1%, \$ .008

For \$630 it will be 630 times as much: \$ .008 × 630 = \$5.04.

The interest is \$665.28 − \$630 = \$35.28.

Dividing \$35.28 by \$5.04, we have 7% *Ans.*

PROOF.—Will \$630, at 7%, amount to \$665.28 in 9 mo. 18 da.?

EXAMPLES FOR PRACTICE.

Prove each example, § 349. At what rate will

1. \$530, in 3 yr. 6 mo., yield \$92.75 interest? *Ans.* 5%.
2. \$4070 yield \$91.575 interest quarterly? *Ans.* 9%.
3. \$100, in 9 mo. 10 da., yield \$3.50 interest? *Ans.* 4½%.
4. £6000, in 5 yr. 20 da., amount to £7820? *Ans.* 6%.
5. £2600 yield £104 interest semi-annually?
6. At what rate will \$1250, in 60 days, amount to \$1264.58⅓?

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347. To find the rate, what must be given? Analyze Example 1. Recite the rule for finding the rate.—348. If the amount is given, in stead of the interest, what must you do?—349. How may this operation be proved? Go through Example 2.

350. To find the Time,

the principal, interest or amount, and rate, being given.

Ex.—In what time will $400 yield $55 interest, at $5\frac{1}{2}$%?

The interest on $400 for *one year*, at $5\frac{1}{2}$%, is $22. To produce $55 interest, will require as many years as $22 is contained times in $55, or $2\frac{1}{2}$.

$400 × .055 = $22, int. 1 yr.
$55 ÷ $22 = 2.5 yr. *Ans.*
Proof. $400 × .055 × 2.5 = $55.

Rule.—*Divide the given interest by the interest of the principal, at the given rate, for* 1 *year.*

A decimal in the quotient must be reduced to months and days, § 284.

Prove by trying whether, in the time found, the principal will produce the given interest at the given rate.

EXAMPLES FOR PRACTICE.

Prove each example. In what time will

1. $4070, at 9%, yield $91.575 interest? *Ans.* 3 mo.
2. $530, at 5%, yield $92.75 interest? *Ans.* 3 yr. 6 mo.
3. $100, at $4\frac{1}{2}$%, yield $3.50 interest? *Ans.* $\frac{7}{9}$ yr. (9 mo. 10 da.)
4. £6000, at 6%, amount to £7820? *Ans.* 5 yr. 20 d.

Find the interest: £7820 − £6000. Then proceed as above.

5. $820, at $5\frac{1}{2}$%, amount to 857.58\frac{1}{3}$? *Ans.* 10 mo.
6. $1250, at 7%, amount to 1264.58\frac{1}{3}$?
7. $700, at 7%, amount to $785.75?
8. £680, at 8%, amount to £950?
9. How long will it take $230, at 4%, to yield $230 interest,—that is, to double itself?

351. In Example 9, the interest equals the principal. Hence, the product of the rate per annum and number of years must be 100 (§ 345); and, to find the years, we may at once divide 100 by the rate. 100 ÷ 4 = 25 yr. *Ans.*

Were the years given, and the rate required, we should divide 100 by the number of years. 100 ÷ 25 = 4%. *Ans.*

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350. To find the time, what must be given? Analyze the example. Recite the rule for finding the time. How may the operation be proved? What must be done with a decimal in the quotient?

10. How long will it take \$600 to amount to \$1200, at 6%?

11. How long will it take \$43.50 to double itself, at 7%? At 3%? At 8%? At 4½%? At 5%? At 9%?

12. In what time will \$150 double itself, at 1% a month? At ¾ of 1% a month? At 2% a month?

13. At what rate will \$60 double itself, in 5 yr.? *Ans.* 20%.

14. At what rate will \$75 amount to \$150, in 12 yr. 6 mo.?

15. At what rate will \$12.125 amount to \$24.25, in 20 yr.? In 16 yr.? In 12 yr.? In 10 yr.?

352. To find the principal,

the interest or amount, time, and rate, being given.

Ex. 1.—What principal will, in 2 yr. 6 mo., at 5½%, yield \$55 interest?

The interest on \$1, for 1 yr., at 5½%, is \$.055; for 2 yr. 6 mo., it is 2½ times \$.055, or \$.1375. To produce \$55 interest, the principal must be as many times \$1 as \$.1375 is contained times in \$55, or 400. *Ans.* \$400.

$$\$1 \times .055 = \$.055$$
$$\$.055 \times 2.5 = \$.1375$$
$$\$55 \div \$.1375 = 400$$

Ans. \$400

PROOF. $\$400 \times .055 \times 2.5 = \$55$

Ex. 2.—What principal will, in 9 months 18 days, amount to \$665.28, at 7%?

½ number of mo.,	\$.045
⅙ number of da.,	\$.003
Int. of \$1, at 6%,	\$.048
Add ⅙,	\$.008
Int. of \$1, at 7%,	\$.056
Amt. of \$1, at 7%,	\$1.056

$\$665.28 \div \$1.056 = 630$

Ans. \$630.

The amount of \$1, for 9 mo. 18 da., at 7%, is \$1.056. To make the amount \$665.28, the principal must be as many times \$1 as \$1.056 is contained times in \$665.28, or 630. *Ans.* \$630.

PROOF.—Will \$630, in 9 mo. 18 da., amount to \$665.28, at 7%?

RULE.—*Divide the given interest (or amount) by the interest (or amount) of* \$1 *for the given time, at the given rate.*

PROVE by trying whether the principal found will produce the given interest or amount in the given time, at the given rate.

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352. To find the principal, what must be given? Analyze Example 1. Analyze Example 2. Recite the rule for finding the principal. How may the operation be proved?

EXAMPLES FOR PRACTICE.

Prove each example. What principal will yield

1. $91.575 interest, every quarter, at 9%? *Ans.* $4070.
2. $92.75 interest, in 3 years 6 months, at 5%? *Ans.* $530.
3. $9.75 interest, in 1 yr. 7 mo. 6 da., at 6%?
4. $79.80 interest, at 4¾%, in 4 years?
5. $.413 interest, at 6%, in 30 days?
6. $1.65 interest, in 90 days, at 6%?
7. $40 interest, in 10 days, at 1½% a month?
8. What principal will amount to £58, in 2 yr., at 8%? *Ans.* £50.
9. What principal will amount to $55, in 1 yr. 8 mo., at 6%?
10. What principal will amount to $12120, in 60 days, at 6%?
11. What principal will amount to $857.31, in 8 mo., at 3%?
12. What principal will double itself in 10 years, at 10%?
13. What principal, placed at interest in California at the legal rate, on the 1st of September, would amount to $1047.87½ on the 16th of the following January? *Ans.* $1010.
14. What sum lent at ½% a month, Feb. 1, 1865, would amount July 22, 1865, to $15427.50? *Ans.* $15000.
15. How much must a lady invest at 6% in her son's name, when he is just twenty years old, that on arriving at twenty-one he may have $10000?
16. How much must a gentleman invest for his daughter at 7%, that she may have $630 a year?
17. What investment at 5% will yield a person a semi-annual income of $500? *Ans.* $20000.
18. What sum invested at 6% will yield $65 a month?
19. What sum invested at 7% will yield $3.50 a day?
20. A person having $100000 invested at 6½%, gives his son sufficient to yield a quarterly income of $650, and his niece 80% of that amount. How much does he retain? *Ans.* $28000.
21. A's property, invested at 7%, yields him $8050 a year. B's income is 90% of A's, and his property is invested at 6%. Which is worth the most, and how much? *Ans.* B, $5750.
22. At what rate will $200, in 2 mo. 12 da., produce $1 interest?

Compound Interest.

353. Compound Interest is that which accrues on interest due and unpaid, as well as principal.

Compound interest can not be collected by law, yet it is often claimed on the ground that, since the debtor has the use of the interest, he should pay for it as well as for that of the principal. Savings Banks pay compound interest to those who do not draw their interest when it is due.

354. Interest may be compounded annually, semi-annually, quarterly, or for any other term, according to the time at which the interest is originally made payable.

Ex. 1.—What is the compound interest of $600, at 7 %, for 2 yr.?

COMPOUNDED ANNUALLY.

As the interest is to be compounded annually, we find the amount of the principal, at 7 %, for 1 yr., by multiplying it by 1.07, the amount of $1, at 7 %, for 1 yr. This becomes a new principal, of which, in like manner, we find the amount for the second year. This amount, diminished by the original principal, will be the compound interest required.

Principal,	$600
	1.07
	4200
	6000
Amt. for 1 yr.,	642.
	1.07
	4494
	6420
Amt. for 2 yr.,	686.94
Less principal,	600.00
Compound int.,	$86.94

COMPOUNDED SEMI-ANNUALLY.

In compounding semi-annually, we find the amount of the principal for 6 mo., by multiplying it by 1.035, the amount of $1, at 7 %, for 6 mo. This becomes a new principal, of which we find the amount for a second period of 6 mo.; then the amount of this result for a third period, and of this last amount for a fourth—making in all 2 yr. We then subtract the original principal.

$600	Principal.
1.035	
621.	Amt. for 6 mo.
1.035	
642.735	Amt. for 12 mo.
1.035	
665.230	Amt. for 18 mo.
1.035	
688.513	Amt. for 24 mo.
600.000	
$88.513	Compound int.

353. What is Compound Interest? Can it be collected by law? On what ground is it claimed? What institutions pay compound interest?—**354.** For what terms may interest be compounded? Go through the example, explaining the steps.

Ex. 2.—Find the compound interest of $600, for 2 yr. 6 mo. 18 da., at 7%, interest payable annually.

We find the amount for 2 yr., the number of entire periods, as above. This we multiply by 1.0385, the amount of $1 for the remaining time, 6 mo. 18 da. The product is the amount at compound interest for 2 yr. 6 mo. 18 da.; from which we obtain the compound interest by subtracting the original principal.

```
    $686.94
     1.0385
     343470
    549552
   206082
  686940
$713.387190
 600.
$113.387  Ans.
```

355. Rule.—*Find the amount of the given principal to the time when the first interest is due. On this amount compute the amount for a like period, and so proceed as many times as payments of interest are due,—always taking the last amount for the new principal. If any time then remains, find the amount for such time; and, to obtain the compound interest, subtract the original principal from the last amount.*

EXAMPLES FOR PRACTICE.

1. What is the compound interest of $1000, for 3 years, at 7%, interest payable annually? *Ans.* $225.043.

2. What, if the int. is payable semi-annually? *Ans.* $229.255.

3. What is the compound interest of $630, for 4 years, at 5%, interest payable annually? *Ans.* $135.769.

4. What, if the int. is payable half-yearly? *Ans.* $137.593.

5. What is the amount of $50, at compound interest for 3 yr., at 8%, interest payable yearly? *Ans.* $62.985.

6. What, if the interest is payable quarterly? *Ans.* $63.412.

7. Find the compound interest of $800, from Jan. 17, 1862, to April 26, 1866, at 6%, interest payable yearly. *Ans.* $226.646.

8. Find the amount of $740, from Dec. 20, 1863, to Nov. 2, 1866, at 6%, interest compounded semi-annually. *Ans.* $876.735.

9. What will $1700 amount to, in 2 yr., at 6%, interest being compounded half-yearly? What, if compounded annually? What, if compounded quarterly? *Last ans.*, $1915.04 +.

Explain Example 2.—355. Recite the rule for finding compound interest.

10. Find the compound int. of $333, at 5 %, from May 15, 1863, to Nov. 15, 1865, interest payable half-yearly. *Ans.* $43.75 +.

356. The following Table may be used with great advantage in calculating compound interest :—

TABLE,

Showing the amount of $1 or £1, for any number of years from 1 to 30, at 3, 4, 5, 6, and 7 %, interest compounded yearly.

For the compound interest, subtract 1 from the numbers in the Table.

YEARS.	3 per ct.	4 per ct.	5 per ct.	6 per ct.	7 per ct.
1	1.030000	1.040000	1.050000	1.060000	1.070000
2	1.060900	1.081600	1.102500	1.123600	1.144900
3	1.092727	1.124864	1.157625	1.191016	1.225043
4	1.125509	1.169859	1.215506	1.262477	1.310796
5	1.159274	1.216653	1.276282	1.338226	1.402552
6	1.194052	1.265319	1.340096	1.418519	1.500730
7	1.229874	1.315932	1.407100	1.503630	1.605781
8	1.266770	1.368569	1.477455	1.593848	1.718186
9	1.304773	1.423312	1.551328	1.689479	1.838459
10	1.343916	1.480244	1.628895	1.790848	1.967151
11	1.384234	1.539454	1.710339	1.898299	2.104852
12	1.425761	1.601032	1.795856	2.012197	2.252192
13	1.468534	1.665074	1.885649	2.132928	2.409845
14	1.512590	1.731676	1.979932	2.260904	2.578534
15	1.557967	1.800944	2.078928	2.396558	2.759032
16	1.604706	1.872981	2.182875	2.540352	2.952164
17	1.652848	1.947900	2.292018	2.692773	3.158815
18	1.702433	2.025817	2.406619	2.854339	3.379932
19	1.753506	2.106849	2.526950	3.025600	3.616528
20	1.806111	2.191123	2.653298	3.207135	3.869684
21	1.860295	2.278768	2.785963	3.399564	4.140562
22	1.916103	2.369919	2.925261	3.603537	4.430402
23	1.973587	2.464716	3.071524	3.819750	4.740530
24	2.032794	2.563304	3.225100	4.048935	5.072367
25	2.093778	2.665836	3.386355	4.291871	5.427433
26	2.156591	2.772470	3.555673	4.549383	5.807353
27	2.221289	2.883369	3.733456	4.822346	6.213868
28	2.287928	2.998703	3.920129	5.111687	6.648838
29	2.356566	3.118651	4.116136	5.418388	7.114257
30	2.427262	3.243398	4.321942	5.743491	7.612255

Ex. 11.—Find, by the Table, the compound interest of $400, at 4%, for 12 years, interest due yearly.

Looking down the column headed 4 per cent., we find the number opposite 12 years to be 1.601032, which is the amount of $1 at compound interest, for the given time, at the given rate. Subtracting 1, we have .601032 for the compound interest of $1. The compound interest of $400 is 400 times as much.

```
     .601032
         400
  240.412800
Ans. $240.41
```

Ex. 12.—What is the compound interest of £90, for 10 yr. 8 mo., at 6%, interest payable yearly?

We find, from the Table, the amount of £1, for 10 yr., at 6%, to be £1.790848. For £90, it will be 90 times as much. 8 months remain; find the amount for this time by multiplying by 1.04, the amount of £1 for 8 mo. Subtracting the original principal from the last amount, we get for the compound interest £77.6233728,—or, reducing the decimal to lower denominations, £77 12s. 5d. 2 far. +

```
         £1.790848
                90
        161.176320
              1.04
         644705280
       1611763200
       167.62337280
       90.
       £77.6233728 =
£77 12s. 5½d. + Ans.
```

Required the interest, compounded annually, of

13. $100, for 17 years, at 6%. *Ans.* $169.277.

14. $625, for 18 years, at 5%. *Ans.* $1504.137.

15. $379, for 30 years, at 3%. *Ans.* $919.932.

16. $49, for 20 yr. 2 mo., at 6%.

17. $875, for 12 yr. 1 mo. 15 da., at 6%.

18. What is the compound interest of $100, for 3 yr., at 6%, interest payable every six months?

In this case, the periods are 6 months each. The interest of $1, for 6 mo., at 6 per cent., equals the interest of $1, for 1 yr., at 3 per cent. There are 6 periods of 6 mo., in 3 yr. Hence we find in the Table the amount opposite to 6 yr. in the 3 per cent. column, subtract 1 since the *interest* is required, and multiply the remainder by the given principal.

19. Find the compound interest of $480, from Jan. 1, 1860, to July 1, 1862, at 8%, interest payable semi-annually.

20. What will $1200 amount to in 8 yr., at 10%, interest compounded half-yearly?

21. What will $1450 amount to in 10 yr. 6 mo., at 6%, interest being compounded semi-annually?

CHAPTER XXI.

NOTES.—PARTIAL PAYMENTS.—ANNUAL INTEREST.

357. A **Note** (also called a Promissory Note or Note of Hand) is a written promise to pay a certain sum to a person specified, or to his order, or to the bearer, at a time named or on demand.

358. The **Drawer** or **Maker** of a note is the one who signs it. The **Payee** is the one to whom it is made payable. The **Holder** is the person who has it in possession.

The **Face** of the note is the sum promised. In the body of the note the number of dollars is written out, and at the top or bottom expressed in figures.

For example, JACOB COOPER is the drawer of Note 1, given below; RUFUS S. BROWN is the payee; the face of the note is $300.

359. PROMISSORY NOTES.

(1)

$300. *Baltimore, April* 9, 1866.

Sixty days after date, I promise to pay Rufus S. Brown, or order, three hundred dollars, value received.

JACOB COOPER.

(2)

Savannah, Jan. 31, 1866.

For value received, thirteen months after date, we promise to pay Messrs. Root & Swan, or order, one hundred and forty-five $\frac{50}{100}$ *dollars, with interest.*

HOMER F. GREEN.

$145.50 MOSES WATERBURY.

A note should always contain the words *value received.* Otherwise, if suit is brought on it, the holder may have trouble in proving that the drawer received a valuable consideration.

Note 2 is signed by two parties, and is therefore called a Joint Note. It contains the words *with interest,* and hence carries interest from its

357. What is a Note?—358. Who is meant by the Drawer or Maker of a note? By the Payee? By the Holder? What is the Face of the note?—359. Learn the forms. What words should a note always contain, and why? What is Note 2 called, and why? What is the effect of the words *with interest?* Can interest be

date, at the legal rate of the State. If these words are omitted, as in Note 1, no interest can be collected,—unless the note is not paid at the time specified, in which case it accrues from that date.

A bank-bill is a note signed by the president and cashier, payable in specie to the bearer on demand,—that is, whenever presented.

360. A note is said *to mature* on the day that it becomes legally due. This is not till the third day after the time specified in the note, three *days of grace*, as they are called, being allowed, unless the words *without grace* are inserted. If the last day of grace is Sunday or a public holiday, the note matures on the preceding day.

The term *months*, used in a note, means *calendar months*. Thus, Note 2 is nominally due at the expiration of thirteen calendar months, that is on the last day, or 28th, of February, 1867; it is legally due on the third day thereafter, March 3d—and interest must be computed for 1 yr. 1 mo. 3 da. It would have matured on the same day, had it been dated Jan. 30, 29, or 28.

361. A note *to bearer* may pass freely from hand to hand. A note *to order*, to be thus passed, must be signed on the back, or *endorsed*, by the payee. Thus endorsed, it is said to be *negotiable*.

An endorser is responsible for the payment of the note, if the maker fails to meet it at maturity, unless the words *without recourse* appear above his name on the back. If there are several endorsers, the holder of the note may look to any or all of them for payment; each is responsible to those that endorsed after him, and the first endorser has his remedy against the drawer.

To make the endorsers responsible, the holder of the note, if it is not paid at maturity, must, on the same day, have it *protested* by a Notary Public, and serve a notice of protest on each endorser.

362. A **Bond** is a written instrument by which a party binds himself to pay to another a certain sum, under a penalty usually twice the face of the bond.

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collected, if these words are omitted? What is a bank-bill?—360. When is a note said to *mature*? What is meant by *days of grace*? What does the term *months*, used in a note, mean? Illustrate this in the case of Note 2.—361. How is a note *to order* rendered negotiable? If the maker fails to meet the note at maturity, who is responsible? If there are several endorsers, in what order are they responsible? What must the holder do, to make the endorsers responsible?—362. What is a Bond?—363. If partial payments are made on notes, &c., where are they entered? What are they called?—364. What rule has been adopted by the courts in most of the States for finding the balance due?

Partial Payments.

363. Partial payments, on account, may be made on notes, bonds, or other obligations that carry interest. They are entered, with their dates, on the back of the instrument, and are therefore called *Endorsements.*

364. When such payments are made, different methods are used for finding the balance due at the time of settlement. The courts in most of the States have adopted the rule prescribed by the Supreme Court of the United States.

United States Rule.

365. According to the United States method, the account is balanced as often as payments are made that equal or exceed the interest due. The interest being first cancelled, the surplus of the payment goes towards discharging the principal, subsequent interest being computed on the balance of principal. No interest is allowed on interest; hence the account is not balanced when payments less than the interest are made.

366. Rule.—*Find the amount of the given principal to the time when a payment or payments were made sufficient to cancel the interest then due, and from this amount subtract such payment or payments. Taking the remainder for a new principal, treat it like the former one; and so proceed to the time of settlement.*

It can generally be determined *mentally* whether a payment exceeds the interest due. If it is clear that it does not, proceed at once to the next payment.—Follow the forms given under Examples 1, 2.

(1) $620. Troy, N. Y., Nov. 1, 1862.

For value received, I promise to pay Thomas Jones, or order, six hundred and twenty dollars, on demand, with interest. Charles Banks.

Endorsed as follows:—Received, Oct. 6, 1863, $61.07.

365. According to the U. S. rule, how often is the account balanced? To what is the payment first applied? To what, the surplus? Why is not the account balanced, when payments less than the interest are made?—366. Recite the rule.

March 4, 1864, \$89.03. Dec. 11, 1864, \$107.77. July 20, 1865, \$200.50. Settled, Oct. 15, 1865; what was due?

It is clear that each payment exceeds the interest due. Hence we must compute the amount to the date of each payment. First find the intervals of time by subtraction, then the multipliers at 6%.

	Yr.	mo.	da.	Intervals.		Multipliers at 6 per cent.
Date of note,	1862	11	1			
1st payment,	1863	10	6	11 mo.	5 da.	$.055\frac{5}{6}$
2d payment,	1864	3	4	4 mo.	28 da.	$.024\frac{2}{3}$
3d payment,	1864	12	11	9 mo.	7 da.	$.046\frac{1}{6}$
4th payment,	1865	7	20	7 mo.	9 da.	.0365
Date of settlement,	1865	10	15	2 mo.	25 da.	$.014\frac{1}{6}$
		Total,		35 mo.	14 da.	$.177\frac{1}{3}$

To prove this work, add the intervals, and see whether their sum equals the interval from the date of the note to the time of settlement; also, add the multipliers, and see whether their sum corresponds with the multiplier that would be obtained from the sum of the intervals.

Date of settlement,	1865	10	15	$\frac{1}{2}$ of 35 mo., .175
Date of note,	1862	11	1	$\frac{1}{6}$ of 14 da., $.002\frac{1}{3}$
	2	11	14 = 35 mo. 14 da.	$.177\frac{1}{3}$

The multipliers being thus proved correct, we use them in computing the several amounts according to the rule, adding $\frac{1}{6}$ as the legal rate for N. Y. is 7%, and carrying the result to three places of decimals, the last of which must be increased by 1 when the next figure is 5 or over.

Face of note, or given principal, ...	\$620.000
Interest on the same to Oct. 6, 1863, date of 1st payment, .	40.386
Amount due Oct. 6, 1863, ...	660.386
First payment, ...	61.070
Balance and new principal, ...	599.316
Int. on new principal to Mar. 4, 1864, date of 2d payment, .	17.247
Amount due March 4, 1864, ...	616.563
Second payment, ...	89.080
Balance and new principal, ...	527.533
Int. on new principal to Dec. 11, 1864, date of 3d payment, .	28.414
Amount due Dec. 11, 1864, ...	555.947
Third payment, ...	107.770
Balance and new principal, ...	448.177
Int. on new principal to July 20, 1865, date of 4th payment, .	19.085
Amount due July 20, 1865, ...	467.262
Fourth payment, ...	200.500
Balance and new principal, ...	266.762
Int. on new principal to Oct. 15, 1865, date of settlement, .	4.409
Balance due at date of settlement, Oct. 15, 1865, . .	\$271.171

(2) $1200. BOSTON, Jan. 1, 1857.

On demand, I promise to pay Eli Hart, or order, twelve hundred dollars, value received, with interest. SAMUEL WOODWORTH.

Attest: GEO. S. GRAHAM.

Endorsements:—Received, Feb. 16, 1857, $200. Apl. 16, 1859, $300. Dec. 24, 1859, $25. May 3, 1860, $15. Nov. 3, 1862, $400. What was the balance due Feb. 3, 1864?

Find the intervals and multipliers as in Ex. 1. It is clear that the 3d and 4th payments are less than the interest due; hence we pass them over, and find the time from the 2d payment to the 5th.—The legal rate in Massachusetts is 6%.

	Yr.	mo.	da.	Intervals.	Multipliers at 6 per cent.
Date of note,	1857	1	1		
1st payment,	1857	2	16	1 mo. 15 da.	.0075
2d payment,	1859	4	16	2 yr. 2 mo. 0 da.	.13
5th payment,	1862	11	3	3 yr. 6 mo. 17 da.	$.212\frac{5}{6}$
Date of settlement,	1864	2	3	1 yr. 3 mo. 0 da.	.075
		Total,		7 yr. 1 mo. 2 da.	$.425\frac{1}{3}$

PROOF.	1864	2	3	$\frac{1}{2}$ of 85 mo.,	.425
	1857	1	1	$\frac{1}{6}$ of 2 da.	$.000\frac{1}{3}$
	7	1	2		$.425\frac{1}{3}$

Face of note, or given principal,		$1200.000
Interest on the same to Feb. 16, 1857, date of 1st payment,		9.000
Amount due Feb. 16, 1857,		1209.000
First payment,		200.000
Balance and new principal,		1009.000
Int. on new principal to April 16, 1859, date of 2d payment,		131.170
Amount due April 16, 1859,		1140.170
Second payment,		300.000
Balance and new principal,		840.170
Int. on new principal to Nov. 3, 1862, date of 5th payment,		178.816
Amount due Nov. 3, 1862,		1018.986
Third payment (less than interest),	$25.	
Fourth payment, " " "	15.	
Fifth payment,	400.	
		440.000
Balance and new principal,		578.986
Int. on new principal to Feb. 3, 1864, date of settlement,		43.424
Balance due at date of settlement, Feb. 3, 1864,		$622.410

(3) $108\frac{41}{100}$ MILWAUKEE, WIS., Dec. 9, 1860.

On demand, we promise to pay to the order of Wm. K. Root one hundred and eight $\frac{41}{100}$ dollars, value received, with interest. BRADBURY, WHITE, & Co.

Endorsements:—Received, March 3, 1861, $50.04. Dec. 10, 1861, $13.19. May 1, 1863, $50.11.

How much was due October 9, 1865? *Ans.* $5.844.

(4) $350. WILMINGTON, N. C., May 1, 1862.

For value received, we jointly and severally promise to pay Conover, Clark, & Co., or order, on demand, three hundred and fifty dollars, with interest. ANSON HAIGHT.

BENJ. W. BLOSSOM.

Endorsements:—Received, Dec. 25, 1862, $50. June 30, 1863, $5. Aug. 22, 1864, $15. June 4, 1865, $100.

What was due on taking up the note, April 5, 1866?

Ans. $251.62.

5. A note for $143.50, dated Aug. 1, 1862, bears the following endorsements:—Received, Dec. 17, 1862, $37.40. July 1, 1863, $7.09. Dec. 22, 1864, $13.13. Sept. 9, 1865, $50.50. How much is due Dec. 28, 1865, the rate being 7%? *Ans.* $60.866.

6. On a note for $3240, dated Dec. 1, 1859, at 5%, the following payments were made:—Dec. 1, 1860, $100. Dec. 1, 1861, $100. Dec. 1, 1862, $100. Nov. 1, 1863, $2500. Oct. 15, 1864, $20. July 20, 1866, $25. What was due Aug. 1, 1866?

Ans. $1177.244.

7. A note for $486, dated Sept. 7, 1863, was endorsed as follows:—Received, March 22, 1864, $125. Nov. 29, 1864, $150. May 13, 1865, $120. What was the balance due April 19, 1866, the rate being 7%? *Ans.* $144.404.

8. A note for $8000, dated June 20, 1858, bore the following endorsements:—Received, Jan. 20, 1861, $2000. March 2, 1861, $1000. Dec. 5, 1861, $175. July 31, 1863, $1500. Aug. 15, 1864, $100. What was the balance due May 31, 1866, the rate being 6%? *Ans.* $6366.333.

MERCANTILE RULE.

367. Merchants, in computing what is due on accounts and notes bearing interest, when partial payments have been made, generally strike a balance for successive periods of *one year*, allowing interest on the original principal and the several balances, and also on payments made during each year, from their date to its close.

368. RULE.—1. *Find the amount of the given principal for one year, and from it subtract the amount of each payment made during the year, from its date to the end of the year; the remainder forms a new principal.*

2. *Proceed thus for each entire year that follows, together with such portion of a year as may intervene between the expiration of the last annual term and the time of settlement.*

Ex. 9.—According to the mercantile rule, what was the balance due on Note 1, p. 225, at the time of settling?

Face of note, or given principal, Nov. 1, 1862,		$620.000
Interest on the same for 1 yr.,		43.400
Amount, Nov. 1, 1863,		663.400
Payment made Oct. 6, 1863,	$61.070	
Interest on same to Nov. 1, 1863, (25 days),	.297	61.367
Balance and new principal, Nov. 1, 1863,		602.033
Interest on new principal for 1 yr.,		42.142
Amount, Nov. 1, 1864,		644.175
Payment made March 4, 1864,	$89.030	
Int. on same to Nov. 1, 1864, (7 mo. 27 da.),	4.103	93.133
Balance and new principal, Nov. 1, 1864,		551.042
Interest to date of settlement, Oct. 15, 1865,		36.859
Amount, Oct. 15, 1865,		587.901
Payment made Dec. 11, 1864,	$107.770	
Int. on same to Oct. 15, 1865, (10 mo. 4 da.),	6.370	
Payment made July 20, 1865,	200.500	
Int. on same to Oct. 15, 1865, (2 mo. 25 da.),	3.314	317.954
Balance due at date of settlement, Oct. 15, 1865,		$269.947

367. How do merchants generally find the balance due, when partial payments have been made?—368. Recite the mercantile rule.

10. According to the mercantile rule, what was the balance due Sept. 30, 1865, on a note for $1475, dated June 2, 1864, on which was paid, Sept. 17, 1864, $200; Jan. 3, 1865, $300; Aug. 2, 1865, $400; interest being allowed at 6%? *Ans.* $664.285.

11. Solve Ex. 4, p. 228, by the mercantile rule. *Ans.* $252.123.

12. Solve Ex. 7, p. 228, by the mercantile rule. *Ans.* $143.553.

369. When the note is settled within a year, *find the amount of each payment from its date to the time of settlement, and subtract their sum from the amount of the face of the note from its date to the time of settlement.*

13. A note for $1000 was given July 18, 1865, at 6%. $200 was paid Sept. 10; $140, Dec. 20; $350, April 21, 1866. What was due on taking up the note, June 2, 1866? *Ans.* $347.428.

14. Requirèd the balance due Aug. 1, 1866, on a note for $1380, at 6½%, dated Oct. 1, 1865, on which a payment of $50 was made Jan. 1, 1866, and a like payment on the 1st day of every month thereafter. *Ans.* $1097.166.

15. A note for $600 is dated Jan. 10, 1865, bearing interest at 7%. Payments of $100 each are made March 15, April 18, Aug. 1, of the same year. What is due Sept. 15, 1865? *Ans.* $321.369.

CONNECTICUT RULE.

370. By the Connecticut rule, the balance is found yearly, when a payment is made within the year; when not, the U. S. method is followed.

371. RULE.—1. *Find the balance due at the close of successive years from the date of the note, if a payment has been made each year, by subtracting from the amount of the principal for the year, the amount of the payment or payments of that year from their date to its end, if such payment or payments exceed the interest; if not, the payment alone, without interest, must be subtracted.*

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369. What is the method almost universally used for finding the balance due on notes settled within a year?—370. By the Connecticut rule, how often is the balance found?—371. Recite the Connecticut rule.

2. *If no payment has been made within a year, find the amount of the principal to the time of the next payment, and subtract the payment.*

3. *Should the time of settlement not coincide with the close of an annual term, compute the last amounts to the time of settlement, and not to the close of the year.*

Ex. 16.—By the Connecticut rule, what was due on Note 2, p. 227?

Face of note, or given principal, Jan. 1, 1857,		$1200.000
Interest on the same for 1 year,		72.000
Amount, Jan. 1, 1858,		1272.000
Payment, Feb. 16, 1857,	$200.00	
Interest on same to Jan. 1, 1858, (10 mo. 15 da.),	10.50	210.500
Balance and new principal, Jan. 1, 1858,		1061.500
Interest on new principal to April 16, 1859, (1 yr. 3 mo. 15 da.),		82.266
Amount, April 16, 1859,		1143.766
Payment, April 16, 1859,		300.000
Balance and new principal, April 16, 1859,		843.766
Interest on new principal for 1 year,		50.626
Amount, April 16, 1860,		894.392
Payment, Dec. 24, 1859, (less than interest then due),		25.000
Balance and new principal, April 16, 1860,		869.392
Interest on new principal for 1 year,		52.164
Amount, April 16, 1861,		921.556
Payment, May 3, 1860,	$15.000	
Interest on same to April 16, 1861, (11 mo. 13 da.),	.858	15.858
Balance and new principal, April 16, 1861,		905.698
Interest on new principal to Nov. 3, 1862, (1 yr. 6 mo. 17da.),		84.079
Amount, Nov. 3, 1862,		989.777
Payment, Nov. 3, 1862,		400.000
Balance and new principal,		589.777
Interest to date of settlement, Feb. 3, 1864,		44.233
Balance due Feb. 3, 1864,		$634.010

According to the Connecticut rule,

17. What was due on Note 3, p. 228? *Ans.* $5.793.

18. Find the answer to Ex. 8, p. 228. *Ans.* $6405.66.

19. What is due July 4, 1866, on a note for $9500, at 6 %, dated June 15, 1863,—$3000 having been paid on account, Aug. 1, 1864; $50, June 30, 1865; $175, May 30, 1866? *Ans.* $7763.841.

Notes with interest annually.

372. Notes sometimes contain the words *with interest annually*. In such cases, if the interest is not paid, the law in New Hampshire allows the creditor simple interest on each item of annual interest from the time it accrued to the date of settlement.

Ex. 1.—A note for $2000 is given March 17, 1863, with interest at 6%, payable annually. No interest having been paid, what is due May 3, 1866, according to the law of N. H.?

Face of note, on interest from March 17, 1863, . . .		$2000.000
Interest on same to date of settlement, May 3, 1866, . .		375.333
Annual interest, $120, has accrued 3 times.		
Interest on $120 from March 17, 1864, to date of settlement, . . .	2 yr. 1 mo. 16 da.	
Int. on $120 from March 17, 1865, .	1 yr. 1 mo. 16 da.	
Int. on $120 from March 17, 1866, .	1 mo. 16 da.	
Total time, . . .	3 yr. 4 mo. 18 da.	
Interest on $120, for 3 yr. 4 mo. 18 da., ...		24.360
Amount due May 3, 1866, ...		$2399.693

In stead of computing the interest separately on each item of annual interest, it is shorter to add the periods, as above, and find the interest on 1 year's interest for a time equal to their sum.

Rule.—*Add the given principal, its interest from date to the time of settlement, and the interest on 1 year's interest for a term equal to the sum of all the periods during which successive payments of interest have been due. Their sum is the amount at annual interest.*

This amount will be less than the amount at compound interest, as only simple interest on the interest is allowed.

373. If partial payments have been made on notes "with interest annually", the balance due is found, according to usage in New Hampshire, by the Mercantile Rule, § 368, which is therefore sometimes called the New Hampshire Rule.

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372. What words do notes sometimes contain? If no interest is paid on such notes, what does the law in New Hampshire allow the creditor? Go through Example 1. What short method is suggested? Recite the rule for finding the amount

EXAMPLES FOR PRACTICE.

1. What amount is due July 5, 1866, on a note for \$820, dated Jan. 3, 1864, at 5%, interest annually, no interest having been paid? *Ans.* \$926.851.

2. Find the amount due on a note for \$1125, interest payable annually at 6%, said note having run 3 yr. 9 mo. 9 da. without any payment. *Ans.* \$1401.379.

3. What is due on a note promising to pay \$560 five years after date without grace, with interest at 5½%, payable annually, no payment having been made till maturity? *Ans.* \$730.94.

4. Required the amount of \$290.50, for 6 yr. 2 mo., at 6%, interest payable annually. *Ans.* \$414.718.

5. Find the amount of \$425, for 4 years, at 4%, interest payable annually.

6. Required the amount of \$850.75, for 3 yr. 10 mo. 6 da., at 6%, interest payable annually.

7. A note for \$715, dated Dover, N. H., Oct. 4, 1863, bearing interest at 6% payable annually, is endorsed as follows: Received, April 4, 1864, \$75; Oct. 1, 1865, \$10; Dec. 3, 1865, \$100. What is due, April 28, 1866? (See § 368.) *Ans.* \$633.258.

CHAPTER XXII.

DISCOUNT.

374. Discount is an allowance made for the payment of money before it is due.

375. Discount is often computed without reference to time, at a certain per cent. on the amount due, and may exceed legal interest. This, however, is not *true discount.*

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of a note with interest payable annually. How does this amount compare with the amount at compound interest?—373. If partial payments have been made on notes with interest payable annually, how is the balance found in N. H.?

374. What is Discount?—375. How is discount often computed?

For example: A merchant buys \$1000 worth of goods on 6 months' credit. The money being worth more to the seller than its mere interest, he will make a *discount* of 5 % on the face of the bill for cash; that is, the buyer can discharge his debt of \$1000, due in 6 months, by paying \$950 down.

Present Worth.—True Discount.

376. The **Present Worth** of a sum due at a future time without interest, is such a sum as put at interest for the given time will amount to the debt.

The **True Discount** is the difference between the present worth and the face of the debt. In other words, it is the interest on the present worth for the given time.

If I owe \$106 a year hence without interest, and money is bringing 6 %, the *present worth* is \$100, because that sum at 6 %, for 1 year, would amount to \$106. The *true discount* is \$106—\$100, or \$6; which is the interest on \$100, at 6 %, for 1 year.

377. It will be seen that the debt corresponds to the amount, of which the present worth is the principal. Hence, to obtain the present worth from the debt, the rate and time being given, we have only to apply the rule in § 352.

378. RULE.—1. *To find the present worth, divide the debt by the amount of* \$1, *for the given time, at the given rate.*

2. *To find the true discount, subtract the present worth from the debt.*

EXAMPLE.—What is the present worth of \$124.20, due in 6 months without interest, the current rate being 7 %? What is the true discount?

Amount of \$1, for 6 mo., at 7 %, \$1.035.

$$\left.\begin{array}{l}\$124.20 \div 1.035 = \$120, \text{ present worth.} \\ \$124.20 - \$120 = \$4.20, \text{ true discount.}\end{array}\right\} \textit{Answers.}$$

Give an example of discount computed without reference to time.—376. What is the Present Worth of a sum due at a future time without interest? What is the True Discount? Illustrate these definitions.—378. Recite the rule for finding the present worth and true discount. Solve the given example.

EXAMPLES FOR PRACTICE.

1. What is the present worth of $4161.575, due three months hence, when money brings ¾ % a month? *Ans.* $4070.

2. Of $622.75, due 3½ years hence, at 5 %? *Ans.* $580.

3. What is the true discount on $100, due in 6 months, when money is worth 6 %? *Ans.* $2.913.

4. On $750, due 9 months hence, at 7 %? *Ans.* $37.411.

5. Find the present worth of $7102.72, due 4 yr. 12 da. hence, at 8 %. What is the true discount?

6. A debt of $150 is due Oct. 1, 1866; what amount would pay it, June 13, 1866, reckoning at 6 %? *Ans.* $147.347.

7. Bought, May 1, $50 worth of goods, on 6 months' credit. What sum paid Aug. 1 will discharge the debt, money being worth 4½ % per annum? *Ans.* $49.443.

8. A owes B $961.13, due 1 year 5 months hence, and $3471.20, due in 3 years 9 months, without interest. Money being worth 7 %, what discount should be allowed on both debts, if paid at once? *Ans.* $808.448.

9. What sum paid down Jan. 1 is equivalent to $37.40 paid on the 1st of the next August, money being worth 6 %?

10. A merchant buys a bill of $1500, on 6 months' credit, but settles it by paying cash, a discount of 5 % on the face of the bill being allowed. What does the discount amount to, and by how much does it exceed the true discount, money being worth 7 %? *Ans.* $75; $24.28.

11. When money brings ½ % a month, a merchant settles a bill of $840, due 60 days hence, for cash, at a discount of 2½ %. What does he pay down, how much discount does he get, and by how much does it exceed the true discount? *Last ans.* $12.68.

12. Sold $1500 worth of goods, on 7½ months' credit. What is the present worth of the bill, computed at 7 %?

13. Bought, on 6 months' credit, muslins for $123, hosiery to the amount of $100.50, and $750 worth of cloth. If cash is paid for the whole bill, what amount should be deducted, reckoning at 6 %? How much, at 7 %? *First ans.* $28.35.

14. Sold $1500 worth of hardware, half on 6 months' and half on 9 months' credit. What sum paid down would discharge the whole debt, the current rate of interest being 7%?

15. A man buys a farm of 97 A., at $110 per acre, on a credit of 9 mo. What discount should be allowed if the money is paid down, reckoning at 5%? At 6½%? *Last ans.* $495.984.

16. Bought goods to the amount of $1200, one third payable in 3 mo., one third in 6 mo., and the rest in 9 mo. What sum paid down would discharge the whole debt, money being worth 6 per cent? *Ans.* $1165.21.

17. Which is worth most, $500 cash down, $516 six months hence, or $530 in twelve months, money being worth 7%?

18. A merchant, having bought some goods, has his choice between paying the face of the bill, $1050, in 90 days, or paying cash at a discount of 2%. If money is worth 7%, which had he better do, and what will he gain by so doing?

Ans. Pay cash; gain, $2.94.

Bank Discount.

379. A **Bank** is an institution chartered by law, for the purpose of receiving deposits, loaning money, and issuing notes, or bills, payable on demand in specie,—that is, in gold or silver.

380. Banks loan money on notes. Deducting a certain part of the face of the note in consideration of advancing the money, the bank pays over the rest to the borrower. The note is then said to be *discounted.* It thus becomes the property of the bank, which, when it matures, receives from the drawer the amount of its face.

The portion deducted, or allowance made to the bank, is called the **Bank Discount.** The sum paid to the holder is called the **Proceeds** or **Avails** of the note.

A merchant holds a note for $200, payable in 90 days. Wishing to

379. What is a Bank?—380. When is a note said to be *discounted?* What is meant by Bank Discount? What is meant by the Proceeds or Avails of the note? Illustrate this process and these definitions.

use the money immediately, he endorses it, takes it to his bank, and places it in the discount box. If both maker and endorser are considered responsible, the bank retains the note, and, deducting $3.10, pays over the balance $196.90 to the holder. The Bank Discount is $3.10; the Proceeds are $196.90.

381. *Bank discount* is greater than *true discount*,—the former being computed on the face of the note or amount, the latter on the present worth or principal. It is equivalent to simple interest paid in advance, for three days more than the time specified in the note,—three days of grace being always allowed in computing bank discount.

382. CASE I.—*To find the bank discount and proceeds of a note, its face being given.*

Ex. 1.—A holds a note for $1000, dated Feb. 1,1866, payable in 4 mo. April 1, he gets it discounted at 6%. What are the bank discount and proceeds?

Two months having expired at the date of discount, interest must be computed for 2 mo. 3 da.

Interest of $1 for 2 mo. 3 da. = .0105

$\left.\begin{array}{l}.0105 \times 1000 = \$10.50\text{, Bank Discount.} \\ \$1000 - 10.50 = \$989.50\text{, Proceeds.}\end{array}\right\}$ *Answers.*

RULE.—1. *For the bank discount, find the interest on the face of the note, at the given rate, for three days more than the specified time.*

2. *For the proceeds, subtract the bank discount from the face of the note.*

383. If the note bears interest, cast interest as above on *the amount due at maturity*, in stead of on *the face* of the note.

Ex. 2.—At 7%, what is the bank discount on a note for $600, payable in 6 mo. with interest at 6%?

Amount of $600, for 6 mo. 3 da., at 6%, $618.30.
Interest on $618.30, for 6 mo. 3 da., at 7%, $22.
Ans. $22.

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381. How does *bank discount* compare with *true discount*? Why is bank discount the greater? To what is it equivalent?—382. What is Case I.? Explain Example 1. Recite the rule.—383. If the note bears interest, how must we proceed? Solve Example 2. How does it differ from Ex. 1?

EXAMPLES FOR PRACTICE.

1. What is the bank discount on a note for $1000, for 3 mo., at 7%? *Ans.* $18.083.

2. On a note for $150, for 6 mo., at 6%?

3. On a note for $375, for 3 mo. 9 da., at 7%?

4. On a note for $400, for 9 mo. 27 da., at 6½%?

5. Find the proceeds of a note for $472, nominally due Nov. 15, discounted the 15th of the previous January, at 7 per cent. *Ans.* $444.19.

6. A note for $1800, payable in 60 days, was discounted at a bank at 6%; how much did the holder receive? *Ans.* $1781.10.

7. A merchant gets three notes discounted, the first two at a broker's for 6%, the third at a bank for 7%. What does he receive on all three, the first being for $837.50 payable in 30 days, the second for $650 in 60 days, the third for $6720 in 90 days? *Ans.* $8074.55.

8. A farmer buys 43 A. 1 R. of land at $80 an acre. Getting a note for $4280.75, payable in 90 days, discounted at a bank at 6%, he pays for his land out of the proceeds; how much has he left? *Ans.* $754.40.

9. A builder buys 23250 ft. of boards, at $30 per M., paying the bill with his note at 15 days. The seller gets the note discounted at a bank three days afterwards, at 7%; how much does he realize for it? *Ans.* $695.47.

10. What is the difference between the bank discount and the true discount on a note dated Feb. 1, 1866, for $400, payable in 90 days, at 7%? *Ans.* 13c.

11. What are the proceeds of a note for $426.10, payable in 57 days, with interest at 6%, discounted at a bank for 6%? (§ 383) *Ans.* $426.06.

12. A owes B for 46 bundles of paper, at £1 10s. a ream. He pays B the proceeds of a note for £100, payable in 30 days, which he gets discounted at a bank for 6%. How much is he then in B's debt? *Ans.* £38 11s.

13. A person having a six-month note for $1200, dated May

2, 1866, on the 1st of June gets it discounted at a bank for 5%, and invests the proceeds in land at $1 per acre. How much land does he buy? *Ans.* 1174¼ A.

14. If I get a note for $720, payable 4 mo. 15 da. hence, with interest at 7%, discounted at 6%, what will the discount be?

384. CASE II.—*To find for what sum a note must be drawn, for a given time and rate, to yield certain proceeds.*

Ex.—For what sum must a note be drawn at 90 days, that, when discounted at a bank at 6%, it may yield $200 proceeds?

Find the proceeds of $1, for the given time and rate.
Bank discount on $1 for 90 + 3 days, at 6%, $.0155.
Proceeds of $1, discounted for 93 days, at 6%, $.9845.
Since $1 yields $.9845 proceeds, to yield $200 proceeds will require as many times $1 as $.9845 is contained times in $200, or $203.149.

PROOF.—Bank discount on $203.149, at 6%, for 93 days, $3.149.
$203.149 — $3.149 = $200, *Proceeds.*

RULE.—1. *Divide the given proceeds by the proceeds of* $1 *for the given time and rate.*

2. *Prove by finding whether the proceeds of the result equal the given proceeds.*

EXAMPLES FOR PRACTICE.

1. For what sum must a note be drawn, that, when discounted for 3 mo., at 6%, its proceeds may be $600? *Ans.* $609.45.

2. What must be the face of a note, that, when discounted at 5% for 10 mo., the avails may be $1000? *Ans.* $1043.93.

3. For what amount must I draw my note at 12 mo., that, when discounted at 7%, it may yield $100?

4. For what sum must a note dated May 3, payable Nov. 3, be drawn, to yield $365, when discounted at 6%? *Ans.* $376.48.

5. A man bought a house for $3287 cash. How large a note, payable in 90 days, must he have discounted at 6%, to realize that amount? *Ans.* $3338.75.

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384. What is Case II.? Explain and prove the given example.

6. I had three notes discounted at 6%, for 3 mo., 4 mo., and 6 mo., respectively. The proceeds were \$600, \$400, and \$300. What was the face of each? *Sum of ans.* \$1327.26.

7. A merchant had three six-month notes discounted at 5, 6, and 7%, respectively. The proceeds of each were \$1000. What was the face of each? *First ans.* \$1026.08.

CHAPTER XXIII.

COMMISSION.—BROKERAGE.—STOCKS.

385. Commission is a percentage allowed to an agent for the purchase or sale of property, the collection or investment of money, or the transaction of other business.

A party attending to such business for a commission is called an Agent, a Factor, Commission-merchant, or Broker.

386. A Broker is one who buys or sells goods for another, without having them at any time in his possession, or who exchanges money, obtains loans, or deals in stocks. The commission paid to a Broker is called **Brokerage.**

The rate of commission and brokerage differs according to the business transacted and the amount involved, ranging from $\frac{1}{8}$ to 5%. A commission-merchant usually gets $2\frac{1}{2}$% for selling goods, and an additional $2\frac{1}{2}$% if he guarantees the payment.

387. A **Consignment** is a lot of goods sent by one party to another for sale. The party sending them is called the Consignor; the one receiving them, the Consignee.

The Gross Proceeds of a consignment are the whole

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385. What is Commission? What is a party attending to business on commission called?—386. What is a Broker? What is Brokerage? Between what limits does the rate of commission and brokerage generally range? What does a commission-merchant usually get for selling goods?—387. What is a Consignment? Who is the Consignor? Who is the Consignee? What is meant by the Gross Proceeds

amount realized by the sale. The Net Proceeds are what is left for the owner, after deducting commission and other charges.

388. Stocks is a general term applied to Government or State bonds, and the capital of companies incorporated or chartered by law. There are state stocks, bank stocks, railroad stocks, &c.

When a company is formed for building a railroad, constructing a telegraph line, establishing a bank, carrying on extensive manufacturing operations, or any other enterprise, those interested subscribe a certain amount needed for conducting the business, which constitutes the Capital, or Stock, of the company. This stock is divided into portions called Shares, which may be of any amount, but are usually $100 each, and are represented by Certificates or Scrip.—Stock is bought and sold by brokers. It is constantly fluctuating in value, rising or falling according to the demand for it, the profits of the company, and other influences.

Those who own any particular stock, whether by original subscription or purchase, are called Stockholders. They constitute the Company, and elect Directors, by whom a President and other officers are chosen.

389. A broker who deals in stocks is called a Stock-broker. His commission for buying or selling is reckoned at a certain per cent. (usually $\frac{1}{4}\%$) on the nominal value of the stock, without reference to the market price.

390. Commission is a *percentage*.

The money collected, realized, or invested, is the *base*.

The per cent. allowed as commission is the *rate*.

Hence, by the principles of Percentage (§ 321), these

RULES.—I. *To find the commission, multiply the base by the rate.*

II. *To find the rate, divide the commission by the base.*

III. *To find the base, divide the commission by the rate.*

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of a consignment? By the Net Proceeds?—388. What is meant by Stocks? When a company is formed, how is the necessary capital obtained? How is this capital, or stock, divided? By whom is stock bought and sold? What makes it fluctuate in value? Who are called Stockholders? Whom do they elect? Who are chosen by the directors?—389. What is a broker who deals in stocks called? How is a stock-broker's commission reckoned?—390. Commission being a percentage, what is the base? What is the rate? Recite the rules.

EXAMPLES FOR PRACTICE.

[In all the examples relating to stocks, take $100 for a share, unless otherwise directed.]

1. What commission must be paid an agent for collecting bills to the amount of $2460, at 5%? *Ans.* $123.

2. A broker buys for me 100 shares of Erie R. R. stock, and sells the same the next day. What is his brokerage, ¼% being charged for each transaction?

3. A lady, having $22000 on bond and mortgage at 6%, employs an agent to collect 1 year's interest and invest it. What commission must she pay, the rate being 2½% for collecting and ½% for investing? *Ans.* $39.60.

4. What brokerage must a person pay to have $1475 uncurrent money exchanged, at an average rate of ¾%, and how much should he receive in current funds?

5. An auctioneer, who charges 2%, receives $225 for selling some paintings; how much did they sell for? *Ans.* $11250.

6. What are the net proceeds of a consignment sold for $4250, on which there are charges of $27 cartage, $103 storage, and 2½% commission? *Ans.* $4013.75.

7. Sold 412 bales of cotton, averaging 405 lb. each, @ 27c. a lb. What was the commission, at 2½%? *Ans.* $1126.305.

8. What % does a commission-merchant charge, who receives $223 for buying $5575 worth of goods? *Ans.* 4%.

9. A factor in Mobile received from a planter 514 bales of cotton; after paying on it $840 expenses, he sold it at $120 a bale. He then bought for the planter $1525 worth of hardware, and groceries to the amount of $3018.20. His commission being 2% on sales, and 3% on purchases, how much must he remit to the planter? *Ans.* $54926.90.

10. An agent collects for a society 250 bills, of $6 each. How much must he pay over, if he gets 5% commission?

11. A broker sells for a customer 250 shares of N. Y. Central R. R. stock, and buys for him 300 shares of Michigan Southern. At ¼%, what is the brokerage?

12. Wishing to buy 85 A. of land, I obtained the necessary amount through a broker, who charged 1 % for negotiating the loan. His commission amounted to \$63.75; what did the land cost per acre? *Ans.* \$75.

13. A commission-merchant, for selling \$12000 worth of grain and guaranteeing payment, charged \$600, and for purchasing a bill of \$4220 charged \$63.30. What % did he charge for each transaction?

14. A factor, having sold 1250 barrels of flour at \$8 a barrel, invested his commission, which was at the rate of 1¾ %, in a new company that was forming. How many shares, at \$25 each, did he take? *Ans.* 7.

391. *To find the base, the rate and the sum or difference of the commission and base being given.*

A party sometimes remits to an agent a certain amount to be invested, after deducting his commission. Here the sum of the commission and base is given, and the base, or amount invested, is required.

Again, when the net proceeds and rate are known, it is sometimes required to find the gross proceeds of a sale. Here the difference between the base and the commission is given, and the base is required.

These cases are analogous to those presented in § 323, under Percentage.

Ex. 1.—B sends a commission-merchant \$6000 to invest in cotton, after deducting his commission of 2 %. How much must be invested, and what is the commission?

Every \$1 invested will cost B \$1 + 2c. commission, or \$1.02. Hence there will be as many times \$1 invested as \$1.02 is contained times in \$6000. \$6000 ÷ 1.02 = \$5882.35, *Amount invested.*

The commission will be the difference between the whole amount sent and the sum invested. \$6000 — \$5882.35 = \$117.65, *Commission.*

PROVE by finding whether the commission on \$5882.35, at 2 %, is \$117.65.

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391. What cases are sometimes presented, analogous to those in § 323, under Percentage? Explain Example 1.

Ex. 2.—A real estate agent, having sold a house, pays himself 1% commission, and hands over to the owner \$13365. What did the property bring, and what is the commission?

The commission being 1%, every \$1 of the purchase price will net the owner 99c. The house, therefore, brought as many times \$1, as 99c., the net proceeds of \$1, is contained times in \$13365, the net proceeds the owner received. \$13365 ÷ .99 = \$13500, *Selling price.*

The commission will be the difference between the selling price and the net proceeds. \$13500 − \$13365 = \$135, *Commission.*

PROVE by finding whether the commission on \$13500, at 1%, is \$135.

392. RULE.—1. *For the base, divide the given number of dollars by* 1 *increased or diminished by the rate expressed decimally, according as the sum or difference of the commission and base is given.*

2. *For the commission, take the difference between the base and the given number of dollars.*

3. *Prove by finding whether the commission obtained by multiplying the base by the given rate, equals the commission as just found.*

EXAMPLES FOR PRACTICE.

1. A broker receives \$30000 to invest in real estate, after deducting his brokerage of ¼%. What will be the amount invested, and what his commission? *First ans.* \$29925.19.

2. A person sends his commission-merchant \$15000 to invest in corn. The commission, 1%, being taken out of the sum sent, and the corn costing 75c. a bushel, how many bushels were purchased? *Ans.* 19801 bu. +

3. An agent, having sold some property, paid the owner \$11137.50, which remained after deducting his commission of 1%. What did the property sell for? *Ans.* \$11250.

4. A commission-merchant paid \$1000.50 charges on a consignment, retained 2½% commission, and remitted \$38487 to the owner; what were the gross proceeds? *Ans.* \$40500.

Explain Example 2.—392. Recite the rule for finding the base and commission, when the sum or difference of the commission and base is given.

5. An agent, who gets 5%, collects a number of bills of $10 each, for a society. He pays over to the treasurer $1149.50; how many bills were collected? *Ans.* 121.

6. What are the gross proceeds of a consignment, if the commission is 2½%, the charges are $1000.85, and the net proceeds $12772?

7. A gentleman who has $30000 invested on bond and mortgage at 7%, employs an agent to collect six months' interest, and directs him to invest in grain what is left after paying himself his commission,—which was 1% on the amount collected, and 2% on the amount invested. How much was invested? *Ans.* $1019.12.

Account of Sales.

393. An **Account of Sales** is a statement rendered by a commission-merchant to a consignor, setting forth the prices obtained for the goods sent and the amount realized, the charges paid and the net proceeds due the consignor. They are made out in the following form:—

Sales of Flour for acct. of R. Day & Co., Buffalo.

1866.	Sold to	Description.	Bar.	@	
June 1	Beck & Co.	Extra Ohio.	83	$9.00	$747.00
" 2	I. R. Shaw.	Canadian.	20	9.20	184.00
" 4	S. Bennett.	Phenix Mills.	95	8.75	831.25
" 5	David Orr.	" "	75	8.70	652.50
" "	Roe & Son.	City Mills.	160	7.95	1272.00
					$3686.75

CHARGES.

Freight on 433 bar., @ 75c.,	$324.75	
Cartage, ...	21.50	
Storage, ...	43.30	
Commission on $3686.75, @ 2½%,	92.17	
Total charges,		$481.72
Net proceeds to credit of R. Day & Co.,		$3205.03

E. & O. E.*

N. Y., June 6, 1866. BUTTERWORTH, HUDSON & Co.

* Errors and omissions excepted.

1. Make out the following Account of Sales, and find the net proceeds due the consignor:—

Sales of 4265 *Bushels Wheat, for acct. of Asa F. White, Oswego.*

1866.	Sold to	Description.	Bu.	@	
July 2	H. Brown.	Red Winter.	750	$1.90	$
" "	City Mills.	" "	600	1.89	
" 3	Bruce & Co.	" "	600	1.92	
" "	Farr Bro's.	New Mich.	500	2.62	
" "	I. R. Moe.	" "	940	2.61	
" 5	H. S. Hunt.	" "	875	2.59	
					$

CHARGES.

Freight on 1950 bu., @ 12c.,	$	
" " 2315 bu., @ 12½c.,		
Advertising, ...	$5.45	
Commission on $, at 2½%,		
Total charges,		$
Net proceeds to credit of Asa F. White,		$

E. & O. E.

N. Y., July 7, 1866. HARRISON & BARROW.

2. Make out an Account of Sales, in proper form, from the following data:—

Messrs. Meyer & Herzog, commission-merchants, of New York, received a consignment of provisions from Henry L. Jones & Co., of Rome, N. Y., as follows:—10 firkins of butter, 940 lb.; 37 cwt. of cheese; 30 barrels mess pork; 16 cwt. hams; 40 packages shoulders, 2700 lb.

They paid charges on the consignment as follows:—Freight, $75.40; drayage, $5.75; storage, $12.25; insurance, $6.50; advertising, $12.75. Their commission was 2½%.

They sold the butter, June 19, 1866, @ 37½c. a lb.; the cheese, same date, @ 19c. June 20, they sold the pork @ $31 per bar., the hams @ 18¾c. a lb., the shoulders @ 14c.

Ans. Net proceeds, $2484.26.

Stocks.

394. The **Market Value** of a stock is what it sells for.

395. When the market value of a stock is the same as its nominal value, it is said to be **at par.**

When its market value is greater than its nominal value, it is said to be **above par** or **at a premium**; and when less, to be **below par** or **at a discount.**

When a hundred-dollar share sells for $100, the stock is *at par;* at $101, it is *above par*, or *at a premium of* 1 %; at $99, it is *below par*, or *at a discount of* 1 %. The premium or discount is always reckoned on the par value as a base.—Stock is generally quoted at the market value of one share. In the three cases just specified, it would be quoted respectively at 100, 101, and 99.

396. When the capital for a new company has been subscribed, if it is not all needed immediately it is called for in portions, or **Instalments**—a certain per cent. at a time.

397. Stockholders are sometimes called on to meet losses or make up deficiencies, by paying a certain amount on each share they hold. The term **Assessment** is applied to a sum thus called for.

398. The **Gross Earnings** of a company consist of all the moneys received in the course of their business. The **Net Earnings** are what is left after deducting expenses. When there are net earnings to any considerable amount, they are divided, in whole or in part, among the stockholders, according to their respective amounts of stock.

399. A **Dividend** is a sum paid from the earnings of a company to its stockholders.

Let the capital of a company be $2500000 ; let its gross earnings for

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394. What is meant by the Market Value of a stock?—395. When is a stock said to be *at par*? When, *above par*? When, *below par*? When, *at a premium*? When, *at a discount*? Illustrate these definitions.—396. What is meant by Instalments?—397. What is meant by an Assessment?—398. What is meant by the Gross Earnings of a company? By the Net Earnings? When there are net earnings to any considerable amount, what is done with them?—399. What is a Dividend?

six months be $250000, and its expenses for the same time $100000: the net earnings will be $150000. Of this sum it is thought best to retain $50000 as a *surplus*, to meet any unforeseen expenses, and to divide the rest, $100000, among the stockholders. To find the rate, the percentage must be divided by the base, § 321. $100000 ÷ 2500000 = .04. *A dividend of* 4 *per cent. is declared;* and each stockholder's dividend will be found by taking 4 % of the par value of his stock.

400. When a company need money, they sometimes borrow it on their property as security, issuing Bonds, which bear a certain fixed rate of interest without reference to the profits. The income from the stock, on the other hand, depends on the net earnings,—the interest on the Bonds, as well as other expenses, having been first paid.

401. Cities, counties, and states, may also issue Bonds to raise money. These Bonds are named according to the interest they bear. Thus, Tennessee 6's are Bonds bearing 6 per cent., issued by the state of Tennessee.

402. The United States Government has issued several different classes of Bonds and Treasury Notes, which constitute what are called "U. S. Securities" or "Federal Securities".

U. S. 5's of '71 and '74 are bonds payable respectively in 1871 and 1874, bearing interest at 5 % in gold.

U. S. 6's of '67, '68, and '81, are bonds payable respectively in 1867, 1868, and 1881, bearing interest at 6 % in gold.

5–20's are bonds bearing interest at 6 % in gold, so called from their being payable in not less than 5 or more than 20 years from their date, at the pleasure of the Government.

10–40's are bonds bearing interest at 5 % in gold, so called from their being payable in not less than 10 or more than 40 years from their date, at the pleasure of the Government.

7–30's or 7 3–10's [Seven-thirties or seven and three-tenths] are treasury notes payable in three years from their date; they are so called from their bearing interest at $7\frac{3}{10}$ % in currency, or lawful money.

403. In the case of sales, brokers have to use a revenue stamp equal in value to 1 cent on each $100 (or fraction of $100) of the currency value of the stocks or bonds sold; this is charged to the parties for whom they sell.

Illustrate the mode of finding the rate of a dividend to be declared. How is each stockholder's dividend found?—400. How is money sometimes raised by a company? How does the income from bonds differ from that arising from stock?—401. What besides companies may issue bonds? How are these bonds named? Give an example.—402. Name the several classes of United States Securities.

EXAMPLES FOR PRACTICE.

[Unless otherwise directed, take $100 for a share, and ¼% for the rate when brokerage is paid.]

1. What is the market value of 200 shares of N. Y. Central R. R. stock, at 97?

If 1 share is worth $97, 200 shares are worth 200 times $97.

2. What will I have to pay for 200 shares of N. Y. Central, at 97, and brokerage on the same?

1 share will cost $97 + ¼ per cent. of $100 (brokerage), or $97.25.
200 shares will cost 200 times $97.25.

3. What will I realize on 200 shares of N. Y. Central sold at 97, over and above brokerage and cost of revenue stamp?

200 shares, at 97, $19400.00.
Deduct brokerage, ¼ per cent. on $20000, $50.00.
" for stamp, 1c. on 194 hundred dollars, 1.94.
$50 + $1.94 = $51.94. $19400 — $51.94 = 19348.06. *Ans.*

4. What is the market value of 100 shares of Michigan Central, at a premium of 3½%? *Ans.* $10350.

5. What will 125 shares of Western Union Telegraph stock cost, at 30% discount, with brokerage? *Ans.* $8781.25.

6. What will be realized, over and above brokerage and cost of revenue stamp, on 500 shares, of $25 each, sold at a premium of 2½%? *Ans.* $12748.72.

7. Bought through a broker 100 shares of Alton and Terre Haute at 31⅞; what do they cost? *Ans.* $3212.50.

8. Sold Virginia 6's to the amount of $20000, at a discount of 30%; and 2000 three-dollar shares of a petroleum stock, at 45% discount. No brokerage being paid, how much is realized from the sale? *Ans.* $17300.

9. Bought 50 shares of Ocean Bank stock at par, and sold them at 105. What is the profit, brokerage being paid on each transaction, and the cost of revenue stamp being deducted?

Find the profit on 1 share, by deducting 50c. brokerage from $5, the advance in price. Multiply the profit on 1 share by the number of shares, and from the product subtract the cost of stamp.

10. What is the loss on 250 fifty-dollar shares, bought for 102 and sold at 99¾, taking brokerage and cost of stamp into account?

11. I buy through a broker 175 shares of bank stock at 97⅛, and sell them through the same at a premium of 4%; what is my profit?

12. If a person buys 40 fifty-dollar shares at 13% above par, and sells them at 11½% below par, does he make or lose, and how much?

13. A person exchanges 150 shares of Erie at 60, for stock of a Quicksilver Co. at 25% premium. How many shares should he receive? *Ans.* 72 shares.

14. Bought some stock at 92, sold it at 94¾. Brokerage was paid on each transaction. The profit being $398.11, how many shares were there?

Brokerage on 1 share $.50. Cost of stamp on 1 share sold at 94½, $.00945. $.50 + $.00945 = $.50945. Profit on 1 share, $2.50 — $.50945 = $1.99055. As many shares were sold as $1.99055 is contained times in $398.11.

15. How much stock, at 10% discount, can be bought for $4500, brokerage being left out of account? *Ans.* 50 shares.

What will 1 share cost, at 10 per cent. discount?
How many shares, at that price, can be bought for $4500?

16. How much stock, at a premium of 3¼%, can be bought for $10350, brokerage being paid? *Ans.* 100 shares.

17. A merchant wishes to sell sufficient stock to realize $15000. The stock being at 75¼, and brokerage ¼%, how many shares must he sell?

18. Bought 100 shares of Nassau Bank stock at 105. They were sold at a profit of $350, leaving brokerage out of account; what premium did they bring? *Ans.* 8½%

19. A broker receives $19100 to invest in Kentucky 6's, brokerage to be paid out of the amount sent. The stock stands at 95¼; how many thousand-dollar bonds can he buy, and what income will be received from them every year?

20. A company with a capital of $750000, having earned $22500, put aside $3750 as a surplus. What per cent. dividend can they declare? (See § 399.) *Ans.* 2½%.

21. In the above company, A holds $10000 worth of stock; B, $20000; C, $17500. What will their dividends respectively amount to? *Ans.* A's, $250, &c.

22. A railroad company having declared a dividend of 3%, how much will a person who holds 400 fifty-dollar shares receive?

23. A mining company, whose shares at par are $25, declare a dividend of 1% every month. How much will a party who holds 1000 shares receive in one year?

24. I hold $5000 worth of 6% bonds in a certain company, and 50 shares of the capital stock. The company declare semi-annual dividends of 3½%. What is my yearly income from both?

In these Examples, the government tax of five per cent. on dividends and interest accruing on all bonds (except those of the U. S.) is left out of account.

25. D bought 100 shares of stock at 84, and sold them at 87, receiving meanwhile a dividend of 3%. What was his profit?

26. A company with a capital of $10000000 have $200000 net earnings; what dividend can they declare? What dividend will a party receive who holds $10000 of their stock?

27. How much stock in the above company does a party hold, who receives a dividend of $10000?

Dividend = Stock × Rate.
Hence, *Stock = Dividend ÷ Rate.* *Rate = Dividend ÷ Stock.*

28. What % dividend does a person get, who receives $350 and owns 50 shares of stock?

29. When gold is at a premium of 29%, what is $1000 in gold worth in currency?

$1 gold = $1.29 currency. $1000 gold = $1.29 × 1000 currency.

The banks having suspended specie payments in 1861, gold and silver have since that time commanded a premium; that is, $1 in gold or silver has been worth more than $1 in currency.

30. When gold is at 129, how much gold will $1290 in currency buy? *Ans.* $1290 ÷ $1.29 = $1000.

31. When gold is at 141, how much in current funds will $12000 in gold cost?

32. When gold is at a premium of 25%, how much gold will $20000 in currency buy?

33. A lady holds $8000 worth of U. S. 5-20's; what will she receive annually from these bonds in currency, if gold commands a premium of 30%? (See § 402.)

$8000 × .06 = $480 in gold. $480 × 1.30 = $624 in currency.

34. What is the semi-annual income in currency from $15000 worth of U. S. 5–20's, when gold brings 133?

35. What is the yearly income in currency from $10000 in U. S. 10–40's, when gold is worth 126? *Ans.* $630.

36. What is the yearly income from $20000 in U. S. 7–30's?

37. What yearly income will one who subscribes for $10000 of a seven per cent. loan, at par, receive from it?

38. If a person invests $8245 in 6% bonds, at 97, what will be his annual income from the investment?

Each dollar of stock bought costs 97c. Hence, for $8245 can be bought as many dollars of stock as 97c. is contained times in $8245. Then find the interest on the amount bought, at 6 per cent.

39. What income will be annually received from certain 7% bonds, bought at 103, and costing $14420? *Ans.* $980.

40. A person invests $19600 in 10–40's, at 98. What income in currency will he annually receive from the bonds purchased, if gold sells at 140? *Ans.* $1400.

41. When gold is worth 129, what half-yearly income in current funds will a person receive who invests $7540 in U. S. 5–20's, then selling at 104? *Ans.* $280.575.

42. When Missouri 6's are at 75, what sum must be invested in them, to yield an annual income of $2700?

Stock required = Income ÷ Rate. $2700 ÷ .06 = $45000.
$45000 stock, at 75, will cost $33750 *Ans.* Hence the following rule:—

404. RULE.—*To find what sum must be invested in bonds, selling at a given rate, to secure a given income,*

1. *Find the par value of the stock required, by dividing the given annual income by the annual income of $1 of the stock.*

2. *Multiply this par value by the market value of $1 of the stock.*

43. How much must one invest in Brooklyn 6's, at 90, to secure an annual income of $1500? *Ans.* $22500.

44. If I sell $10000 U. S. 6's, at 107, and with part of the proceeds buy N. Y. Central 6's, at 90, sufficient to yield $300 annually, how much will I have left? *Ans.* $6200.

45. When U. S. 7–30's are selling at 103, what sum must be invested in them to yield $1460 a year? What sum invested in them will yield a semi-annual income of $109.50?

46. When N. Carolina 6's are 15% below par, what will be the cost of bonds sufficient to yield $1200 yearly?

47. Holding a large amount of Erie R. stock, I wish to sell part of it and buy Tennessee 6's sufficient to yield me $1800 a year. Erie standing at 60, and Tennessee 6's at 90, how many shares of Erie must I sell to make the change, leaving brokerage out of account? *Ans.* 450 shares.

48. What % income will a person realize on his investment, who buys 6 per cent. bonds at 96?

$1 of the stock yields 6c. and costs 96c. The question therefore becomes, What per cent. is 6c. of 96c.? Divide the percentage by the base, § 321.

.06 ÷ .96 = .0625. *Ans.* $6\frac{1}{4}$ per cent. Hence the following rule:—

405. RULE.—*To find what % annual income is realized on an investment in stocks at a given price,*

Divide the annual income of $1 *by the cost of* $1 *of the stock.*

49. What % income will be realized on 7% bonds bought at 91? At 98? At 105? *First ans.* $7\frac{9}{13}$%.

50. If I get an annual dividend of 7% on stock that cost me 70, what % do I receive on my investment?

51. What % on the investment will a stock bought at 90 yield, if a dividend of 3% is paid every six months? *Ans.* $6\frac{2}{3}$.

52. What % on his investment will a person receive, who buys U. S. seven-thirties at 104? *Ans.* $7\frac{1}{52}$%.

53. What % on his investment will a person receive, who buys U. S. 6's at 107, when gold stands at 150?

.06 × 1.50 = .09   .09 ÷ 1.07 = $8\frac{44}{107}$ per cent. *Ans.*

54. When U. S. 10–40's are at 97, and gold is worth 125, what per cent. will an investment in these bonds yield?

55. A person desiring to make a permanent investment, hesitates between buying U. S. 7–30's at 103 and Kentucky 6's at 95. Which will pay him the better % on his investment, and how much? *Ans.* Seven-thirties, $\frac{1510}{1957}$%.

56. Which investment will pay the better %—and how much—5-20's at $104\frac{1}{8}$, or 10–40's at $97\frac{1}{4}$?

57. A person having his money invested on bond and mortgage, at 6%, calls it in, and buys Michigan Central 8's, at 110. How does his rate of income on the latter investment compare with what it was before? *Ans.* $1\frac{3}{11}$% better.

58. Which is the best for permanent investment—5's at 75, 6's at 85, or 7's at par?

59. A party investing in 6 per cent. bonds realizes 8% income on his investment. How did the bonds stand when he bought?

$1 of the bonds yields 6c. The question therefore becomes, 6c. is 8 per cent. of what? Divide the percentage by the rate, § 231:—

$.06 \div .08 = .625$, cost of $1 of the bonds.

$.625 \times 100 = 62\frac{1}{2}$, cost of $100 of the bonds. *Ans.* $62\frac{1}{2}$.

60. What must one buy a 7% stock for, to realize an income of 8% on his investment? *Ans.* $87\frac{1}{2}$.

61. How much above par does an 8% stock sell for, when it pays an interest of 7% on the investment? What must it sell for, to pay an interest of 9% on the investment?

62. When gold stands at 130, what must a party buy 5–20's for, to realize 7% on his investment?

$1 of the bonds yields $ .06 in gold, or ($.06 \times 1.30$) $ .078 in currency. Then proceed as in Example 59.

63. When gold is at 135, what must 10–40's sell for, to yield 8% interest on the investment? *Ans.* $84\frac{3}{8}$.

64. What must gold sell for, that a party investing in 5–20's, at 105, may realize 8% interest on his investment?

$1 of 5–20's yields $ .06 in gold, and costs $1.05.

Hence, § 231, $.06 \div 1.05 = .05\frac{5}{7}$. The interest on the investment, in gold, is therefore $5\frac{5}{7}$; and, to pay 8 per cent. in currency, gold must sell for as much as $5\frac{5}{7}$ is contained times in 8, or 1.40. *Ans.* 40 per cent. premium, or 140.

65. What must gold sell for, that an investment in 10–40's at 97 may yield an interest of 7%? *Ans.* $135\frac{4}{5}$.

66. Which is the better investment, U. S. 5–20's at 104, gold standing at 125, or Virginia 6's at 70—and how much?

67. If I sell 200 shares of stock at 49, paying brokerage, and invest the proceeds in 10–40's at $97\frac{1}{2}$, what will be my annual income when gold is 130? *Ans.* $650.

CHAPTER XXIV.

BANKRUPTCY.

406. A **Bankrupt** is one who fails in business, or is unable to meet his obligations. Such a party is said to be **insolvent.**

The **Assets** of a bankrupt are the property in his hands. His **Liabilities** are his debts, or obligations.

407. When a person becomes bankrupt, an Assignee is usually appointed, who takes possession of the assets, turns them into cash, and, after deducting his own charges, divides the net proceeds among the creditors in proportion to their claims.

EXAMPLE.—A merchant fails, owing A $3000, B $6250, C $800, and D $11950. His assets are $8650, and the expenses of settling $650. What can he pay on the dollar, and how much will each creditor receive?

We must first find the *rate* of dividend. The total of liabilities is the *base;* the net proceeds of the assets, the *percentage.* Dividing the percentage by the base, § 321, we find the rate to be 40%, or 40 cents on the dollar. Each creditor's share is then found by multiplying his claim by this rate.

Prove by finding whether the sum of the several dividends corresponds with the net proceeds to be divided.

LIAB'S,	A $3000	ASSETS, $8650	A $3000 × .40 = $1200
	B 6250	Expenses, 650	B 6250 × .40 = 2500
	C 800	Net pro., $8000	C 800 × .40 = 320
	D 9950	8000 ÷ 20000 = .40	D 9950 × .40 = 3980
Total,	$20000	Rate, 40%.	PROOF, $8000

408. RULE.—1. *Find the rate of dividend, by dividing the net proceeds of the assets by the total of liabilities.*

2. *Find each creditor's dividend, by multiplying his claim by this rate.*

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406. What is a Bankrupt? What is meant by the Assets of a bankrupt? By his Liabilities?—407. When a person becomes bankrupt, what is usually done? Go through the example, explaining the several steps and proof.—408. Recite the rule.

EXAMPLES FOR PRACTICE.

1. A merchant becomes insolvent, owing A $875.50, B $1100, C $4168.75, D $3725, and E $8630.75. His assets realize $11400, and the assignee's charge is $600. What is the rate of dividend, and what each creditor's share? *Ans.* Rate, 60 %.

2. Harrison & Co. having failed, their liabilities are found to be $71600. Their assets consist of goods that sell for $9815; debts collectible, $17005; house and lot, worth $7250. The assignee's charge is 5 % on the assets, and other expenses amount to $146.50. What % can they pay, and how much will Ira Jones receive, to whom they owe $12500? *Last ans.* $5625.

3. S becomes insolvent, owing $62000, and having $14200 assets; the expenses of settling are $560. How much can he pay on a dollar? What is P's dividend on a claim of $1400? Q receives $275; what was his claim? *Last ans.* $1250.

4. A bankrupt settled with his creditors for 35c. on a dollar. B received a dividend of $5075, and C 5 % of that amount; what were their respective claims? *Ans.* C's, $725.

5. The assets of a bankrupt are $42000. He owes V $17000, W $24150, X $37140.75, Y $28000.50, and Z $10708.75. Y becomes assignee, and receives 4 % on the assets for his services; the other expenses of settling are $1320. What is each creditor's share—Y's to include his percentage as assignee? *Ans.* Y's, $11013.50.

CHAPTER XXV.

INSURANCE.

409. Insurance is a contract by which, in consideration of a certain sum paid, one party agrees to secure another against loss or risk.

410. There are different kinds of Insurance:—

Fire Insurance secures against loss or damage by fire; Marine Insurance, against the dangers of navigation;

Accident Insurance, against casualties to travellers and others. Health Insurance secures a weekly allowance during sickness. Life Insurance secures a certain sum, on the death of the insured, to some party named in the contract.

411. The **Underwriter** is the insurer,—the person or company that takes the risk.

The **Policy** is the written contract.

The **Premium** is the sum paid the underwriter for taking the risk. In the case of Fire and Marine Insurance, it is reckoned at a certain % on the sum insured.

412. The rate is sometimes given at so many cents on $100, in stead of on $1. In that case, be careful to write the decimal properly. 20 cents on $1 is written .2; on $100, .002. 45c. on $1 is .45; on $100, .0045.

Insurance is usually effected with companies. Some companies, to guard against fraud, will not insure to the full value of the property. Different rates are charged, according to the risk. In case of loss, the underwriters may either replace the property or pay its value. Only the amount of actual loss can be recovered.

413. The principles of Percentage apply to Insurance (Fire and Marine). The sum insured is the *base;* the premium is the *percentage,* reckoned at a certain *rate.* Hence, according to § 321, the following

RULES.—I. *To find the premium, multiply the sum insured by the rate.*

II. *To find the rate, divide the premium by the sum insured.*

III. *To find the sum insured, divide the premium by the rate.*

EXAMPLES FOR PRACTICE.

1. Insured a house for $10000, and furniture for $5000, at the rate of 30c. on $100; $1 being paid for the policy and survey, what does the insurance cost? *Ans.* $46.

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409. What is Insurance?—410. Name the different kinds of Insurance, and state against what each secures the insured.—411. What is meant by an Underwriter? What is the Policy? What is the Premium?—412. What caution is given as to writing the rate? How do some companies try to guard against fraud? In case of loss, what may the underwriters do?—413. Recite the rules.

2. At ½ of 1%, what is the premium on $8000? On $7250? At ⅝%, what is the premium on $2200? *First ans.* $40.

3. A factory and its contents, worth $72000, are insured for ⅔ of their value, at 3⅓ per cent. The whole is consumed. How much will the owner receive, and what will be the actual loss to the underwriters? *Last ans.* $46400.

The actual loss is the sum they have to pay, less the premium.

4. A merchant insures 1200 bar. of flour, worth $8 a barrel, for their full value, at ¼%. A fire occurring, only 450 barrels are saved. What premium does the merchant pay, how much will he receive from the company, and what will be their actual loss? *Second ans.* $6000.

5. A vessel valued at $90000, and its cargo worth $55000, are insured for half their value, at 2½%. What is the premium, including $1 for policy?

6. Insured $9000 worth of goods for ⅔ of their value, at ⅝%. They were damaged by fire to the extent of $1250. What was the premium, how much did the underwriters pay the insurer, and what was their actual loss? *Last ans.* $1212.50.

7. The premium on a house, at ⅓ of 1%, cost me $20; what was the sum insured? (See Rule III., § 413.) *Ans.* $6000.

8. Paid for insuring a hotel for ⅔ of its value, $151. The rate being 75c. on $100, and the policy costing $1, what was the hotel worth? *Ans.* $30000.

As the policy cost $1, the premium was $151—$1, or $150. 75c. on $100 = .0075, rate. Apply Rule III., to find the sum insured, and this will be ⅔ of the value of the hotel.

9. Paid $18 for insuring $9000; what was the rate? (See Rule II., § 413.) *Ans.* ⅕ of 1 per cent.

10. Paid $400 for insuring a factory, worth $48000, for ⅚ of its value; what was the rate?

11. An underwriter agrees to insure a hotel, worth $24000, for a sufficient sum to cover its value and the premium. The rate being 1%, for how much must he insure it?

Analogous to Example 2, § 323. As the rate is 1 per cent. of the sum to be insured, the value of the hotel, $24000, must be 99 per cent. of this sum. Then by Rule III., § 321, $24000 ÷ .99 = $24242.42 *Ans.*

12. For how much must a schooner be insured, to cover its value, $15000, and the premium, the rate being $1\frac{1}{2}\%$? What will the premium amount to? *Last ans.* $228.43.

13. Paid for insuring the full value of a ship and cargo, at 1 %, $450. If the cargo was worth half as much as the ship, what was the value of the ship?

414. Accident Insurance.—Insurance against accidents is effected by paying (in advance) an annual premium, in consideration of which the underwriters give the insured a certain allowance per week in case he is disabled by an accident, or pay his heirs a specified sum if he is killed.

14. A party paying $12 premium annually, in the third year for which he insures, is disabled by an accident for 13 weeks, during which time he receives $10 a week. How much more does he receive than he paid for premiums? *Ans.* $94.

15. A person who has paid five annual premiums of $30 each, is killed by an accident. His family receive $5000. Not reckoning interest, what is the loss to the underwriters?

16. A railroad conductor insures for $60 a year, his weekly compensation in case of a disabling accident to be $50. In the tenth year, he is laid up by an accident for 4 weeks; does he gain or lose by insuring, and how much, leaving interest out of account? *Ans.* Loses $400.

415.—Life Insurance.—Life Insurance is effected by paying (in advance) an annual premium during life or for a term of years, in consideration of which the underwriters, on the death of the insured, pay a certain sum to his heirs or some party named in the policy.

416. The rates of life insurance depend on the age at which one begins to insure, and are fixed at a certain sum on every $100 or $1000 insured. They differ but little in different companies, being based on the Expectation of

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414. How is Accident Insurance effected?—415. How is Life Insurance effected?—416. On what do its rates depend? How are they fixed? On what are they based?

Life,—that is, the average number of years that persons at different ages live, as shown by statistics.

417. RULE.—*To find the premium in life insurance, multiply the premium on* $100 *or* $1000 *by the number of hundred or thousand dollars insured.*

17. What annual premium must a person, aged 30 when he begins to insure, pay for a life policy of $5000, the rate being $2.3023 on $100? *Ans.* $115.12.

18. At the age of 40, a gentleman insures his life for $3000, payment of premiums to cease in ten years. The rate is $57.959 on $1000. If he dies at 55, how much more will his family receive than he paid for premiums? *Ans.* $1261.20.

19. On his 40th birth-day, a clergyman insures his life for $6000, payment of premiums to cease when he is 65. The rate is $35.12 on $1000. If he dies aged 45 years 1 month, how much more than the premiums paid will his heirs receive?

Ans. $4735.68.

20. A farmer insured his life for $1750, at the rate of $3.66 on $100. Just 9 months afterwards he died. Taking interest on the premium (at 6%) into account, how much was gained by insuring? *Ans.* $1683.07.

CHAPTER XXVI.

TAXES.

418. A **Tax** is a sum assessed on the person, property, or income of an individual, for the support of government.

When assessed on the person, it is called a Poll-tax, and is a uniform sum on each male citizen, except such as may be exempted by law.

When assessed on the property, it is called a Property-tax, and is reckoned at a certain rate on the estimated value.

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417. Recite the rule for finding the premium in life insurance.—418. What is a Tax? Name and define the three kinds of taxes.

When assessed on the income, it is called an Income-tax, and is computed at a certain %.

419. Taxable property is either Real or Personal.

Real Estate is fixed property; as, lands, houses.

Personal Property is that which is movable; as, cash, notes, ships, furniture, cattle, &c.

420. An **Assessor** is an officer appointed to estimate the value of property and tax it in proportion.

421. ASSESSMENT OF TAXES.—In assessing a property-tax, an Inventory, or list, of all the taxable property, real and personal, with its estimated value, must first be made out. If there is, besides, a poll-tax, a list of polls (that is, of persons liable to said tax) must also be drawn up. The poll-tax having been fixed, the rate of property-tax must then be found, and lastly each man's tax.

Ex. 1.—A tax of $6402 is to be raised in a certain town, containing 480 polls, which are assessed $1 each. The real estate of said town is valued at $878500, the personal property at $108500. What will be the rate on $1,—and what will be A's tax, who pays for 4 polls, and whose real estate is inventoried at $5500, his personal property at $1250?

$878500 + $108500 = $987000, total taxable property.
$1 × 480 = $480, total poll-tax.
$6402 − $480 = $5922, property-tax to be assessed.
By Rule II., § 321, $5922 ÷ 987000 = .006, rate.
$5500 + $1250 = $6750, A's taxable property.
$6750 × .006 = $40.50, A's property-tax.
$1 × 4 = $4, A's poll-tax.
$40.50 + $4 = $44.50, total A's tax.

422. RULE.—1. *To find the rate of property-tax, divide the sum to be raised, less the amount assessed on polls, by the value of the taxable property, real and personal.*

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419. How many kinds of taxable property are there? What is Real Estate? What is Personal Property?—420. What is the business of an Assessor?—421. In assessing a property-tax, what must first be made out? If there is, besides, a poll-tax, what must be done? What are the next steps? Go through the given example, explaining the steps.—422. Recite the rule. If there is no poll-tax, what must be done?

2. *To find each man's tax, multiply his taxable property by the rate, and to the product add his poll-tax.*

If there is no poll-tax, the whole amount to be raised must be divided by the value of the taxable property.

423. If the given amount to be raised does not include the expense of collecting, the whole sum needed, including this expense, must first be found, *by dividing the given amount by $1 diminished by the rate % to be paid for collecting.*

Thus, in Example 1, let the expense of collecting, 2½%, not be included in the $6402 named; then, as $1 raised would net but $.975, there would have to be raised as many times $1 as $.975 is contained times in $6402. In other words, we should have to divide $6402 by $1 diminished by .025, the rate paid for collecting.

424. After finding the rate as above, assessors usually construct a Table, from which, by adding the amounts standing opposite to the thousands, hundreds, tens, and units of any given sum, they can readily determine the tax it must bear—more readily, as a general thing, than by multiplying by the rate.

Assessor's Table for a rate of .006.

$1	$.006	$10	$.06	$100	$0.60	$1000	$ 6.
2	.012	20	.12	200	1.20	2000	12.
3	.018	30	.18	300	1.80	3000	18.
4	.024	40	.24	400	2.40	4000	24.
5	.030	50	.30	500	3.00	5000	30.
6	.036	60	.36	600	3.60	6000	36.
7	.042	70	.42	700	4.20	7000	42.
8	.048	80	.48	800	4.80	8000	48.
9	.054	90	.54	900	5.40	9000	54.

2. Find by the Table what tax B must pay on $7560.

Opposite	$7000	we	find	$42.00
"	500	"	"	3.00
"	60	"	"	0.36
Total for	$7560,			$45.36 *Ans.*

3. What is C's tax on $425, and 3 polls, at $1 each? D's, on $900 real estate, $650 personal property? E's, on $2820 real estate, $710 personal, 1 poll?

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423. If the given amount to be raised does not include the expense of collecting, how must we proceed? Illustrate this in the case of Ex. 1.—424. After finding the rate as above, what do assessors usually construct? Show how the Table is used.

4. The people of a certain town have to raise a tax of $4656, besides the expense of collecting, which is 3% (see § 423). The inventory shows real estate valued at $401250, and personal property at $98750. There are 400 polls, assessed at 75c. each.

Find the rate on $1, draw out a Table like that on p. 262, and from it determine the tax of the following parties:—

G, who pays on $3460 and 2 polls. *Ans.* $32.64.

H, on $1975 and 4 polls.

I, on $2000 real, $800 personal, and 3 polls.

J, on $1750 real, $640 personal, and 1 poll.

425. NATIONAL TAX.—By Act of Congress, a tax for the National Government is laid on incomes, &c., as follows:—

5% on amounts of annual income in excess of $600 and not exceeding $5000, amount paid for rent and certain other allowances being first deducted.

10% on amounts of annual income in excess of $5000.

5c. an oz. on silver plate kept for use, exceeding 40 oz.

50c. an oz. on gold plate kept for use.

Carriages, gold watches, pianos and other musical instruments valued at not less than $100 (not including those in churches or public edifices), yachts, and billiard-tables, are also taxed.

Ex. 5.—Find K's national tax for 1865; his income is $7420, his rent $700, and he has 55 oz. silver plate.

Income, ...	$7420	
Deduct $600 exempt, $700 rent,	1300	
Taxable income,	$6120	
$5000 — $600 = $4400, at 5%, ...		$220.00
$6120 — $4400 = $1720, at 10%, . . .		172.00
55 oz. — 40 oz. = 15 oz., at 5c., ...		.75
Total tax,		$392.75 *Ans.*

6. What is M's national tax on an income of $6542, 72 oz. of silver plate, 1 carriage at $2, 1 piano at $4, 1 gold watch at $2? *Ans.* $383.80.

7. What is N's tax on an income of $1500, and 63 oz. of silver plate, his rent being $400? *Ans.* $26.15.

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425. What is the rate of income tax imposed by the National Government? What is the rate on silver plate? On gold plate? What other articles are taxed? Explain Ex. 5.

CHAPTER XXVII.

DUTIES.

426. Duties, or **Customs**, are taxes on goods imported from foreign countries, levied for the support of the National Government.

427. A **Custom-house** is an office established by government for the collection of duties. A port containing a custom-house is called a Port of Entry.

428. Duties are either Specific or Ad valorem.

A **Specific Duty** is a fixed sum imposed on each ton, pound, yard, gallon, &c., of an imported article, without regard to its cost.

An **Ad valorem Duty** is a percentage on the cost of an imported article in the country from which it was brought. *Ad valorem* means *on the value.*

429. An **Invoice** is a statement in detail of goods shipped, their measure or weight, and cost in the currency of the country from which they were brought.

430. Before computing duties, certain Allowances, or Deductions, are made:—

Tare is an allowance for the weight of the box, cask, &c., containing the goods; Leakage, for waste of liquids imported in casks; Breakage, for loss of liquids imported in bottles.

Tare is estimated either at the rate specified in the invoice accompanying the goods, or according to rates adopted by Act of Congress, differing for different articles.

For Leakage 2% is allowed; for Breakage, 10% on beer, ale, and porter, in bottles; 5% on other liquids,—a dozen "quart" bottles being estimated to contain 2$\frac{2}{5}$ gallons.

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426. What are Duties, or Customs?—427. What is a Custom-house? What is a Port of Entry?—428. Name the two kinds of duties. What is a Specific Duty? What is an Ad valorem Duty?—429. What is an Invoice?—430. Name and define the allowances made before computing specific duties. How is Tare estimated? How much is allowed for Leakage? How much for Breakage?

In stead of computing by these fixed rates, the weight of the box, &c., and the amount lost by leakage and breakage, are sometimes ascertained by actual trial and allowed for accordingly.

In these allowances, reject a fraction less than ½; reckon ½ or more as 1.—In custom-house computations, allow 112 lb. to a cwt.

431. Gross Weight is the weight of goods, together with that of the box, cask, bag, &c., containing them.

Net Weight is the weight of goods after allowances have been deducted.

432. RULES.—I. *To find a specific duty, deduct allowances, and multiply the number of tons, pounds, yards, gallons, &c., remaining, by the duty on one ton, pound, yard, gallon, &c.*

II. *To find an ad valorem duty, multiply the invoice-value of the goods by the given rate.*

Duties are required to be paid in gold.

EXAMPLES FOR PRACTICE.

1. What is the duty on a lot of silks, costing in our currency $14056, at 60%? When gold is at a premium of 40%, what sum in currency will pay said duty? *Last ans.* $11807.04.

2. Imported 75 casks of raisins, weighing 112 lb. each. The tare being 12%, and the duty 5 cents a pound, what is the duty on the whole in gold? When gold is at 130, what sum in currency will pay it? *Last ans.* $480.48.

3. Required the duty on 42 barrels of spirits of turpentine, containing 31 gallons each, leakage being allowed, and the rate being 30 cents per gal. *Ans.* $382.80.

4. At 40% ad valorem, what is the duty on 346 lb. of sewing-silk, bought for $12 a pound?

5. What is the duty on 6 casks of claret, holding 43 gal. each, invoiced at $1 a gal., allowing for leakage, the rate being 50c. a gallon and 25% ad valorem? *Ans.* $189.75.

How are these allowances sometimes determined? How many pounds are allowed to 1 cwt., in custom-house computations?—431. What is Gross Weight? What is Net Weight?—432. Recite the rules.

6. The duty on tea being 25 cents a pound, what must be paid on 175 chests, each weighing 60 lb., a tare of 6 lb. being allowed on each chest?

7. What is the duty on 12 cases of brandy, containing 1 dozen bottles each, the usual allowance being made for breakage, and the rate being $3.60 a gal.? *Ans.* $111.60.

8. At 5 cents a pound, what is the duty on 50 bags of coffee, averaging 100 lb. gross weight, tare 2%?

9. A merchant imported 10 hhd. of sugar averaging 1185 lb., and 8 hhd. of molasses holding 63 gal. each. A tare of 12½% is allowed on the sugar, and leakage on the molasses. What is the duty on the whole, the rate on the sugar being 3c. a lb., and on the molasses 8c. a gal.? *Ans.* $350.59.

CHAPTER XXVIII.

EQUATION OF PAYMENTS.

433. Equation of Payments is the process of finding when two or more sums due at different times may be paid at once, without loss to debtor or creditor. The time for such payment is called the Equated Time.

Ex. 1.—A owes B $1000, of which $100 is due in 2 months, $250 in 4 mo., $350 in 6 mo., and $300 in 9 mo. If A pays the whole sum at one time, how long a credit should he have?

The use of $100 for 2 mo. = use of $1 for	100 × 2,	or 200 mo.
" " " $250 for 4 mo. = " " $1 "	250 × 4,	or 1000 mo.
" " " $350 for 6 mo. = " " $1 "	350 × 6,	or 2100 mo.
" " " $300 for 9 mo. = " " $1 "	300 × 9,	or 2700 mo.
Hence A is entitled to the use of $1 for 6000 mo., or $1000 for $\frac{1}{1000}$ of that time, or 6 mo.	1000)	6000 mo.
		Ans. 6 mo.

433. What is Equation of Payments? What is meant by the Equated Time? Go through Ex. 1.

434. RULE.—*To equate two or more payments, multiply each payment by its time, and divide the sum of the products by the sum of the payments.*

The times of the several payments must be in the same denomination, and this will be the denomination of the answer.

Less than ½ day in the answer is rejected; ½ day or more counts as 1.

435. If the *date* is required, *reckon the equated time forward from the given date.*

Ex. 2.—July 9, 1866, C becomes indebted to D for a certain sum; ⅓ is to be paid in 6 months, ¼ in 8 mo., and the rest in 12 mo. At what date may he equitably pay the whole?

$$\begin{array}{r}
\frac{1}{3} \times 6 = 2 \\
\frac{1}{4} \times 8 = 2 \\
\frac{5}{12} \times 12 = 5 \\
\hline
1)\quad 9 \\
\hline
\textit{Ans. } 9 \text{ mo.}
\end{array}$$

Use the fractions representing the amounts as in Ex. 1. The equated time being 9 months, payment should be made 9 months from July 9, 1866,—that is, April 9, 1867.

EXAMPLES FOR PRACTICE.

1. A merchant has the following sums due from a customer: $300 in 2 mo., $800 in 5 mo., and $400 in 10 mo. Find the equated time. *Ans.* 5 mo. 22 da.

2. E owes F $1200, $200 of it payable in 2 mo., $400 in 5 mo., and the rest in 8 mo. What is the equated time?

3. A trader bought goods, Aug. 1, 1866, to the amount of $2400: for ¼ of the bill he was to pay cash; ⅓ of it he bought on 6 months' credit, and the rest on 10 months. On what day may he equitably pay the whole? *Ans.* Feb. 6, 1867.

The cash payment must be added with the others, but its product is 0. $\Big\}\ \frac{1}{4} \times 0 = 0$

4. One person owes another a certain sum, ⅕ of which is due in 3½ mo., ¼ in 4½ mo., ⅓ in 5 mo., and the balance in 8 mo. What is the equated time? *Ans.* 5 mo. 7 da.

5. Jan. 1st, I owe a friend $100 cash; $150, payable Feb. 5; and $300, payable April 10. It being leap year, on what day may I fairly pay the whole at once? *Ans.* Mar. 5.

The Table on p. 156 will assist in finding the number of days.

434. Recite the rule.—435. If the *date* is required, what must be done?

6. Equate the following payments: $400 due in 15 days, $600 in 20 days; $1000 in 60 days; $350 in 90 days.

7. A farmer, on the 1st of March, bought some land for $1000. He agreed to pay $250 cash; $250 on the 3d of the following May; $250, July 4; and $250, Sept. 15. He prefers paying the whole at once; when should it be? *Ans.* June 6.

Ex. 8.—Suppose $700 to be due in 6 mo. At the expiration of 3 mo., $100 is paid on account; and at the end of 5 mo., $300. How long after the six months expire should the balance be allowed to stand, in consideration of these prepayments?

On the principle applied in Ex. 1, the creditor gets the use of what is equivalent to $1 for 600 mo.; the debtor is, therefore, also entitled to the use of $1 for 600 mo., or $300 (the balance) for $\frac{1}{300}$ of 600 mo., or 2 mo.

$$\begin{array}{r} 100 \times 3 = 300 \\ 300 \times 1 = \underline{300} \\ \overline{400} \qquad\quad 600 \\ 700 - 400 = 300 \\ 600 \div 300 = 2 \text{ mo. } Ans. \end{array}$$

436. RULE.—*When partial payments have been made on a debt before it is due, to find how long the balance should remain unpaid, multiply each payment by the time it was made before falling due, and divide the sum of these products by the balance.*

9. A person owes $1000, due in 12 mo. At the end of 3 mo. he pays $100, and one month afterwards $100. How long beyond the 12 mo. should the balance stand? *Ans.* 2 mo. 4 da.

10. $1496.41 is due in 90 days. 84 days before it falls due, $500 is paid, and 52 days after the first payment $502.50. How long after the 90 days, before the balance of the debt should be paid? *Ans.* 118 days.

11. A lent B $200 for 8 months, and on another occasion $300 for 6 months. How long should B lend A $800, to balance these favors? *Ans.* 4¼ mo.

12. A credit of 6 mo. on $500, one of 4 mo. on $1000, and one of 8 mo. on $400, are equivalent to a credit on how many dollars for 12 mo.? *Ans.* $850.

Analyze Ex. 8.—436. Recite the rule for finding how long a balance should stand, when partial payments have been made on a debt before it is due.

13. T. Hoe buys goods of G. A. Rand, as follows:—

1. May 1, bill of $600, on 3 mo. credit.
2. May 15, " " $800, " 4 mo. "
3. June 1, " " $500, " 6 mo. "
4. June 9, " " $900, for cash.

Rand agrees to take Hoe's note for the whole, for 30 days, with interest. When should the note be dated?

Here the terms of credit begin at *different* dates. We must first find when each bill falls due, by reckoning forward from its date the term of credit.

Term of credit.	*Due.*	*Pay't.*	*Days.*	*Product.*
1. 3 mo. from May 1,	Aug. 1, .	$600 ×	53 =	31800
2. 4 mo. from May 15,	Sept. 15,	$800 ×	98 =	78400
3. 6 mo. from June 1,	Dec. 1,	$500 ×	175 =	87500
4. Cash payment,	June 9,	$900 ×	0 =	0
		$2800		197700

$$197700 \div 2800 = 70\tfrac{17}{28}.$$

Equated Time, 71 days.

Since there is no uniform date to reckon from, as in the former examples, we take the earliest date on which a payment falls due, June 9, and find the number of days from that time to the date when each payment falls due, writing it opposite the payment it belongs to, as in the 4th column above. Then finding the products and dividing as before, we get 71 days for the equated time, which must be reckoned forward from the standard date, June 9.

21 days remaining in June.
31 " in July.

$71 - 52 = 19$. *Ans.* August 19.

We might have assumed the latest date at which a payment fell due, Dec. 1, as a standard, proceeded as above, and reckoned the equated time so found back from that date. The result would have been the same. The operation may always thus be proved.

437. Rule.—*To equate payments when the terms of credit begin at different times, find the dates when the several payments become due. From the earliest of these dates, as a standard, reckon the number of days to each of the others. Then find the equated time as before,* § 434, *and reckon it forward from the standard date.*

Explain Ex. 13. How does this differ from the preceding examples? Why do we assume the earliest date as a standard? What other date might have been assumed? How may the operation be proved?—437. Recite the rule for equating payments, when the terms of credit begin at different times.

We may shorten the multiplication, without materially affecting the result, by rejecting less than 50 cents in any payment, and calling 50 cents or over, $1.

14. Bought goods of Parsons & Co., on different terms of credit, to the following amounts: March 6, $275.50, on 30 days; March 31, $560, on 3 months; April 10, $820.10, on 60 days; May 3, $515, on 4 months; May 9, $1225.40, on 6 months. At what date may the whole be discharged at once? *Ans.* Aug. 14.

15. Harvey Bolton is indebted to a silk-house for goods bought, as follows:—June 1, $842, on 6 months; June 2, $1500, on 4 months; June 3, $1875.75, on 3 months; June 4, $400, on 6 months; June 5, $750, cash. In stead of paying the items separately when due, Bolton gives his note, without interest, for the whole; for how much should his note be drawn, and when should it mature? *Last ans.* Sept. 19.

16. Cranston & Miner have sold goods to Henry S. Owens, as follows:—Nov. 1, 1865, on 6 months, $1200; Nov. 5, on 4 mo., $800; Nov. 30, on 3 mo., $440.96; Dec. 3, on 90 days, $650; Dec. 10, on 2 mo., $1120.25; Dec. 24, on 6 mo., $347. Owens proposes to discharge the whole at one payment; when should it be made? *Ans.* March 22, 1866.

17. Sold a customer the following goods: Aug. 2, 2 dozen overcoats, @ $25 each, on 60 days' credit; Aug. 4, 6 dozen boys' sacks, @ $8.50, and 12 dozen boys' pants, @ $5, on 90 days; Aug. 5, 4 dozen cassimere pants, @ $12, on 90 days; Aug. 6, 6 dozen vests, @ $3.25, on 4 months. When should a note for the whole amount, without interest, mature? *Ans.* Oct. 29.

Averaging Accounts.

438. An **Account** is a statement of mercantile transactions, its left side (marked *Dr.*) being appropriated to debits, and its right side (marked *Cr.*) to credits. The difference between the sum of the debits and that of the credits is the Balance of the Account.

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438. What is an Account? What is meant by the Balance of an Account?

439. Averaging an Account is the process of finding the equitable time for the payment of the balance.

Those accounts only need averaging, in which items occur bearing interest from their date, or from the expiration of their terms of credit.

440. Finding the **Cash Balance** of an account is finding what sum will balance the account at any given time, interest being allowed on the several items.

Ex. 1.—Average the following account, supposed to be taken from the Ledger of Stephen Stewart :—

Dr. MOSES T. MARSH. *Cr.*

1866			1866		
May 8	To Merchandise	$900	Apr. 3	By Merchandise	$200
" 12	" "	850	" 10	" "	400
" 15	" "	610	" 17	" "	500
June 1	" "	400	May 15	" Cash	450

Marsh owes Stewart $1210, as is found by balancing the account. When is it equitably due? Or, if Stewart gives his note for the balance, when should it be dated?

Take the earliest date on either side of the account, April 3, as the standard. Then, according to the principle already explained, the interest on all the debits from this standard date to the times they severally fall due would equal the interest of $1 for 113870 days (see operation below); that on the credits would equal the interest of $1 for 28700 days. There is, therefore, an excess of interest in favor of the debits, equal to the interest of $1 for 85170 days—or of $1210 (the balance of account, on the debit side) for $\frac{1}{1210}$ of 85170 days, or 70 days. Hence Marsh is entitled to retain the balance he owes, till the expiration of 70 days from the standard date, April 3,—or June 12.

Debits,	900 × 35 =	31500	Credits,	200 × 0 =	0
	850 × 39 =	33150		400 × 7 =	2800
	610 × 42 =	25620		500 × 14 =	7000
	400 × 59 =	23600		450 × 42 =	18900
	2760	113870		1550	28700
	1550	28700			
Balance,	1210)	85170	Excess of debit products.		

Averaged time, 70 days. Date, June 12. *Ans.*

Had the excess of interest and the balance of account stood on *opposite* sides, we should have had to count the 70 days *back* from the standard date.

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439. What is Averaging an Account? What accounts need averaging?—440. What is meant by finding the Cash Balance of an account? Explain Ex. 1. Under what circumstances would we have had to count the 70 days *back*?

441. If a credit were allowed on each of the merchandise items, we should have found when each item became due, and used those dates in stead of the dates of the transactions. Thus :—

Ex. 2.—Average the account presented in Ex. 1, allowing each merchandise item a credit of 3 months.

Find when each item falls due. May 15 is the standard date.

Debits.				Credits.			
Due Aug. 8,	$900 ×	85 =	76500	Due July 3,	$200 ×	49 =	9800
" Aug. 12,	850 ×	89 =	75650	" July 10,	400 ×	56 =	22400
" Aug. 15,	610 ×	92 =	56120	" July 17,	500 ×	63 =	31500
" Sept. 1,	400 ×	109 =	43600	" May 15,	450 ×	0 =	0
	2760		251870		1550		63700
	1550		63700				
Balance,	1210)		188170	Excess of debit products.			

Averaged time, 156 days. Date, Oct. 18. *Ans.*

442. Cash Balance.—What is the cash balance of the account presented in Ex. 1, due Aug. 20, allowing 3 months' credit on each merchandise item, and interest at 6 % ?

We have just found, in Ex. 2, that the balance of $1210 is due Oct. 18. The cash balance on August 20th is therefore the present worth of $1210, due Oct. 18,—that is in 1 mo. 28 days. This, by § 378, is found to be $1198.42. *Ans.*

Had the given date of settlement fallen *after* the averaged date, Oct. 18, we should have *added interest* to $1210 for the interval.

With Interest Tables, which accountants universally use, the second method given in the rule below will be found the more convenient.

From the above examples we derive the following rules :—

443. Rules.—I. *To average an account, take the earliest date on either side as a standard, and multiply each item by the number of days between the time when it falls due and the standard date. Divide the difference between the sum of the debit and that of the credit products by the balance of the account. The quotient will be the averaged time. Reckon this forward from the standard date, if the excess of products is on the same side with the balance of account ; if not, backward.*

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441. What must we do when a credit is allowed on the merchandise items? Explain Ex. 2.—442. How may we find the cash balance of the account presented in Ex. 1, due Aug. 20?—443. Recite the rule for averaging an account. For finding the cash balance.

II. *To find the cash balance, average the account, and, if the given date of settlement falls before the averaged time, find the present worth of the balance of account for the interval; if after, add interest for the interval.*

Or, find the interest on each item from the date it falls due to the time of settlement; write it on the same side of the account as its item, if the item falls due before the date of settlement,—if not, on the opposite side. Find the balance of interest, and add it to the balance of the account if the two balances stand on the same side; if not, subtract it.

3. Average, and find cash balance Mar. 1, 1866, at 7%.

Dr. REUBEN THOMPSON. *Cr.*

1866				1866			
Jan.	2	To Cash	$1200	Jan. 10		By Merch., 4 mo.	$1000
"	5	" Merch., 60 da.	1400	" 18		" Merch., 6 mo.	1160
"	8	" Merch., 3 mo.	1500	Feb. 2		" Merch., 3 mo.	1250
"	10	" Merch., 60 da.	2000	" 6		" Merch., 6 mo.	1300

Ans. { Balance of acct., $1390, due June 23, 1865.
Cash balance, March 1, 1866, $1441.08.

4. Average, and find cash balance Jan. 1, 1866, at 6%.

Dr. ALBERT B. CONNER. *Cr.*

1865				1865		
Sept.	3	To Merch., 3 mo.	$750	Oct. 1	By Merch., 4 mo.	$200
"	20	" Merch., 4 mo.	610	" 20	" Cash	800
Oct.	12	" Merch., 4 mo.	900	Nov. 5	" Merch., 3 mo.	325
Nov.	1	" Merch., 6 mo.	220	" 12	" Merch., 6 mo.	540
"	10	" Merch., 3 mo.	400			

Ans. { Balance of acct., $1015, due Feb. 10, 1866.
Cash balance, Jan. 1, 1866, $1008.45.

MISCELLANEOUS QUESTIONS.—Recite the rules relating to Percentage, § 321. Apply these rules to Interest, showing what corresponds to the base, and what to the percentage. Show how the rules of Percentage apply to Bank Discount. To Commission. To Bankruptcy, in ascertaining the rate of dividend, and in finding each creditor's share. To Insurance. To Assessment of Property Taxes, in determining the rate, and in finding each individual's tax. To ad valorem Duties.

CHAPTER XXIX.

RATIO.

444. Ratio is the relation that one quantity bears to another of the same kind. It is represented by the quotient arising from dividing one by the other. The ratio of 8 to 2 is 4.

445. Two quantities are necessary to form a ratio; these are called its Terms.

The **Antecedent** is the first term of a ratio; the **Consequent**, the second.

446. A ratio is either Direct or Inverse. It is **Direct**, when the antecedent is divided by the consequent; **Inverse**, when the consequent is divided by the antecedent. When the word *ratio* is used alone, a direct ratio is meant.

The direct ratio of 8 to 2 is 4. The inverse ratio of 8 to 2 is $\frac{1}{4}$. In either case, 8 is the antecedent, and 2 the consequent.

447. Ratio is expressed in two ways:—1. By two dots, in the form of a colon, between the terms; as, 8 : 4. 2. In the form of a fraction; as, $\frac{8}{4}$.

The two dots and the fractional line both come from the sign of division $\div$. When the two dots are used, the line between is omitted; when the fractional line is used, the two dots are omitted.

8 : 4 is read *the ratio of* 8 *to* 4.

448. A ratio being expressed by a fraction, of which the antecedent is the numerator and the consequent the denominator, it follows that the principles which apply to the terms of a fraction, § 137, apply also to the terms of a ratio. That is,

Multiplying the antecedent multiplies the ratio, and dividing the antecedent divides the ratio.

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444. What is Ratio? By what is it represented?—445. How many quantities are necessary to form a ratio? What are they called? Which is the Antecedent? Which, the Consequent?—446. What is the difference between Direct and Inverse Ratio? Give an example.—447. In how many ways is ratio expressed? Describe them. What is the origin of the two dots and the fractional line?—448. State the three principles that apply to multiplying or dividing the terms of a ratio.

Multiplying the consequent divides the ratio, and dividing the consequent multiplies the ratio.

Multiplying or dividing both terms by the same number does not alter the ratio.

449. Fractions having a common denominator are to each other as their numerators.

$\frac{7}{10} : \frac{9}{10} = 7 : 9$, or $\frac{7}{9}$. For, as we have just seen, dividing both terms of the second ratio, 7 and 9, by the same number, 10, does not alter their ratio.—The ratio between two fractions that have not a common denominator, may be found by reducing them to others that have, and taking the ratio of their numerators.

450. There is no ratio between quantities of different kinds; as, 8 yd. and 4 lb. But a ratio subsists between quantities of the same kind, though of different denominations.

Thus, the ratio of 8 yd. (= 24 ft.) to 4 ft. is 6. In such cases, to find the ratio, the terms must be brought to the same denomination.

451. A **Simple Ratio** is one into which but two terms enter. A **Compound Ratio** is the product of two or more simple ratios, the first term being the product of the antecedents, the second that of the consequents.

Simple Ratios, 8 : 4	The ratio compounded of these
9 : 3	three simple ratios is
2 : 6	$8 \times 9 \times 2 : 4 \times 3 \times 6$.

EXERCISE.

1. Express the ratio of 27 to 9; of 7 to 16; of 43 to 100.
2. Read the following ratios:—

144 : 12	$\frac{2}{3} : \frac{3}{4}$	6 lb. : 12 lb.	4 cwt. : 16 lb.
16 : 288	$\frac{5}{6} : \frac{7}{8}$	9 gr. : $4\frac{1}{2}$ gr.	3 mi. : 20 rd.
.005 : 100	$.7 : \frac{3}{5}$	2 mo. : 7.5 mo.	2 pt. : 16 gal.
240 : .8	$\frac{5}{9} : .1$	\$5 : \$.001	6 qt. : 50 bu.

3. Find the value of the above ratios, when direct.
4. Find the value of the above ratios, when inverse.

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449. What ratio do fractions having a common denominator sustain to each other? Prove this. Hence, how may the ratio between two fractions that have not a common denominator be found?—450. How may we find the ratio between two quantities of the same kind, but different denominations?—451. What is a Simple Ratio? What is a Compound Ratio? Give an example.

CHAPTER XXX.

PROPORTION.

452. Proportion is an equality of ratios.

The ratio of 8 to 4 is 2; the ratio of 6 to 3 is also 2. Hence the proportion, 8 : 4 = 6 : 3.

453. Proportion is expressed in two ways:—1. By the sign of equality between the ratios. 2. By four dots, in the form of a double colon, between the ratios.

8 : 4 = 6 : 3 } Read, 8 *is to* 4 *as* 6 *is to* 3.
8 : 4 :: 6 : 3 } Or, *the ratio of* 8 *to* 4 *equals the ratio of* 6 *to* 3.

454. Four quantities forming a proportion are called Proportionals. The first two are called the First Couplet; the last two, the Second Couplet. The first and fourth are called the Extremes; the second and third, the Means.

In the proportion 8 : 4 :: 6 : 3, 8 and 4 form the first couplet, 6 and 3 the second. 8 and 3 are the extremes, 4 and 6 the means.

455. Three quantities are in proportion when the 1st is to the 2d as the 2d to the 3d. 8 : 4 :: 4 : 2.

A term so repeated is called a Mean Proportional between the other two. 4 is a mean proportional between 8 and 2.

456. *The product of the extremes, in every proportion, equals the product of the means.* Thus, in the last proportion, $8 \times 2 = 4 \times 4$. Hence the following rules:—

457. Rules.—I. *To find an extreme, divide the product of the means by the given extreme.*

II. *To find a mean, divide the product of the extremes by the given mean.*

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452. What is Proportion?—453. In how many ways is proportion expressed? Describe them.—454. What are four quantities forming a proportion called? What are the first two called? The last two? Which are the Extremes? Which, the Means?—455. When are three quantities in proportion? What is meant by a Mean Proportional?—456. What principle holds good in every proportion?—457. Give the rule for finding an extreme. For finding a mean.

Ex. 1.—Find the 4th term of the proportion 8 . 4 : : 26 : ?

Find the product of the means : $4 \times 26 = 104$.
Divide by the given extreme : . $104 \div 8 = 13$. *Ans.*

Ex. 2.—Find the 2d term of the proportion 8 : ? : : 26 : 13.

Find the product of the extremes : $8 \times 13 = 104$.
Divide by the given mean : $104 \div 26 = 4$. *Ans.*

EXAMPLES FOR PRACTICE.

Complete the following proportions :—

1. 18 : 54 : : 200 : ? *Ans.* 600.
2. 60 : 90 : : ? : 1.83
3. ¾ : ? : : 12 : 8
4. ? : 80 : : ¼ : 1
5. 3 pt. : 12 pt. : : 2 bu. : ? *Ans.* 8 bu.
6. 1 qt. : ? : : 1 hr. : 1 da. *Ans.* 3 pk.
7. 15 gr. : 1 dr. : : ? : 3 sc.
8. 2 cwt. : 20 lb. : : $16 : ?
9. ? : 2 mi. : : £1 : 4d.
10. 600 : ? : : 3° : 20′
11. 1 rd. : ½ ft. : : ? : 50c.
12. 450 : 30 : : 1200 : ?

Simple Proportion, or Rule of Three.

458. A Simple Proportion expresses the equality of two simple ratios. Simple Proportions may be used to solve many questions in which three proportionals are given and the fourth is required.

As *three* terms are given, the rule for Simple Proportion is often called the Rule of Three.

Ex. 1.—If 8 yd. of cloth cost $40, what will 24 yd. cost?

The terms of a couplet must be of the same kind. Hence, in forming a proportion from the above question, as the answer, or fourth term, is to be dollars, we take $40 for the third term. Then, since 24 yd. will cost more than 8 yd., we arrange the other two numbers so as to form a ratio greater than 1, by taking 24, the greater, for the second term, and 8, the less, for the first. The proportion then stands,

8 yd. : 24 yd. : : $40, the cost of 8 yd. : the cost of 24 yd.
The 4th term is required ; we find it by Rule 1, § 457.
$24 \times 40 = 960$ $960 \div 8 = 120$ *Ans.* $120.

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458. What does a Simple Proportion express? To what questions do Simple Proportions apply? What is the rule often called? Explain Ex. 1.—459. How may cancellation be brought to bear?—460. Recite the rule.

459. In solving questions in Proportion, equal factors, if there are any, in the 1st and 2d, or 1st and 3d terms, should be cancelled. Thus, in Ex. 1 :—

$$\cancel{8}\text{ yd.} : \underset{3}{\cancel{24}}\text{ yd.} :: \$40.$$

$$\$40 \times 3 = \$120 \textit{ Ans.}$$

460. RULE.—1. *Take for the third term the number that is of the same kind as the answer. Of the two remaining numbers, make the larger the second term, when from the nature of the question the answer should exceed the third term; when not, make the smaller the second term.*

2. *Cancel equal factors in the first and second terms, or the first and third. Then multiply the means together, and divide their product by the given extreme.*

The first and second terms must be of the same denomination. If the third term is a compound number, it must be reduced to the lowest denomination it contains, and this will be the denomination of the answer.

EXAMPLES FOR PRACTICE.

1. What cost 8 cords of wood, if 2 cords cost $9? *Ans.* $36.

2. If 25 lb. of coffee cost $4.50, what cost 312 lb.? *Ans.* $56.16.

3. If a railroad car goes 17 miles in 45 minutes, how far will it go in 5 hours at the same rate? *Ans.* 113⅓ mi.

4. How long will it take $100 to produce $100 interest, if it produces $7 in one year?

5. If 15 men can build a wall 12 ft. high in 1 wk., how many will be needed to raise it 20 ft. in the same time? How long would it take 5 men to raise it 20 ft.? *Last ans.* 5 wk.

6. What cost 9 hats, if 5 hats cost £4 5s.? *Ans.* £7 13s.

7. If 7 tons of coal, of 2000 lb. each, last 3½ months, of 30 days each, how much will be consumed in 3 weeks?

8. If 9 bu. 2 pk. of wheat make 2 barrels of flour, how many bushels will be required to make 13 barrels?

9. If 5 bu. of potatoes last 8 adults and 2 children 40 days, how long, at the same rate, will they last 18 adults and 9 children, each adult consuming as much as 2 children? *Ans.* 16 days.

10. How long will it take a steamboat to move its own length, if it goes 15 miles an hour and is 242 feet long? *Ans.* 11 sec.

11. How many times its own length will a steamboat move in eleven hours, if it is 242 ft. long and goes 15 miles an hour?

12. A reservoir has two pipes that can discharge respectively 30 gal. and 15 gal. in one minute. How long will they be in discharging 15 hogsheads? *Ans.* 21 min.

13. If a man can mow 9 acres in $3\frac{1}{2}$ days, of 10 hours each, how many such days will it take him to mow 21 acres?

14. An insolvent debtor owes $7560, and has only $3100 with which to make payment. How much should a creditor receive, whose claim is $378? *Ans.* $155.

15. If $\frac{3}{10}$ of a ship is worth $2853, and $\frac{4}{5}$ of the cargo is worth $6080, how much are both ship and cargo worth?

16. How many yards of oil-cloth, $1\frac{1}{4}$ yd. wide, will be needed to cover a certain floor, if 30 yd., $\frac{3}{4}$ yd. wide, will cover it?

17. If the earth moves through 360° in $365\frac{1}{4}$ days, how far will it move in a lunar month of $29\frac{1}{2}$ days? *Ans.* $29\frac{37}{487}$°.

Compound Proportion, or Double Rule of Three.

461. A Compound Proportion expresses the equality of a compound and a simple ratio.

Compound Proportions are used in solving questions that involve two or more simple proportions; hence this rule is often called the Double Rule of Three.

Ex.—If 6 men can mow 30 acres of grass in 5 days, working 8 hours each day, how many acres can 4 men mow in 9 days, of 10 hours each?

As the answer is to be acres, we write 30 acres as the third term. We then take the other terms in pairs of the same kind—6 men and 4 men, 5 days and 9 days, 8 hours and 10 hours, and form a ratio with each pair as if the answer depended on it alone, as in simple proportion. As 4 men will mow less than 6 men, we take the smaller number for the sec-

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461. What does a Compound Proportion express? What is the rule for Compound Proportion often called? Why so? Explain the Example.

ond term of the ratio, 6 : 4. As in 9 days they can mow more than in 5 days, we take the greater for the second term, 5 : 9. As working 10 hours a day they can mow more than working 8 hours a day, we take the greater for the second term, 8 : 10. The proportion then stands,

6 men : 4 men :: 30 acres
5 days : 9 days
8 hr. : 10 hr.

Cancel equal factors, and proceed as in Simple Proportion.

Cancelling,

$\cancel{6}$	$\cancel{4}$
$\cancel{5}$	9
$\cancel{8}$	$\cancel{10}$ 5
$\cancel{2}$	$\cancel{30}$

$9 \times 5 = 45$ acres *Ans.*

462. Rule.—1. *Take for the third term the number that is of the same kind as the answer. For the first and second terms, form the remaining numbers, taken in pairs of the same kind, into ratios, making the larger number the consequent when the answer, if it depended solely on the couplet in question, should exceed the third term; when not, make the smaller the consequent.*

2. *Cancel as in Simple Proportion. Multiply together the second and third terms that remain, and divide their product by the product of the first terms.*

The first and second terms of each ratio must be brought to the same denomination. If the third term is a compound number, it must be reduced to the lowest denomination it contains, and this will be the denomination of the answer.

EXAMPLES FOR PRACTICE.

1. If a person travels 300 miles in 17 days, journeying 6 hours each day, how many miles will he travel in 15 days, journeying 10 hours a day? *Ans.* $441\frac{3}{17}$ mi.

2. What will be the weight of a slab of marble, 8 ft. long, 48 in. wide, and 5 in. thick, if a slab of the same density 10 ft. long, 3 ft. wide, and 3 in. thick, weighs 400 lb.? *Ans.* $711\frac{1}{9}$ lb.

3. If the expenses of a family of 10 persons amount to $500 in 23 weeks, how long will $600 support eight persons at the same rate? *Ans.* $34\frac{1}{2}$ wk.

4. 15 men, working 10 hr. a day, have taken 18 days to build

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462. Recite the rule for Compound Proportion. What reductions may have to be made?

450 yd. of stone fence. How many men, working 8 days, of 12 hours each, will it take to build 480 yd.? *Ans.* 30 men.

5. If it takes 1200 yd. of cloth, $\frac{5}{4}$ wide, to clothe 500 men, how many yards, $\frac{7}{8}$ wide, will be needed for 960 men? *Ans.* $3291\frac{3}{7}$ yd.

6. How many pounds of wool will make 150 yd. of cloth, 1 yd. wide, if 12 ounces make $2\frac{1}{4}$ yd., 6 qr. wide?

7. If the wages of 6 men, for 14 days, are $126, what will be the wages of 9 men, for 16 days? *Ans.* $216.

8. If 100 men, in 40 days of 10 hours each, build a wall 30 ft. long, 8 ft. high, and 24 in. thick, how many men will it take to build a wall 40 ft. long, 6 ft. high, and 4 ft. thick, in 20 days, working 8 hours a day? *Ans.* 500 men.

9. If $400, at 7%, in 9 mo., produce $21 interest, what will be the interest on $360, for 8 mo., at 6%? *Ans.* $14.40.

10. From the milk of 30 cows, each furnishing 16 qt. daily, 24 cheeses of 55 lb. each are made in 36 days; how many cows, giving $4\frac{1}{2}$ gal. daily, will be required, to produce, in 30 days, 33 cheeses of 1 cwt. each? *Ans.* 80 cows.

11. How many persons can be supplied with bread 8 months, for $50, when flour is $5 a barrel, if, when it is $7 a barrel, $21 worth of bread will supply 6 persons 4 months? *Ans.* 10.

CHAPTER XXXI.

ANALYSIS.

463. Analysis, in Arithmetic, is the process of arriving at a required result, not by formal rules, but by tracing out relations and reasoning from what is known to what is unknown. We generally reason from the given number to 1, and from 1 to the required number.

The rules in this book have been in many cases deduced from examples solved by Analysis. Analysis may also be applied to examples in Simple and Compound Proportion, and in Reduction of Currencies, as well as to a great variety of miscellaneous questions.

Ex. 1—If 8 yd. of cloth cost $40, what will 24 yd. cost?

This example has already been solved by Simple Proportion, p. 277. By Analysis, we should reason thus:—If 8 yd. cost $40, 1 yd. will cost $\frac{1}{8}$ of $40, or $5; and 24 yd. will cost 24 times $5, or $120. *Ans.* $120.

Ex. 2.—If 6 men can mow 30 acres of grass in 5 days, working 8 hours each day, how many acres can 4 men mow in 9 days, of 10 hours each?

This example has already been solved by Compound Proportion, p. 279. By Analysis, we should reason thus:—

If 6 men, in 5 days, working 8 hr. a day, can mow 30 acres,
1 man, in 5 days, working 8 hr. a day, " " 5 "
1 man, in 1 day, working 8 hr. a day, " " 1 acre.
1 man, in 1 day, working 1 hr. a day, " " $\frac{1}{8}$ "
4 men, in 1 day, working 1 hr. a day, " " $\frac{1}{2}$ "
4 men, in 9 days, working 1 hr. a day, " " $\frac{9}{2}$ "
4 men, in 9 days, working 10 hr. a day, " " $\frac{90}{2}$ = 45 acres. *Ans.*

EXAMPLES FOR PRACTICE.

Solve the first 8 examples by both Analysis and Simple Proportion, the next 8 by both Analysis and Compound Proportion.

1. If 12 barrels of cider cost $54, what will 15 barrels cost? 20 barrels? 100 barrels? *First ans.* $67.50.

2. How long will it take 2 men to hoe a field of corn, if 6 men can do it in 7 days?

3. How many times will a wheel revolve in going 1 mi. 2 fur., if it revolves 12 times in going 10 rd.? *Ans.* 480 times.

4. At the rate of $6 for 20 square feet, what will an acre of land cost? *Ans.* $13068.

5. If a locomotive can run 40 mi. 1 fur. 20 rd. in one hour how far can it go in 10 minutes?

In stead of reasoning from 1 hr. to 1 min., and from 1 min. to 10 min., we may say at once, 10 min. is $\frac{1}{6}$ of 1 hour; therefore in 10 min. it can go $\frac{1}{6}$ of 40 mi. 1 fur. 20 rd.

6. If $\frac{3}{8}$ of a farm is worth $1860, what is the whole worth?

7. A person bequeathed $4800, which was $\frac{3}{16}$ of his property, to charitable societies. How much was he worth?

8. If the freight on 2 cwt. 1 qr. is 22½d., at the same rate what will be the freight on 2 T. 14 cwt.? *Ans.* £2 5s.

9. A miller had to transport 21600 bushels of grain from a railroad depot to his mill. In 8 days, 10 horses had removed 7200 bushels; at this rate, how many horses would be required to remove what remained, in 10 days? *Ans.* 6 horses.

10. If 2 loads of hay will serve 3 horses 4 weeks, how many days will 5 loads serve 6 horses? *Ans.* 35 days.

11. An oblong field 8 rd. wide, 330 ft. long, contains an acre; how wide is a field that is 80 rd. long and contains 5 A.?

12. If the freight on 18 hhd. of sugar, each weighing $9\frac{1}{2}$ cwt., for a certain distance, costs $51.30, how much, at the same rate, will it cost to transport 32 hogsheads, each weighing $10\frac{1}{2}$ cwt., twice that distance? *Ans.* $196.80.

13. How much will 46 men and 24 boys earn in 60 days, if the wages of 5 men for 5 days are £7 10s., and the wages of 10 boys for 10 days are £10? *Ans.* £972.

14. A garrison of 800 men have food enough to last them 60 days, allowing each man 2 lb. a day. After 20 days, a detachment of 200 men leave; how long will the remaining provisions supply the men that remain? *Ans.* $53\frac{1}{3}$ days.

15. A garrison of 900 men have food enough to last them 40 days, allowing each man 2 lb. a day. After 10 days, they are reinforced by 300 men, and their allowance is reduced to $1\frac{1}{2}$ lb. a day; how long will their supplies then last? *Ans.* 30 days.

16. A body of 450 men have to march 430 miles. The first ten days, marching 6 hours a day, they go 150 miles; how long will it take them, marching 8 hours a day at the same rate, to complete the distance?

17. If a farmer buys 4 cows, at $45 apiece, and pays for them with hay, at $18 a ton, how many tons must he give? *Ans.* 10.

18. How many bushels of potatoes, at 80c. a bushel, will it take to pay for 12 pair of hose, at 50c.?

19. Bought some land, at $4.50 an acre; paid for it with 270 Cd. of wood, valued at $5 a cord. How many acres of land were bought? *Ans.* 300 A.

20. How much butter, at 30c. a lb., will pay for 2 boxes of tea, containing 54 lb. each, at $1.30 a lb.?

21. A can do a piece of work in 3 days, B in 5 days, C in 4 days. In how many days can they do it, working together?

In 1 day, A can do $\frac{1}{3}$, B $\frac{1}{5}$, C $\frac{1}{4}$; and all three can do $\frac{1}{3}+\frac{1}{5}+\frac{1}{4}$, or $\frac{47}{60}$. If in 1 day they can do $\frac{47}{60}$, to do $\frac{60}{60}$, or the whole, will require as many days as 47 is contained times in 60, or $1\frac{13}{47}$ days. *Ans.*

22. A can mow a field in 6 days, B in 5, C in $4\frac{1}{2}$, D in 3. How long will it take all four to do it? *Ans.* $1\frac{7}{83}$ da.

23. A, B, and C, can clear a piece of land in 10 days; A and B can do it in 16 days; how long will it take C? *Ans.* $26\frac{2}{3}$ da.

24. The head of a fish is $\frac{1}{4}$ of its whole length; its tail is $\frac{1}{6}$ of its length; its body is 7 inches. How long is the fish?

Head and tail together are $\frac{1}{4}+\frac{1}{6}$, or $\frac{5}{12}$, of the whole length. The body, therefore, is $\frac{12}{12}-\frac{5}{12}$, or $\frac{7}{12}$, of the whole length. If 7 inches are $\frac{7}{12}$, $\frac{1}{12}$ is $\frac{1}{7}$ of 7 inches, or 1 inch; and $\frac{12}{12}$, or the whole, is 12 times 1 inch, or 12 inches. *Ans.*

25. A person, being asked his age, replied that $\frac{1}{4}$ of his life had been passed in Baltimore, $\frac{1}{20}$ of it in Richmond, and the remainder, which was 28 years, in New York; how old was he?

26. At 12 the hour and minute hand of a clock are together; when are they next together?

In the course of 12 hours, the minute hand overtakes the hour hand 11 times; to overtake it once, therefore, will require $\frac{1}{11}$ of 12 hours, or $1\frac{1}{11}$ hours. $1\frac{1}{11}$ hours past 12 will be 5 min. $5\frac{5}{11}$ sec. past 1. *Ans.*

27. At what time between 5 and 6 will the hour and minute hands stand together? At what time between 8 and 9? At what time between 10 and 11?

28. A agreed to work for B 60 days, on condition that he should receive \$3.20 for every day he worked, and forfeit \$1 for every day he was idle. At the expiration of the 60 days, he received \$129. How many days did he work?

Had he worked every day, he would have received 60 times \$3.20, or \$192; therefore he lost by idleness \$192 − \$129, or \$63. Every day he was idle, he failed to make \$3.20 and forfeited \$1, thus losing \$4.20; hence, to lose \$63, he must have been idle as many days as \$4.20 is contained times in \$63, or 15 days. If he was idle 15 days, he must have worked 60 − 15, or 45 days. *Ans.*

29. D contracted to work 30 days for C; he was to have \$1.75 for every day he worked, and to forfeit 60c. for every day he was idle. If, at the end of the time, D received \$43.10, how many days was he idle? *Ans.* 4 days.

Reduction of Currencies.

464. Reduction of Currencies is the process of finding what a sum expressed in one currency is equivalent to in another.

465. COLONIAL CURRENCIES.—Sterling money was formerly the legal currency of this country. Federal money took its place in 1786; but the old denominations were long retained, and we sometimes still hear the prices of articles given in shillings and pence.

The word *shilling* does not denote the same value in all the states. This is because the colonial paper currency in some had depreciated more than in others; that is, the colonial pound, shilling, and penny, were not worth so much in dollars and cents in one state as in another.

GEORGIA CURRENCY.	In Georgia and S. Carolina,	4s. 8d. = \$1	1s. = $21\frac{3}{7}$c.	£1 = \$$4\frac{2}{7}$
NEW ENG. CURRENCY.	In New England, Virginia, Kentucky, and Tennessee,	6s. = \$1	1s. = $16\frac{2}{3}$c.	£1 = \$$3\frac{1}{3}$
PENN. CURRENCY.	In Pennsylvania, New Jersey, Delaware, Maryland,	7s. 6d. = \$1	1s. = $13\frac{1}{3}$c.	£1 = \$$2\frac{2}{3}$
NEW YORK CURRENCY.	In New York, Ohio, Michigan, and N. Carolina,	8s. = \$1	1s. = $12\frac{1}{2}$c.	£1 = \$$2\frac{1}{2}$

Ex.—What will 2 dozen tumblers cost, at 9d. apiece, New England currency?

By Analysis:—In N. E. currency, 6s. or 72c. = \$1; hence 9d. is $\frac{1}{8}$ of \$1. 24 tumblers, at \$$\frac{1}{8}$ apiece, will cost 24 times \$$\frac{1}{8}$, or \$3. ***Ans.*** **\$3.**

EXAMPLES FOR PRACTICE.

1. At the rate of 9s. a day, New England currency, what will be the wages of 4 men, for 10 days? *Ans.* \$60.

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464. What is Reduction of Currencies?—465. Why do we sometimes still hear the prices of articles named in shillings and pence? How did the word *shilling* come to denote different values in different states? Name the different colonial currencies. What was the value of the shilling and pound in each?

2. At 6d. apiece, N. Y. currency, what cost 3 dozen pencils?

3. What cost 36¼ yd. linen, at 7s. 6d., Penn. currency?

4. Reduce £42 10s., Georgia currency, to Federal money. Reduce £14 2s. 4d. *Sum of ans.* $242.64.

5. At 9d. a yard, New England currency, what will 4 pieces of calico, averaging 48 yd. each, cost? *Ans.* $24.

466. Foreign Currencies.—The value of certain foreign currencies in Federal Money is fixed by Act of Congress or by commercial usage, as follows:—

Value of Foreign Currencies in U. S. Money.

Banco Rix Dollar of Denmark,	$0.53	Millrea of Azores,	$0.83⅓
Banco Rix Dollar of Sweden and Norway,	0.39¾	Millrea of Madeira,	1.00
Dollar Thaler of Bremen,	0.71	Millrea of Portugal,	1.12
Dollar of Rome,	1.05	Ounce of Sicily,	2.40
Ducat of Naples,	0.80	Pagoda of India,	1.94
Florin of Austria, Bohemia, Augsburg,	0.48½	Piaster of Turkey,	0.05
Florin of Basle,	0.41	Pound Sterling, Gr't Britain,	4.84
Florin (Guilder) of Netherlands and S. Germany,	0.40	Pound Sterling, British Provinces, Canada, Nova Scotia, &c.,	4.00
Florin of Prussia,	0.22¾	Real Plate of Spain,	0.10
Franc of France and Belgium,	$0.18\frac{6}{10}$	Real Vellon of Spain,	0.05
Guilder of Brabant,	0.33¾	Rix Dollar of Bremen,	0.78¾
Lira of Sardinia,	$0.18\frac{6}{10}$	Rix Dollar of Prussia and Northern Germany,	0.69
Lira of Tuscany,	0.16	Ruble of Russia, silver,	0.75
Livre of Genoa,	$0.18\frac{6}{10}$	Rupee of British India,	0.44½
Livre of Leghorn,	0.16	Scudo of Malta,	0.40
Livre of Neufchatel,	0.26½	Scudo of Rome,	0.99 / 0.99½
Livre of Switzerland,	0.27	Specie Dollar, Denmark,	1.05
Livre Tournois, France,	0.18½	Specie Dollar of Sweden and Norway,	1.06
Marc Banco, Hamburg,	0.35	Tael of China,	1.48
		Tical of Siam,	0.61

Ex. 1.—Reduce 75 rix dollars of Bremen to U. S. currency.

By the Table, 1 rix dollar of Bremen = $0.78¾.
75 rix dollars = 75 times $0.78¾, or $59.0625. *Ans.*

Ex. 2.—Reduce $560 to millreas of Portugal.

By the Table, $1.12 = 1 millrea of Portugal.
$560 will equal as many millreas as $1.12 is contained times in $560, or 500. *Ans.* 500 millreas.

EXAMPLES FOR PRACTICE.

1. How many dollars equal 1000 francs? *Ans.* $186.
2. Reduce $725 to Austrian florins. *Ans.* $1494\frac{82}{97}$ fl.
3. What is the value of 6000 Swiss livres? *Ans.* $1620.
4. How many Canada pounds are 20 eagles worth? *Ans.* £50.
5. 5s. Halifax money equals how much in U. S. gold?
6. What is the value of 16 half-eagles in ducats? In piasters? In silver rubles? In marcs banco?
7. Bought some East Indian goods for 200 rupees; what did they cost in Federal money? *Ans.* $89.
8. How many sovereigns (the coin that represents the pound sterling of Great Britain) will pay the duty on a lot of worsted hose, costing $1452, the rate being 35% ad valorem? *Ans.* 105 sov.
9. Reduce 600 specie dollars of Denmark to U. S. money.

CHAPTER XXXII.

EXCHANGE.

467. Exchange is a method by which a person in one place makes payments in another by means of written orders, without the transmission of money.

468. A Bill of Exchange, or **Draft,** is a written order on one party to pay another a certain sum, at sight or some specified time.

469. The parties to the transaction are, the **Drawer,** or **Maker,** who signs the bill; the **Drawee,** to whom it is ad-

467. What is Exchange?—468. What is a Bill of Exchange?—469. Name the parties to the transaction.

dressed; the **Payee**, to whom it is ordered to be paid; and the **Buyer** or **Remitter**, who buys or remits it, and who may be the payee or not.

470. When a draft is presented to the drawee, if he acknowledges the obligation, he writes the word *Accepted*, with the date and his name, across the face of the bill, and thus makes himself responsible for the payment. This is called *accepting* the draft.

471. As in the case of notes, three days of grace are allowed for the payment of drafts. But in New York, Pennsylvania, Maryland, and some other states, it is customary to pay sight drafts on presentation, and of course no acceptance is then necessary.—As regards protesting and the responsibility of endorsers, the same rules apply to drafts as to notes, § 361.

472. Suppose Aaron Brooks, of St. Louis, owes Cobb & Deming, of N. Y., $1000. He buys of Eugene Ford & Co., bankers in St. Louis, a draft for $1000 on their correspondents, Gregory & Co., of N. Y., as follows:—

$1000. *St. Louis, July* 20, 1866.

Ten days after sight pay to the order of Aaron Brooks one thousand dollars, value received, and charge the same to account of

EUGENE FORD & Co.

To Messrs. Gregory & Co., N. Y.

Brooks endorses this draft, "Pay to the order of Cobb & Deming," affixes his signature, and remits it to the latter. They, on its receipt, present it to Gregory & Co., who accept it July 27th, and pay it thirteen days afterwards.—Here, Ford & Co. are the drawers; Gregory & Co. are drawees and also acceptors; Brooks is payee, endorser, and remitter; Cobb & Deming are holders, as long as they retain the draft in legal possession. If they desire to pass it before maturity, they endorse it, and thus render it negotiable.

473. When a draft costs its exact face, exchange is said to be *at par*. When it costs more than its face, exchange is said to be *above par*, *at a premium*, or *against* the place where the draft is drawn; when less, exchange is *below par*, *at a discount*, or *in favor of* the place where the draft is drawn.

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470. What is meant by *accepting* a draft?—471. What is the custom as regards allowing days of grace for the payment of drafts?—472. Give the form of a draft, illustrate its use in making a remittance, and name the parties concerned.—473. When is exchange said to be *at par*? When, *above par*? When, *below par*? When is it *against* a place, and when *in its favor*?

Domestic Bills of Exchange.

474. Domestic, or **Inland, Bills of Exchange** (commonly called Drafts) are those that are payable in the country in which they are drawn.

475. Operations in Domestic Exchange are similar to those in Stocks.

Ex. 1.—Bought in Louisville a thirty-day draft on New York for \$300, at $\frac{1}{4}$% premium. What did it cost?

> \$1, at $\frac{1}{4}$% premium, cost \$1 + \$.0025 = \$1.0025.
> \$300 cost 300 times \$1.0025, or \$300.75. *Ans.*

Ex. 2.—How large a draft on Milwaukee can a person in N. Y. buy for \$1000, when exchange is at a discount of $\frac{1}{2}$ per cent?

> \$1, at $\frac{1}{2}$% discount, will cost \$1 − \$.005 = \$.995.
> For \$1000 can be bought a draft for as many dollars as \$.995 is contained times in \$1000, or \$1005.03. *Ans.*

476. Rules.—I. *To find the cost of a domestic bill, multiply the cost of \$1 at the given rate of premium or discount, by the face of the bill.*

II. *To find the face of a bill that a given sum will buy, divide the given sum by the cost of \$1.*

EXAMPLES FOR PRACTICE.

1. What is the cost of a sight draft on Mobile for \$1800, at $1\frac{3}{4}$ per cent. premium? *Ans.* \$1831.50.

2. How large a draft on Cincinnati can a person in St. Paul buy for \$2500, when exchange is 2% against St. Paul?

3. The course of exchange on Baltimore being $\frac{1}{4}$% premium for sight, and $\frac{3}{4}$% discount for sixty days, what must I pay for a sight draft on Baltimore for \$1000 and a sixty-day draft for \$750? *Ans.* \$1749.375.

4. A person living in Portland sold some property in Galveston for \$10500. Would it be better for him to draw on Galveston for

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474. What are Domestic, or Inland, Bills of Exchange?—475. To what are operations in Domestic Exchange similar? Explain Exs. 1 and 2.—476. Recite the rules.

this amount and pay 2 % for collection, or to have a draft on Portland bought with said amount and remitted, exchange on Portland being at a premium of 3 per cent.?

Ans. Gain by drawing on Galveston, $95.38.

5. B, living in Detroit, holds 100 shares of the Phenix Bank, of New York. The bank declares a dividend of 4 %. B draws for his dividend, and sells the draft at 1 % premium. What does he realize? *Ans.* $404.

Foreign Bills of Exchange.

477. Foreign Bills of Exchange are those that are drawn in one country and payable in another.

478. By a **Set of Exchange** are meant two or more bills of the same date and tenor, only one of which is to be paid. They are sent by different mails; and the object of drawing more than one is to save time in case one is lost.

479. EXCHANGE ON ENGLAND.—Exchange on England is always at a premium in the United States, and thus the balance of trade always *appears* to be against this country. This is because the base of computation is made the old value of the pound sterling, $$4\frac{4}{9}$, or $$4.44\frac{4}{9}$; whereas the intrinsic value of the new Victoria sovereign is about $$4.86\frac{2}{3}$, which is $109\frac{1}{2}$ % of $$4.44\frac{4}{9}$. *When, therefore, sight exchange on England is quoted at* $109\frac{1}{2}$, *or* $9\frac{1}{2}$ % *premium, it is really at par.*

Ex. 1.—What is the cost (in gold) of the following foreign bill, at $9\frac{1}{4}$ % premium?

Exchange for £250. *Boston, July* 24, 1866.

Sixty days after sight of this First of Exchange (*Second and Third of the same date and tenor unpaid*) *,

* The Second Bill of the Set would read, "of this Second of Exchange (First and Third of the same date and tenor unpaid)". The Third would run, "of this Third of Exchange (First and Second, &c.)".

477. What are Foreign Bills of Exchange?—478. What is a Set of Exchange? What is the object of drawing more than one bill?—479. How does exchange on England always stand in the U. S.? Why is this? When is exchange on England really at par? Give the form of a foreign bill of exchange. Explain Ex. 1.

pay to the order of J. M. Mosely two hundred and fifty pounds sterling, value received, with or without further advice.

WARD & SUNDERLAND.

To HAMILTON BROTHERS, *London.*

£1 = $\$\frac{40}{9}$, nominal par. At $9\frac{1}{4}\%$ premium, £1 costs $\$\frac{40}{9} \times 1.0925$; and £250 will cost 250 times as much, or $\$\frac{40}{9} \times 1.0925 \times 250 =$ $1213.89. *Ans.*

Ex. 2.—For what amount will $1213.89 purchase a bill on London, when exchange is $109\frac{1}{4}$?

In Ex. 1 we found that, at $109\frac{1}{4}$, £1 = $\$\frac{40}{9} \times 1.0925$, or $\$4.85\frac{5}{9}$. Hence $1213.89 will buy a bill for as many pounds as $\$4.85\frac{5}{9}$ is contained times in $1213.89, or 250. *Ans.* £250.

480. RULES.—I. *To find the cost of a bill on England (in gold), multiply together* $\$\frac{40}{9}$, 1 *increased by the premium, and the face of the bill in pounds.*

II. *To find the face of a bill that a given sum (in gold) will buy, divide the given sum by the product of* $\$\frac{40}{9}$ *and* 1 *increased by the premium.*

In examples under Rule I., shillings and pence must be reduced to the decimal of a pound; and the decimal of a pound, in answers of examples under Rule II., must be reduced to shillings and pence.

481. EXCHANGE ON OTHER COUNTRIES.—Exchange on France is quoted at so many francs and centimes to the dollar. A franc, at par, = $18\frac{6}{10}$ cents; a centime is $\frac{1}{100}$ of a franc.

Exchange on other countries is quoted at so many cents to some coin taken as a standard: thus, on Hamburg, $35\frac{1}{2}$ cents to the marc banco; on Amsterdam, 39 cents to the florin, &c.

In these cases, the cost of a bill, and the face of a bill that a given sum will buy, are readily found by Analysis, as in Reduction of Currencies.

Explain Ex. 2.—480. Recite the rule for finding the cost of a bill on England. Recite the rule for finding the face of a bill that a given sum will buy. What reductions must be made?—481. How is exchange on France quoted? How is exchange on other countries quoted? In these cases, how are the cost of a bill, and the face of a bill that a given sum will buy, found? Explain Ex. 8.

Ex. 3.—What is the value of a bill on Havre for 1200 francs, exchange being 5 francs 18 centimes to the dollar?

If 5 francs 18 centimes = \$1, a bill for 1200 francs will cost as many dollars as 5.18 is contained times in 1200, or 231.66. *Ans.* \$231.66.

EXAMPLES FOR PRACTICE.

1. What is the cost, in gold, in N. Y., of a set of exchange on Dublin for £450 10s., at $9\frac{3}{4}$% premium? *Ans.* \$2197.44.

2. What is the cost, in gold, of a bill on Paris for 7500 francs, when exchange is 5 fr. 10 cen. to the dollar? *Ans.* \$1470.59.

3. When the course of exchange is $75\frac{1}{4}$c. to the ruble, what will a bill on St. Petersburg for 2400 rubles cost?

4. How large a bill on Bremen can be bought for \$2000, when exchange is 79c. to the rix dollar?

5. Exchange on Liverpool standing at 109, what will a bill on that city for £1500 2s. 6d. cost? *Ans.* \$7267.27.

6. A New York merchant, owing a debt in London, can purchase gold at 145, and with it buy exchange at $9\frac{1}{2}$% premium; or can remit U. S. 10–40's, and sell the same in London at $60\frac{1}{2}$. How low must he buy the bonds (for currency), to make a saving by remitting them in stead of a bill of exchange?

Each \$1 of bonds transmitted would be worth \$.605 × 1.095, in gold. Reducing this value to a currency basis, we have \$.605 × 1.095 × 1.45 = \$.96 +. If, therefore, the bonds can be bought for less than 96, there will be a saving in remitting them.

Arbitration of Exchange.

482. Arbitration of Exchange is the process of finding the rate of exchange between two countries, when there have been intermediate exchanges through other countries. In Arbitration, we use what is called Conjoined Proportion or the Chain Rule.

A merchant, for example, may remit from New York to Hamburg, by remitting from New York to London, from London to Paris, from Paris to Amsterdam, and from Amsterdam to Hamburg. The rate of this Circuitous Exchange, as it is called, will probably differ somewhat from that of a direct remittance from New York to Hamburg; to find whether it will cost more or less, is the object of Arbitration.

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482. What is Arbitration of Exchange? Give an illustration of Circuitous Exchange.—483. Recite the Chain Rule.

483. Chain Rule.—1. *Write the equivalents by pairs, each with its denomination, on opposite sides of a vertical line, commencing on the left with the denomination of the required sum, and on the right with the given sum to be remitted; and arranging the terms so that each denomination on the right may correspond with the one next below it on the left.*

2. *Cancel common factors on the left and right, and divide the product of the remaining terms on the right by that of the remaining terms on the left.*

If the terms are properly arranged, the last denomination on the right will correspond with the first on the left.

Ex.—When exchange at New York on London is at 10% premium, at London on Paris 27 francs 20 centimes to £1, at Paris on Amsterdam 9 stivers to 1 franc, and at Amsterdam on Hamburg 18 stivers to 1 marc banco, what will it cost to remit 5000 marcs banco from N. Y. to Hamburg, through London, Paris, and Amsterdam? Would it be better to remit in this way, or direct from N. Y. to Hamburg, the rate being 36 cents to the marc banco—and how much?

$?	5000 marcs b.
1 marc b.	18 stivers
9 stivers	1 franc
27.2 fr.	£1
£9	$40 × 1.10

Cancelling,
$$\begin{array}{rr|ll} & \cancel{9} & \cancel{5000} & 625 \\ \cancel{3.4} & \cancel{27.2} & \cancel{18} & \cancel{2} \\ 1.7 & 9 & 44 & \end{array}$$

$$625 \times 44 = 27500$$
$$1.7 \times 9 = 15.3$$
$$27500 \div 15.3 = \$1797.39$$

Direct Exchange, $0.36 × 5000 =	$1800.00
Circuitous Exchange,	1797.39
Gain by Circuitous Exchange,	$2.61 *Ans.*

In this example, £1 = $\$\frac{40}{9} \times 1.10$; hence £9 = $40 × 1.10, as given above.—The relative value of different measures, weights, and goods, may be found, on the same principle, by the Chain Rule.

EXAMPLES FOR PRACTICE.

1. A person in Philadelphia desires to pay £1800 in Liverpool. Exchange on Liverpool is at 9¾% premium, on Paris 5 francs 15

centimes to a dollar. Exchange on Liverpool in Paris is 25 francs 15 centimes to the pound sterling. Is it better for him to remit direct to Liverpool, or through Paris, and how much?

Ans. Gain by direct remittance, $10.29.

2. A New York merchant orders £1000 due him in London to be remitted by the following route: to Hamburg, the course of exchange being 14 marcs banco to the pound; thence to Copenhagen, at $1\frac{1}{2}$ marcs banco to the rix dollar; thence to Bordeaux, at 2 francs 80 centimes to the rix dollar; thence to N. Y., at 5 francs 30 centimes to the dollar. How many dollars did he receive? *Ans.* $4930.82.

Would he have gained or lost by drawing directly for the amount on London, and selling his draft at $109\frac{1}{2}$, leaving interest out of account?

3. If 16 barrels of cider are worth 64 bushels of corn, and 15 bu. of corn are worth 2 barrels of flour, and 3 tons of coal are worth 4 barrels of flour, and 16 lb. of tea are worth 2 tons of coal, how many pounds of tea are equal in value to 7 barrels of cider?

Ans. $22\frac{2}{5}$ lb.

CHAPTER XXXIII.

PARTNERSHIP.

484. A **Partnership** is a business association between two or more persons, who agree to share the profits or losses. Persons so associated are called **Partners**.

Capital is money invested in business.

Different agreements are made between partners as to the division of profits. One may contribute the capital, and another his services, and they may divide equally. Or all may contribute capital and labor equally, and make an equal division. When different amounts of capital are furnished, and little or no labor is required, or all contribute equally of

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484. What is a Partnership? What is Capital? What is said about the division of profits among partners?

their labor, the profit or loss is usually divided according to the amounts of capital furnished.

485. CASE I.—*To find each partner's share, when they furnish capital for the same length of time.*

Ex. 1.—A, B, and C, engaged in a speculation. A put in \$180, B \$240, C \$480. They gained \$300; what was each partner's share?

The whole capital employed was \$180 + \$240 + \$480, or \$900. Since \$900 capital gained \$300, \$1 of capital gained $\frac{1}{900}$ of \$300; and A's capital of \$180 was entitled to $\frac{180}{900}$, B's \$240 to $\frac{240}{900}$, and C's \$480 to $\frac{480}{900}$, of \$300. The operation is proved by adding the shares found, and seeing whether their sum equals the whole gain.

A's capital, \$180	A's share, $\frac{180}{900}$ of \$300 = \$60	*Ans.*
B's " 240	B's " $\frac{240}{900}$ " " = 80	
C's " 480	C's " $\frac{480}{900}$ " " = 160	
Total capital, \$900	PROOF: Gain, \$300	

RULE.—*Make each partner's capital the numerator of a fraction, and the total capital the denominator; for each partner's share, take his fraction, thus formed, of the whole gain or loss.*

Ex. 2.—Two brothers, the one 18 years old and the other 21, contribute \$468 for the support of a parent, in the ratio of their ages. What does each give?

This example is analogous to a question in Partnership. There are in all 18 + 21, or 39, parts; of which one furnishes 18, the other 21.

1st	18	$\frac{18}{39}$ of \$468 = \$216	*Ans.*
2d	21	$\frac{21}{39}$ of \$468 = \$252	
	39	PROOF: \$468	

EXAMPLES FOR PRACTICE.

1. The profits of Mason, Dean, & Co., for one year, are \$9275. Mason contributes \$20000 capital; Dean, \$12500; and Graham (who is the Co.), \$4600. What is each partner's share of the profits? *Ans.* Mason's, \$5000; Dean's, \$3125; Graham's, \$1150.

2. A and B buy a house for \$2500, A furnishing \$1200, B \$1300. They receive \$210 rent; how should it be divided?

3. Ames, Boorman, & Crane, buy a hotel for \$18500, of which

485. What is Case I.? Explain Ex. 1. Recite the rule. Explain Ex. 2.

Ames contributes $8000, Boorman $6200, and Crane the rest. They sell it for $16975, and their expenses are $325. How much of the loss must each bear? *Ans.* A., $800; B., $620; C., $430.

4. Two persons hire a pasture for $30. The first turns in 8 cows; the second, 5. How much ought each to pay?

5. A, B, C, D, and E, are to divide $2400 among themselves. A is to have $\frac{1}{6}$, B $\frac{1}{4}$, C $\frac{3}{8}$; D and E are to divide the remainder in the ratio of 5 to 7. How much should each receive?

Last answers: D, $208.33⅓; E, $291.66⅔.

6. A person wills to his elder son $1200, to his younger $1000, to his daughter $600. But it is found that his whole property is worth only $800. How much should each receive?

7. X, Y, and Z, embark in a speculation, X furnishing $\frac{1}{2}$ the capital, Y $\frac{2}{3}$ of the remainder, and Z the rest. Their profit is $1900, and X is allowed $100 for attending to the business. How much does each receive? *Ans.* X, $1000; Y, $600; Z, $300.

486. CASE II.—*To find each partner's share, when they furnish capital for different lengths of time.*

Ex. 1.—Three partners, O, P, and Q, furnished capital as follows: O put in $400 for 2 mo.; P, $300 for 4 mo.; Q, $500 for 3 mo. They gained $350; what was the share of each?

O's $400 for 2 mo. = $800 for 1 mo.
P's $300 " 4 mo. = $1200 " 1 mo.
Q's $500 " 3 mo. = $1500 " 1 mo.

The whole capital is therefore equivalent to $3500 for 1 month; and, as O put in what is equivalent to $800 for 1 mo., he is entitled to $\frac{800}{3500}$ of $350, or $80. In like manner, P is entitled to $\frac{1200}{3500}$ of $350, or $120; and Q, to $\frac{1500}{3500}$ of $350, or $150.

RULE.—*Multiply each partner's capital by its time. Treat this product as his capital, and proceed as in Case* I.

Ex. 2.—Three partners were in business for 12 months, and cleared $2919. The first had $4000 in the whole time. The second put in $5000 three months after the partnership commenced, and three months afterwards $3000 more. The third put in $3000 on starting, but with-

486. What is Case II.? Explain Ex. 1. Recite the rule. Explain Ex. 2.

drew $2000 four months before the partnership expired. Divide the profit.

1st $4000 × 12 =		48000	Share, $\frac{48}{189}$ of $2919 = $1008.
2d $5000 × 9 =	45000		
$3000 × 6 =	18000		
		63000	Share, $\frac{63}{189}$ " " = $1323.
3d $3000 × 8 =	24000		
$1000 × 4 =	4000		
		28000	Share, $\frac{28}{189}$ " " = $588.
		189000	PROOF: $2919.

EXAMPLES FOR PRACTICE.

1. A and B enter into partnership, A furnishing $325 for 6 months, and B $200 for 8 months. There is a loss of $100; what is the share of each? *Ans.* A, $54.93; B, 45.07.

2. Two partners received $300 for constructing a piece of road. The first furnished 5 laborers for 9 days; the second, 7 laborers for 11 days. What was the share of each?

3. Three farmers hired a pasture for $55.50. The first put in 6 cows for 8 mo.; the second, 8 cows for 2 mo.; the third, 10 cows for 4 mo. What must each pay?

4. For the transportation of some flour 93 miles, I have to pay $116.25. A carried 50 bar. 70 miles; B, 10 bar. 93 miles; C, 40 bar. 53 miles; D, 50 bar. 23 miles; E, 40 bar. 40 miles. How much must I pay each? *Ans.* A, $43.75, &c.

5. A, B, and C, began business Jan. 1 with $650, furnished by A; April 1, B put in $500; July 1, C put in $450. The profit for the year was $875; divide it. *Ans.* A, $195, &c.

6. D, E, and F, were interested in a coal mine, and cleared the first year $3285. D had $10000 invested for 9 mo., when he withdrew half of that sum; E put in $20000, 2 mo. after the partnership was formed; and F put in $12000, 5 mo. before it expired. Divide the profit. *Ans.* D, $945; E, $1800; F, $540.

7. Two partners, G and H, cleared in 6 mo. $2150. G's capital at first was to H's as 2 to 1. After 2 months, G withdrew $\frac{1}{3}$ of his capital, and H $\frac{1}{4}$ of his. Divide the profit.

Ans. G, $1400; H, $750.

CHAPTER XXXIV.

ALLIGATION.

487. Alligation is the process of solving questions as to the mixing of ingredients of different values. There are two kinds of Alligation, Medial and Alternate.

Alligation means *connecting*, and the process is so called from *connecting* or *linking* the prices of the ingredients together, as shown in § 490.

Alligation Medial.

488. Alligation Medial is the process of finding the average value of a mixture, when the value and quantity of each ingredient are known.

Ex. 1.—A grocer mixes 70 lb. of tea worth $1 a lb., 100 lb. worth $1.25, and 30 lb. worth $1.50. What is a pound of the mixture worth?

70 lb., at $1, are worth $70; 100 lb., at $1.25, are worth $125; 30 lb., at $1.50, are worth $45. The whole mixture, therefore, is worth $70 + $125 + $45, or $240; and it contains 70 + 100 + 30 lb., or 200 lb. If 200 lb. are worth $240, 1 lb. is worth $\frac{1}{200}$ of $240, or $1.20. *Ans.*

$$\begin{array}{rcl} 70 \times 1 & = & 70 \\ 100 \times 1.25 & = & 125 \\ \underline{30} \times 1.50 & = & \underline{45} \\ 200) & & 240 \\ & & \overline{\textit{Ans. } \$1.20} \end{array}$$

489. Rule.—*Divide the total value of the ingredients by the sum of the quantities.*

If an ingredient is put in that costs nothing (as water, chaff), its quantity must be added in with the rest, though its value is 0.

The principle of this rule applies to many questions that involve the finding of an average, besides those relating to values or prices.

EXAMPLES FOR PRACTICE.

1. A liquor-merchant mixes 32 gal. of wine at $1.60 a gallon, 15 gal. at $2.40, 45 gal. at $1.92, and 8 gal. at $6.80. What is the value of a gallon of the mixture? *Ans.* $2.008.

2. If a ship sails 5 knots an hour for 3 hours, 7 knots for 5 hours, and 8 knots for 4 hours, what is her average rate per hour?

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487. What is Alligation? Name the two kinds of Alligation. Why is the process so called?—488. What is Alligation Medial? Explain Ex. 1. Recite the rule.

3. A dishonest grocer mixed 3 lb. of sand with 10 lb. of sugar worth 12c., 20 lb. worth 14c., and 30 lb. worth 16c. What did the mixture cost him per pound? *Ans.* $13\frac{61}{63}$c.

4. A goldsmith melts together 11 oz. of gold 23 carats fine, 8 oz. 21 carats fine, 10 oz. of pure gold, and 2 lb. of alloy. How many carats fine is the mixture? *Ans.* $12\frac{25}{53}$ carats.

A carat is $\frac{1}{24}$; that is, gold 21 carats fine is $\frac{21}{24}$ pure metal.

5. If 4 dozen eggs are bought at $18\frac{3}{4}$ cents a dozen, 6 dozen at 21 cents, $3\frac{1}{2}$ dozen at 24c., and $5\frac{1}{2}$ dozen at 25c., what is the average cost per dozen? *Ans.* $22\frac{9}{38}$c.

6. A dairyman owning 30 cows finds, at a certain milking, that 6 give 12 qt. each, 8 give $10\frac{1}{2}$ qt., 10 give $9\frac{1}{2}$ qt., and the rest 8 qt. apiece. What is the average?

7. If a farmer mixes 10 bu. of corn, worth 80 cents a bushel, 20 bu. worth 85c., 25 bu. worth 90c., and 20 bu. worth 95c., what is the mixture worth per bushel? *Ans.* $88\frac{2}{3}$c.

Alligation Alternate.

490. Alligation Alternate is the process of finding the quantities to be taken of two or more ingredients, of given values, to make a mixture of given value.

Ex. 1.—In what relative quantities must coffees worth 15, 16, 20, and 21 cents a pound, be taken, to make a mixture worth 19 cents a pound?

It is clear that the gains and losses on the several ingredients, as compared with the mean value, must balance. Hence we consider a price less than the mean with one greater,—15c. with 21c. On every pound put in at 15c. and sold in the mixture for 19c., there is a gain of 4c.; and on every pound put in at 21c. and sold for 19c., there is a loss of 2c. Therefore, as the gain and loss on equal quantities of these two kinds are as 4 to 2, we must take quantities that are to each other as 2 to 4. In like manner, comparing 1 lb. at 16c., and 1 lb. at 20c., we find that there is a gain of 3c. against a loss of 1c.; hence the quantities taken must be as 1 to 3. The relative quantities, therefore, are 2 lb. at 15c., 1 lb. at 16c., 3 lb. at 20c., and 4 lb. at 21c. *Ans.*

The brief mode of performing this operation is to link the values in pairs, one less than the mean with one greater, to take the difference between the mean and each value, and write it opposite the value with which it is linked.

$$19\left\{\begin{array}{ll} 15 & 2 \\ 16 & 1 \\ 20 & 3 \\ 21 & 4 \end{array}\right.$$

The terms may be linked differently, provided one less than the mean is connected with one greater; the answers, of course, differ, according to the linking. As these answers show merely the *relative* quantities, we may multiply or divide the numbers by any common multiplier or divisor, and thus produce an infinite variety of answers.

19	15	1
	16	2
	20	4
	21	3

Alligation Alternate is proved by Alligation Medial. Thus:—

Proof of 1st answer.

2 lb., at 15c. = 30c.
1 " " 16c. = 16c.
3 " " 20c. = 60c.
4 " " 21c. = 84c.
10 lb. cost $1.90, or 1 lb. 19c.

Proof of 2d answer.

1 lb., at 15c. = 15c.
2 " " 16c. = 32c.
4 " " 20c. = 80c.
3 " " 21c. = 63c.
10 lb. cost $1.90, or 1 lb. 19c.

Ex. 2.—A grocer, having 10 lb. of coffee worth 15c. a pound, wishes to mix it with other kinds worth 16, 20, and 21c., to make a mixture worth 19c. a pound. How many pounds of each must he take?

In Ex. 1, we found the relative quantities of these coffees for a mixture worth 19 cents to be 2, 1, 3, 4, or 1, 2, 4, 3.

Looking at the first answer, we find that the ratio of 10, the given quantity of 15-cent coffee to 2, its difference, is 5; therefore we multiply the numbers throughout by 5.

$$\left.\begin{array}{l} 2 \times 5 = 10 \text{ lb.} \\ 1 \times 5 = 5 \text{ lb.} \\ 3 \times 5 = 15 \text{ lb.} \\ 4 \times 5 = 20 \text{ lb.} \end{array}\right\} \textit{Ans.} \qquad \left.\begin{array}{l} 1 \times 10 = 10 \text{ lb.} \\ 2 \times 10 = 20 \text{ lb.} \\ 4 \times 10 = 40 \text{ lb.} \\ 3 \times 10 = 30 \text{ lb.} \end{array}\right\} \textit{Ans.}$$

In the 2d answer, the ratio is 10 to 1; therefore we multiply by 10.

Ex. 3.—A grocer, having coffees worth respectively 15, 16, 20, and 21 cents, wishes to make with them a mixture of 80 lb., worth 19c. a pound. How many pounds of each kind must he use?

In Ex. 1, we found the relative quantities to be 2, 1, 3, 4 lb., or 1, 2, 4, 3 lb.,—in either case making a total of 10 lb. For a mixture of 80 lb., therefore, he must take 8 times as much of each.

$$\left.\begin{array}{l} 2 \times 8 = 16 \text{ lb.} \\ 1 \times 8 = 8 \text{ lb.} \\ 3 \times 8 = 24 \text{ lb.} \\ 4 \times 8 = 32 \text{ lb.} \end{array}\right\} \textit{Ans.} \qquad \left.\begin{array}{l} 1 \times 8 = 8 \text{ lb.} \\ 2 \times 8 = 16 \text{ lb.} \\ 4 \times 8 = 32 \text{ lb.} \\ 3 \times 8 = 24 \text{ lb.} \end{array}\right\} \textit{Ans.}$$

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490. What is Alligation Alternate? Explain Ex. 1. What is the brief mode of performing the operation? How may different answers be obtained? How is Alligation Alternate proved? Explain Ex. 2. Explain Ex 3. Recite the rule.

491. Rule.—1. *Write the values in a column, and the mean value on the left. Link each value less than the mean with one greater, and each greater with one less. Write the difference between the mean and each value, opposite the value it is linked with. These differences are the relative quantities of the ingredients taken in the order in which their values stand.*

2. *If the quantity of one ingredient is given, to find the corresponding quantities of the others, multiply their differences by the ratio of the given quantity to the difference of the ingredient it represents.*

3. *If the quantity of the mixture is given, to find the quantity of the ingredients, multiply their differences by the ratio of the given quantity to the sum of the differences.*

Ex. 4. A liquor-dealer wishes to mix three kinds of whiskey worth respectively \$3.25, \$3.50, and \$3.75 a gallon, with water, so as to make a mixture worth \$3. What parts of each must he take?

We represent the water by 0. As there are three values greater than the mean and but one less, we have to link the three with the one. There will, therefore, be three differences opposite the 0, and their sum will represent the relative quantity of water. *Ans.* 3 gal. of each kind of whiskey, and $1\frac{1}{2}$ gal. of water.

$$3.\left\{\begin{array}{ll} 3.75 & 3. \\ 3.50 & 3. \\ 3.25 & 3. \\ 0 & .75 + .5 + .25 = 1.5 \end{array}\right.$$

EXAMPLES FOR PRACTICE.

1. In what proportions must gold, 12, 16, 17, and 22 carats fine, be taken, to make a compound 18 carats fine? 20 carats fine? $16\frac{1}{2}$ carats fine? *First ans.* 4, 4, 4, 9.

2. A merchant wishes to mix 90 lb. of sugar, worth $10\frac{1}{2}$c., with three other kinds, worth 10, 12, and 14 cents, respectively. How many pounds of each must he use, that the compound may be worth 11c.? $12\frac{1}{2}$c.? 13c.?

First ans. { 270 lb. at 10c.; 45 lb. at 12c.; 90 lb. at 14c.
Or, 30 lb. at 10c.; 30 lb. at 12c.; 15 lb. at 14c.

3. How many pounds of spices, worth respectively 30, 40, and 50c. a pound, must be mixed with 20 lb. worth 80c. a pound, to form a mixture worth 60c. a pound? Worth 75 cents? Worth 45 cents? *First ans.* 6⅔ lb. of each.

4. A man having 40 bu. of oats that cost him $22, wishes to mix them with two other kinds worth respectively 50 and 65c. How much of each kind must he take, to form a mixture worth 60c. a bushel? *Ans.* 40 bu. at 50c.; 120 bu. at 65c.

5. B, having a contract to furnish 442 lb. of tea worth $1.40 a lb., wishes to make a mixture, of the required value, out of four kinds, worth respectively $1, $1.10, $1.45, and $1.50. How many pounds of each must he take?

Ans. 52 lb. at $1, 26 lb. at $1.10, 156 lb. at $1.45, 208 lb. at $1.50.

6. In what proportions must water, and two kinds of rum worth $2½ and $3 a gallon, be mixed, to form a compound of 40 gallons, worth $2 a gallon?

7. A news-agent sold 198 newspapers, at an average price of 7 cents apiece. How many must he have sold at 3c., 4c., 5c., 6c., and 10c.? *Ans.* 27 at 3c., &c.

CHAPTER XXXV.

INVOLUTION.

492. Involution is the process of multiplying a number by itself. The product is called a **Power** of the number multiplied.

$2 \times 2 = 4$. This process is Involution; 4 is a power of 2.

493. Powers are distinguished as First, Second, Third, Fourth, &c., according to the times that the given number is taken as a factor.

They are *indicated* by a figure, called an **Index** (plural, *indices*) or **Exponent**, placed above the number at the right; as, 2^3, 2^4, 2^5.

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492. What is Involution? What is the product obtained by Involution called? Give an example.—493. How are powers distinguished? How are they indicated?

494. The First Power is the number itself; its index is never written. The Second Power is also called the **Square**, and the Third Power the **Cube**.

First power of 2,	2
Second power, or Square,	$2^2 = 2 \times 2 = 4$
Third power, or Cube,	$2^3 = 2 \times 2 \times 2 = 8$
Fourth power,	$2^4 = 2 \times 2 \times 2 \times 2 = 16$, &c.

495. RULE.—*To involve a number, multiply it by itself as many times, less* 1, *as there are units in the index of the power required.*

$3 \times 3 = 9$. There is *one* multiplication, though 3 is used as a factor *twice*, and 9 is the *second* power.

496. In stead of multiplying by the original number each time, powers already found may be used as multipliers. Thus, for the 7th power, the 4th may be multiplied by the 3d. But observe that the resulting power will be that denoted by the SUM of the indices of the multipliers, not their PRODUCT.

EXAMPLES FOR PRACTICE.

1. Give the squares and cubes of the numbers from 1 to 12.
2. Find the 4th power of $\frac{2}{9}$. Of 4.6. *Ans.* $\frac{16}{6561}$; 447.7456.
3. Square 89. Cube 221. Involve .25 to the 5th power.
4. Find the value of the following indicated powers:—3^6; 14^4; 8^5; 7.2^2; $.01^4$; $(\frac{3}{4})^3$; $(\frac{2}{5})^4$; 1^9. *Sum of ans.* 71966.28747501.

CHAPTER XXXVI.

EVOLUTION.

497. Evolution is the process of resolving a number into two or more equal factors. One of these equal factors is called a **Root** of the number resolved.

$4 = 2 \times 2$. This process is Evolution; 2 is a root of 4.

494. What is the First Power of a number? What is the Second Power also called? The Third Power?—495. Recite the rule for Involution.—496. What may be done, in finding the higher powers? What caution is given?—497. What is Evolution?

Evolution is the opposite of Involution. In the latter, a root is given and a power required; in the former, a power is given and a root required.

498. Roots take their names, Square Root, Cube Root, Fourth Root, Fifth Root, &c., from those of the corresponding powers.

Roots are *indicated* by a character called the **Radical Sign**, $\sqrt{}$, placed before the number whose root is to be extracted.

The **Index** of a root is a figure placed above the radical sign at the left, to denote what root is to be taken,—that is, into how many equal factors the number is to be resolved. To express the square root, the radical sign is used without any index.

$\sqrt{4}$, read *square root of* 4, $= 2$, since $2 \times 2 = 4$.
$\sqrt[3]{8}$, " *cube root of* 8, $= 2$, " $2 \times 2 \times 2 = 8$.
$\sqrt[4]{16}$, " *fourth root of* 16, $= 2$, " $2 \times 2 \times 2 \times 2 = 16$.

The most important operations in Evolution are the extraction of the Square and the Cube Root.

Square Root.

499. Extracting the square root of a number is resolving it into *two* equal factors; as, $4 = 2 \times 2$.

500. Taking the smallest and the greatest number that can be expressed by one figure, by two, three, and four figures, let us see how the number of figures they contain compares with the number of figures in their squares:—

Roots,	1	9	10	99	100	999	1000	9999
Squares,	1	81	1'00	98'01	1'00'00	99'80'01	1'00'00'00	99'98'00'01

We find from these examples that, *if we separate a square into periods of two figures each, commencing at the right, there will be as many figures in the square root as there are periods in the square,—counting the left-hand figure, if there is but one, as a period.*

Of what is Evolution the opposite?—498. From what do roots take their names? How are roots indicated? What is the Index of a root? How is the square root expressed? What are the most important operations in Evolution?—499. What is meant by extracting the square root of a number?—500. How can we find, from a square, the number of figures its square root contains?

501. We derive the method of extracting the square root from the opposite operation of squaring. Square 25, regarding it as composed of 2 tens (20) and 5 units.

	$25 = 20 + 5$	$20^2 = 400$	25
	$20 + 5$	$20 \times 5 = 100$	25
Multiplying by 20,	$20^2 + (20 \times 5)$	$2 \times 100 = 200$	125
Multiplying by 5,	$(20 \times 5) + 5^2$	$5^2 = 25$	50
Adding partial products,	$20^2 + 2\,(20 \times 5) + 5^2 = 25$ squared	$= 625$	625

Hence, *The square of a number composed of tens and units, equals the square of the tens, plus twice the product of the tens and units, plus the square of the units.*

502. Now reverse the process. Find the sq. root of 625.

According to § 500, we separate 625 into periods of two figures each, beginning at the right (6'25), and find that the root will contain two figures,—a tens' and a units' figure.

According to § 501, 625 must equal the square of the tens in its root, plus twice the product of the tens and units, plus the square of the units. The *square of the tens* must be found in the left-hand period, 6(00). The greatest number whose square is less than 6 is 2, which we place on the right as the tens' figure of the root. 2 tens (20) squared = 4 hundreds, which we subtract from the 6 hundreds. Bringing down the remaining period, we have 225, which must equal *twice the product of the tens and units, plus the square of the units.*

$$
\begin{array}{rl}
 & 6'25\ (25 \\
 & \underline{4} \\
20 \times 2 = 40) & 225 \\
(40 \times 5) + 5^2 = & \underline{225}
\end{array}
$$

Hence, to find the units' figure of the root, we divide 225 by twice the tens, or 40. The quotient is 5, which we place in the root as its units' figure. Then, twice the product of the tens and units, plus the square of the units = twice 20 × 5, plus 25 = 225. Placing this under the dividend 225, and subtracting, we have no remainder.

$$
\begin{array}{rl}
 & 6'25\ (25 \\
 & \underline{4} \\
45 & 225 \\
 & \underline{225}
\end{array}
$$

In practice, we write twice the tens' figure (4) on the left as a trial divisor, and complete it by annexing the units' figure of the root. Multiplying this complete divisor by the units' figure, we have the same result, 225.

503. RULE.—1. *Separate the given number into periods of two figures each, beginning at the units' place.*

2. *Find the greatest number whose square is less than the left-hand period, and place it on the right as the first*

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501. Whence do we derive the method of extracting the square root? Square 25, regarding it as composed of 2 tens and 5 units. What principle is deduced from this example?—502. Reverse the process; extract the square root of 625, explaining the steps.—503. Recite the rule for the extraction of the square root.

root figure. Subtract its square from the first period, and to the remainder annex the second period for a dividend.

3. *Double the root already found, and, placing it on the left as a trial divisor, find how many times it is contained in the dividend with its last figure omitted. Annex the quotient to the root already found and to the trial divisor. Multiply the divisor thus completed by the last root figure, subtract, and bring down the next period as before.*

4. *To the last complete divisor add the last root figure for a new trial divisor, and proceed as before till the periods are exhausted.*

If any trial divisor is not contained in the dividend with its last figure omitted, annex 0 to the root already found and to the trial divisor, bring down the next period, and find how many times it is then contained.

If, on multiplying a complete divisor by the last root figure, the product is greater than the dividend, the last root figure must be diminished, and the figure annexed to the trial divisor changed accordingly.

If, when all the periods have been brought down, there is still a remainder, periods of decimal ciphers may be supplied and the operation continued. The root figures corresponding to the decimal periods will be decimals.

504. To point off a decimal for the extraction of the square root, commence at the decimal point and go to the right, completing the last period, if necessary, by annexing a cipher. Root figures resulting from decimal periods are always decimals.

Ex. 2.—Find the square root of 1524.1216.

	15'24.12'16 (39.04
	9
69	624
	621
7804	31216
	31216

505. To find the root of a common fraction, reduce it to its lowest terms, and extract the root of its numerator and denominator separately, if they have exact roots. If not, reduce the fraction to a decimal, and extract its root, carrying the operation as far as may be required. Reduce a mixed number to an improper fraction, and proceed as just directed.

506. To prove the operation, square the root found, and see whether the result equals the given number.

If any trial divisor is not contained in the dividend with its last figure omitted, what must be done? Under what circumstances must a root figure be diminished? If, when all the periods have been brought down, there is still a remainder, what may be done?—504. How is a decimal pointed off for the extraction of the square root? What root figures are always decimals?—505. How is the square root of a common fraction found? How is the square root of a mixed number found?—How is the operation proved?

EXAMPLES FOR PRACTICE.

1. What is the square root of 100180081? *Ans.* 10009.

2. What is the square root of 12321? Of 53824? Of 11390625? Of 16064064? *Sum of ans.* 7786.

3. Find the square root of 4489. Of 531441. Of 16983563041. Of 11019960576. Of 61917364224. *Sum of ans.* 484925.

4. Extract the square root of 6.5536. Of .00390625. Of .0011943936. Of 60.481729. *Sum of ans.* 10.43476.

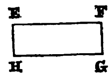
5. Find the square root of $\frac{256}{324}$. Of $4\frac{21}{25}$. Of $4\frac{1}{9}$. Of $\frac{18}{49}$. Of $\frac{10125}{12005}$. Of $\frac{148}{333}$. *Ans.* $\frac{8}{9}$, $2\frac{1}{5}$, 2.027 +, &c.

Applications of Square Root.

507. The areas (§ 252) of similar figures are to each other as the squares of their like dimensions. The areas of two circles whose diameters are 3 and 4 feet, are to each other as 3^2 to 4^2, or 9 to 16.

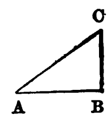
508. When the area of a square is known, extract its square root, to find one of the sides. The answer will be in the denomination of linear measure that corresponds to the denomination of the area. A square field containing 49 *square* rods will be 7 ($\sqrt{49}$) *linear* rods on each side.

509. A Rectangle is a four-sided figure whose angles are all right angles; as, E F G H.



510. A Triangle is a figure bounded by three straight lines.

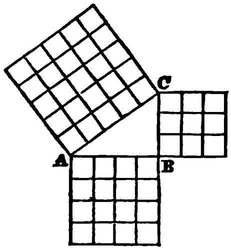
511. A Right-angled Triangle is a triangle that contains a right angle; as, A B C.



The Hypothenuse of a right-angled triangle is the side opposite the right angle; as, A C. Of the two shorter sides, the one on which the triangle stands (as A B) is called the Base, and the other (as B C) the Perpendicular.

507. What principle is laid down respecting the areas of similar figures?—508. How is the side of a square found from its area?—509. What is a Rectangle?—510. What is a Triangle?—511. What is a Right-angled Triangle? What is the Hypothenuse of a right-angled triangle? What is the Base? What is the Perpendicular?

512. It is shown, in Geometry, that *the square on the hypothenuse equals the sum of the squares on the other two sides.*



This principle is illustrated by the figure on the right. The small squares are all equal; it will be seen that the square of the hypothenuse contains 25, that of the base 16, that of the perpendicular 9. $25 = 16 + 9$. Hence these

RULES.—I. *The two shorter sides being given, to find the hypothenuse, add their squares and extract the square root of the sum.*

II. *The hypothenuse and one of the shorter sides being given, to find the other, subtract the square of the given side from that of the hypothenuse, and extract the square root of the remainder.*

Ex.—A liberty pole was broken 30 feet from the top, and the upper piece, falling over, struck the ground 18 ft. from the lower extremity. How high was the pole?

A right-angled triangle was formed, the broken part being the hypothenuse, the upright part the perpendicular, and the distance from the point where the top struck the ground to the foot of the pole the base. Applying Rule II., we find the perpendicular, or upright piece; which, added to the part broken off, gives the whole length.

$$30^2 - 18^2 = 576$$
$$\sqrt{576} = 24$$
$$24 + 30 = 54$$
Ans. 54 ft.

EXAMPLES FOR PRACTICE.

1. A flag-staff 36 ft. high was broken $\frac{1}{3}$ of the way up. How far from its foot did the top strike the ground? *Ans.* 26.8 ft. +

2. If a ladder 35 ft. long is placed 21 ft. from the base of a rock, how high up the rock will it reach? *Ans.* 28 ft.

3. A rope 45 ft. long, attached to the top of a house, extended to a log 36 ft. from its base. How high was the house?

4. Two persons start from the same place, and go, the one due

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512. What does the square on the hypothenuse equal? How is this principle illustrated? Recite the rule for finding the hypothenuse. Recite the rule for finding the base or perpendicular. Explain the example.

north 80 miles, the other due west 60 miles. How far apart are they?

5. What is the side of a square whose area is 121 square feet.

6. What is the distance between two opposite corners of a lot 50 feet by 50 feet? *Ans.* 70.7 ft. +

7. What is the distance between two opposite corners of a square whose area is 900 square feet? *Ans.* 42.426 ft. +

8. What is the distance between two opposite corners of a rectangle 15 rods long by 20 rods wide?

9. What distance will I save by walking directly across, from one corner of a plantation a mile square to the opposite corner, in stead of following the two sides? *Ans.* 187.452 rd.

10. A person lays out two circular plots, one containing 9 times as much land as the other. How do their diameters compare?

Cube Root.

513. Extracting the cube root of a number is resolving it into *three* equal factors; as, $8 = 2 \times 2 \times 2$.

514. Taking the smallest and the greatest number that can be expressed by one figure, by two and three figures, let us see how the number of figures they contain compares with the number of figures in the cubes:—

Roots,	1	9	10	99	100	999
Cubes,	1	729	1'000	970'299	1'000'000	997'002'999

We find from these examples that, *if we separate a cube into periods of three figures each, commencing at the right, there will be as many figures in the cube root as there are periods in the cube,—counting the left-hand figure or figures, if there are but one or two, as a period.*

515. We derive the method of extracting the cube root from the opposite operation of cubing. Cube 25, regarding it as composed of 2 tens (20) and 5 units.

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513. What is meant by extracting the cube root of a number?—514. How can we find, from a cube, the number of figures its cube root contains?—515. Whence do we derive the method of extracting the cube root? Cube 20 + 5.

The square of 20 + 5 was found in § 501; we multiply it by 20 + 5.

Square of 20 + 5 =	$20^2 + 2\,(20 \times 5) + 5^2$
	$20 \;+ 5$
Multiplying by 20,	$20^3 + 2\,(20^2 \times 5) + \;(20 \times 5^2)$
Multiplying by 5,	$(20^2 \times 5) + 2\,(20 \times 5^2) + 5^3$
Adding partial prod's,	$20^3 + 3\;(20^2 \times 5) + 3\,(20 \times 5^2) + 5^3 = 25^3$

As 5 is a common factor of the last three terms, the cube of 25, as just found, may be written as follows:—

$$20^3 + \left[\begin{array}{c} 3 \text{ times } 20^2 \\ + 3 \text{ times } 20 \times 5 \\ + 5^2 \end{array}\right] \times 5$$

$$\begin{array}{rcr} 20^3 & = & 8000 \\ 3\,(20^2 \times 5) & = & 6000 \\ 3\,(20 \;\times 5^2) & = & 1500 \\ 5^3 & = & 125 \\ \hline 25^3 & = & 15625 \end{array}$$

Hence, *the cube of a number composed of tens and units equals the*

$$\textit{cube of the tens} + \left[\begin{array}{c} \textit{3 times the square of the tens} \\ \textit{+ 3 times product of tens and units} \\ \textit{+ the square of the units} \end{array}\right] \times \textit{the units.}$$

516. Reverse the process; find the cube root of 15625.

According to § 514, we separate 15625 into periods of three figures each, beginning at the right (15'625), and find that the root will contain two figures,—a tens' and a units' figure.

The cube of the tens must be found in the left-hand period 15(000). The greatest number whose cube is contained in 15(000) is 2(0), which we place on the right as the tens' figure of the root. 2 tens (20) cubed = 8 thousands, which we subtract from the 15 thousands. Bringing down the remaining period, we have 7625; which, § 515, must equal

$$\begin{array}{l} 15'625 \;(2 \\ \;\;8 \\ \hline \;\;7625 \end{array}$$

$$\left[\begin{array}{c} \text{3 times the square of the tens} \\ \text{+ 3 times the product of the tens and units} \\ \text{+ the square of the units} \end{array}\right] \times \text{the units.}$$

Hence, to find the units' figure of the root, we divide 7625 by 3 times the square of the tens as a trial divisor. It is contained 6 times; but, making allowance for the completion of the trial divisor, we regard the quotient as 5, and write 5 in the root as its units' figure. Now, to complete the divisor, we have to add to 3 *times the square of the tens*, already found, 3 *times the product of the tens and units* (20 × 5 × 3 = 300), and *the square of the units* ($5^2 = 25$),—making 1525. Multiplying this by the units' figure, and subtracting, we have no remainder. *Ans.* 25.

$$\begin{array}{rr} & 15'625 \;(25 \\ & 8 \\ \hline \text{Trial div., } 20^2 \times 3 = 1200 & 7625 \\ 20 \times 5 \times 3 = \;\;300 & \\ 5^2 = \;\;\;\;25 & \\ \hline \text{Complete divisor, } 1525 & 7625 \\ \hline \end{array}$$

How may the cube just found be written? Hence, what does the cube of a number composed of tens and units equal?—516. Reverse the process; extract the cube root of 15625, explaining the steps.—517. Recite the rule.

517. Rule.—1. *Separate the given number into periods of three figures each, beginning at the units' place.*

2. *Find the greatest number whose cube is less than the left-hand period, and place it on the right as the first root figure. Subtract its cube from the first period, and to the remainder annex the second period for a dividend.*

3. *Take three times the square of the root already found; and, annexing two ciphers, place it on the left as a trial divisor. Find how many times the trial divisor is contained in the dividend (making some allowance), and annex the quotient to the root already found. Complete the trial divisor, by adding to it* 30 *times the product of the last root figure and the root previously found, also the square of the last root figure. Multiply the divisor, thus completed, by the last root figure, subtract the product from the dividend, and bring down the next period as before.*

4. *Repeat the processes in the last paragraph, till the periods are exhausted.*

If any trial divisor is not contained in its dividend, place 0 in the root, annex two ciphers to the trial divisor, bring down the next period, and find how many times it is then contained.

If, on multiplying a completed divisor by the last root figure, the product is greater than the dividend, the last root figure must be diminished, and the necessary changes made in completing the divisor.

Separate a decimal into periods, from the decimal point to the right, completing the last period, if necessary, by annexing one or two ciphers.

To find the cube root of a common fraction, see § 505.

To prove the operation, cube the root found.

Ex. 2.—Extract the cube root of 348616. 378872.

		348′616.378′872 (70.38
$7^2 \times 3 =$	147	343
1st trial divisor,	14700	5616
2d trial divisor,	1470000	5616378
$70 \times 3 \times 30 =$	6300	
$3^2 =$	9	
Complete divisor,	1476309	4428927
3d trial divisor,	148262700	1187451872
$703 \times 8 \times 30 =$	168720	
$8^2 =$	64	
Complete divisor,	148431484	1187451872

EXAMPLES FOR PRACTICE.

1. Extract the cube root of 2357947691. *Ans.* 1331.

2. What is the cube root of 91125? Of 7256313856? Of 387420489? Of 10077696? *Sum of ans.* 2926.

3. Extract the cube root of 42875. Of 125450540216. Of 28387903294872. Of 117649. *Sum of ans.* 75128.

4. What is the cube root of 18.609625? Of .065450827? Of .000000008? Of 1.25992105, carried to five decimal places? Of 8, to four decimal places? *Sum of ans.* 5.57725.

5. Find the cube root of $\frac{2197}{4913}$. Of $\frac{3645}{625}$. Of $17\frac{1}{8}$. Of $\frac{32}{108}$. Of $\frac{3456}{686}$. Of $\frac{85169}{170723}$. Of $\frac{2}{3}$. Of $101\frac{1}{7}$. *Ans.* $\frac{13}{17}$, $1\frac{4}{5}$, 2.577 +, &c.

518. The solid contents of similar bodies are to each other as the cubes of their like dimensions. The solid contents of two globes whose diameters are 6 in. and 12 in., are to each other as 6^3 to 12^3, or 216 to 1728.

519. When the solid contents of a cube are known, extract the cube root, to find one of the sides. The answer will be in the denomination of linear measure that corresponds to the denomination of the solid contents. A cubical block whose solid contents are 8 *cubic* inches, will be 2 ($\sqrt[3]{8}$) *linear* inches on each side.

6. If a ball 3 in. in diameter weighs 8 lb., what will a ball of equal density, whose diameter is 4 in., weigh? *Ans.* $18\frac{26}{27}$ lb.

7. What is the side of a cube whose solid contents equal those of a rectangle, 8 ft. 3 in. long, 3 ft. wide, and 2 ft. 7 in. deep? *Ans.* 47.9843 in.

8. What is the side of a cube containing 2197 cu. in.?

9. There are three balls whose diameters are respectively 3, 4, and 5 inches. What is the diameter of a fourth ball, of the same density, equal in weight to the three? *Ans.* 6 in.

10. If a ball 12 in. in diameter weighs 238 lb., what will be the diameter of another ball of the same metal, weighing 32 lb.?

518. What principle is laid down respecting the solid contents of similar bodies? —519. How is the side of a cube found from its solid contents?

CHAPTER XXXVII.

PROGRESSION.

520. Progression is a regular increase or decrease in a series of numbers.

521. There are two kinds of Progression, Arithmetical and Geometrical.

A series of numbers are said to be in **Arithmetical Progression,** when they increase or decrease by a common difference: as, 16, 18, 20, 22; 16, 14, 12, 10.

A series of numbers are said to be in **Geometrical Progression,** when they increase or decrease by a common ratio: as, 16, 32, 64, 126; 16, 8, 4, 2.

522. The numbers forming the series are called **Terms.** The first and the last term are the **Extremes,** the intermediate terms the **Means.**

523. When the terms increase, they form an **Ascending Series;** when they decrease, a **Descending Series.**

Arithmetical Progression.

524. In Arithmetical Progression, there are five things to be considered: the First Term, the Last Term, the Number of Terms, the Common Difference, and the Sum of the Series. Three of these being given, the other two can be found.

To find the relations between these five elements, let us look at the series that follow, in which the first term is 13, the common difference 2, and the number of terms 5:—

Ascending, 13, $13+2$, $13+2+2$, $13+2+2+2$, $13+2+2+2+2$.
Descending, 13, $13-2$, $13-2-2$, $13-2-2-2$, $13-2-2-2-2$.

It will be seen that the *second* term equals the 1st, plus (in the descending series, minus) *once* the common difference; the *third* term

520. What is Progression?—521. How many kinds of Progression are there? When are numbers said to be in Arithmetical Progression? When, in Geometrical Progression? Give examples.—522. What are the numbers forming the series called? What are the Extremes? What are the Means?—523. What is an Ascending Series? What is a Descending Series?—524. How many things are to be considered in Arithmetical Progression? Name them. How many of these must be given, to find the rest?

equals the 1st, plus (or minus) *twice* the common difference; the *fourth* term equals the 1st, plus (or minus) *three* times the common difference. And, generally, any term equals the first term, increased (or diminished) by the common difference taken as many times as the number that represents the term, less 1. Hence the following rule:—

RULE I.—*The first term, common difference, and number of terms being given, to find the last term, multiply the common difference by the number of terms less* 1, *and add the product to (or in a descending series subtract it from) the first term.*

525. Again, looking at the series, we see that the last term equals the first term plus (or minus) the common difference taken as many times as there are terms, less 1. Hence the following rules:—

RULE II.—*The extremes and number of terms being given, to find the common difference, divide the difference of the extremes by the number of terms less* 1.

RULE III.—*The extremes and common difference being given, to find the number of terms, divide the difference of the extremes by the common difference, and to the quotient add* 1.

526. To find the average value of the terms of a series, we add the extremes (the greatest and the least term), and divide their sum by 2. Having thus found the average, if we multiply it by the number of terms, we shall have the sum of the series.

RULE IV.—*The extremes and number of terms being given, to find the sum of the series, multiply half the sum of the extremes by the number of terms.*

527. These principles are embodied in the following formulas:—

a = first term,	Then, $l = a \pm d \times (n-1)$.
l = last term,	$d = \frac{l-a}{n-1}$ or $\frac{a-l}{n-1}$.
n = number of terms,	
d = common difference,	$n = \frac{l-a}{d} + 1$ or $\frac{a-l}{d} + 1$.
s = sum of series.	$s = \frac{a+l}{2} \times n$.

In solving the examples, ask what is given, and what required, and apply the proper rule or formula.

Examining the two series that are given, what do we find the second term equals? The third? The fourth? What general principle is deduced? Recite Rule I.—525. Again, looking at the series, what do we find that the last term equals? Recite Rule II. Recite Rule III.—526. How may we find the average value of the terms of a series? How may we find the sum of the series? Recite Rule IV.

Ex. 1.—A person made 12 deposits in a bank, increasing them each time by a common difference. His first deposit was \$50, and his last \$160; what were the intermediate ones?

Here we have given the extremes, \$50 and \$160, and the number of terms, 12. The means are required, and to form them we need the common difference. Apply Rule II.

$160 - 50 = 110$, difference of extremes.
$110 \div 11 = 10$, common difference.
\$60, \$70, \$80, \$90, &c., intermediate deposits. *Ans.*

Ex. 2.—A falling body moves $16\frac{1}{12}$ ft. during the first second of its descent, and $144\frac{3}{4}$ ft. the fifth second. How far does it fall in five seconds?

Here we have the extremes, $16\frac{1}{12}$ and $144\frac{3}{4}$, and the number of terms, 5. The sum of the series is required. Apply Rule IV.

$16\frac{1}{12} + 144\frac{3}{4} = 160\frac{5}{6}$, sum of the extremes.
$160\frac{5}{6} \div 2 = 80\frac{5}{12}$, half the sum of the extremes.
$80\frac{5}{12} \times 5 = 402\frac{1}{12}$ ft., whole distance. *Ans.*

EXAMPLES FOR PRACTICE.

1. A field of corn containing 50 rows has 20 hills in the first row, 23 in the second, and so on in arithmetical progression. How many hills in the last row? *Ans.* 167 hills.

2. A person travelling 25 days went 11 miles the first day and 135 the last, increasing the number each day by a common difference. How far did he travel each of the intervening days, and how far in all? *Last ans.* 1825 miles.

3. A note is paid in annual instalments, each less than the previous one by \$3. The first payment being \$49, and the last \$7, how many instalments were there? *Ans.* 15.

4. A man has 7 sons, whose ages are in arithmetical progression. The eldest being 23, and the youngest 5, what is the difference in age between the youngest and his next elder brother?

5. Bought 100 yd. of cloth. The first yard cost £1 15s. 6d., and each of the others 4d. less than the preceding one. What did the last yard cost, and what the whole? *First ans.* 2s. 6d.

6. What is the 20th term of the series, 8, 15, 22, &c.?

Geometrical Progression.

528. In Geometrical Progression, there are five things to be considered: the First Term, the Last Term, the Number of Terms, the Ratio, and the Sum of the Series. Three of these being given, the other two can be found.

529. Look at the following series, in which the first term is 6, the ratio 2, and the number of terms 5:—

$$6,\quad 6 \times 2,\quad 6 \times 2 \times 2,\quad 6 \times 2 \times 2 \times 2,\quad 6 \times 2 \times 2 \times 2 \times 2.$$
$$\text{Or, } 6,\quad 6 \times 2,\quad 6 \times 2^2,\quad 6 \times 2^3,\quad 6 \times 2^4.$$

It will be seen that each term consists of the first term, 6, multiplied by the ratio, 2, raised to a power whose index is 1 less than the number of the term. Hence the following rule:—

RULE I. *The first term, ratio, and number of terms being given, to find the last term, multiply the first term by that power of the ratio whose index is* 1 *less than the number of terms.*

530. Suppose the sum of the series 6, 18, 54, 162, 486, is required. Multiplying each term by 3 (the ratio), we form a second series whose sum is 3 times as great. Then, subtracting the 1st series from the 2d, we have a result twice as great as the sum of the 1st series.

	18,	54,	162,	486,	1458	=	3 times.
6,	18,	54,	162,	486,		=	once.
					1458 − 6	=	twice.

Cancelling the intermediate terms, we have 1458 − 6 for the result, which, being twice as great as the sum of the 1st series, we divide by 2. Now 1458 is the last term multiplied by the ratio (486 × 3); and 2, by which we divide, is the difference between the ratio and 1 (3 − 1). Hence,

RULE II.—*The extremes and the ratio being given, to find the sum of the series, multiply the last term by the ratio, find the difference between this product and the first term, and divide it by the difference between the ratio and* 1.

FORMULAS:— $l = a \times r^{n-1}$

$r =$ ratio $\qquad s = \frac{(l \times r) - a}{r - 1}$ or $\frac{a - (l \times r)}{1 - r}$.

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528. How many things are to be considered in Geometrical Progression? Name them. How does the ratio compare with 1 in an ascending series? How, in a descending series?—529. Looking at the given geometrical series, of what will it be seen that each term consists? Recite Rule I.—530. Go through the reasoning by which the rule for the sum of the series is arrived at. Recite Rule II.

531. In a descending *infinite* series, as 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, &c., the last term is infinitely small, and may be regarded as 0.

Ex. 1.—What is the amount of \$250, for 6 years, at 6 %, compound interest?

The principal is the first term of a geometrical series. The amount of \$1, for 1 year, at 6 %, is the ratio. 6 (years) + 1 (the principal being the first term) is the number of terms. The amount required is the last term. Apply Rule I., § 529.

$$1.06^6 = 1.418519112256$$
$$1.418519112256 \times 250 = \$354.629 \textit{ Ans.}$$

Ex. 2.—What is the sum of a series of 8 terms, commencing 200, 50, $12\frac{1}{2}$, &c.?

Here the first term, the ratio ($50 \div 200 = \frac{1}{4}$), and the number of terms are given, and the sum of the series is required. We first apply Rule I., § 529, to find the last term; and then Rule II., § 530, to find the sum.

$$(\tfrac{1}{4})^7 = \tfrac{1}{16384} \qquad \tfrac{1}{16384} \times 200 = \tfrac{25}{2048}, \text{ last term.}$$
$$\tfrac{25}{2048} \times \tfrac{1}{4} = \tfrac{25}{8192}$$
$$200 - \tfrac{25}{8192} = 199\,\tfrac{8167}{8192}$$
$$199\,\tfrac{8167}{8192} \times \tfrac{4}{3} = 266\,\tfrac{1857}{2048} \textit{ Ans.}$$

EXAMPLES FOR PRACTICE.

1. A person goes $2\frac{1}{2}$ miles the first day, 5 the second, and so on in geometrical progression. If he travels thus for 8 days, how far will he go the last day? How far in all? *Last ans.* $637\frac{1}{2}$ mi.

2. B invested \$1000 so that it would double itself every four years. What did his capital amount to at the end of the twelfth year? At the end of the twentieth year?

3. What is the amount of \$800, for 5 years, at 7 %, compound interest? *Ans.* \$1122.04.

4. If ten stones are laid in a line, the first 3 ft. from a basket, the second 9, the third 27, and so on in progression, how far must a person starting from the basket walk, to pick them up singly and place them in the basket? *Ans.* $83\frac{121}{220}$ mi.

5. First term, 100; ratio, $\frac{1}{2}$; number of terms, 9. Required the sum of the series. *Ans.* $199\,\frac{39}{64}$.

6. Find the sum of the infinite series 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, &c. (§ 531).

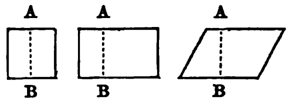
7. Find the sum of the infinite series 1, $\frac{1}{4}$, $\frac{1}{16}$, &c. *Ans.* $1\frac{1}{3}$.

CHAPTER XXXVIII.

MENSURATION.

532. Mensuration is that branch which gives rules for finding the length of lines, the areas of surfaces, and the solidity of bodies. These rules are derived from Geometry.

Several rules of Mensuration have been already given; as, those relating to the sides of right-angled triangles, § 512. Some of the others that are most important are given below.



533. PARALLELOGRAMS.—A Parallelogram is a four-sided figure that has its opposite sides equal and parallel. A square and a rectangle are parallelograms.

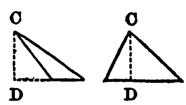
The Base of a parallelogram is the side on which it stands. Its Altitude is the perpendicular distance from its base to the opposite side; as, A B in the figures.

RULE.—*To find the area of a parallelogram, multiply the base by the altitude.*

1. How many square feet of surface will be covered by 12 boards 18 ft. long and 18 in. wide? *Ans.* 324 sq. ft.

2. Find the cost of a piece of land 40 ch. 15 l. square, at $30 an acre. *Ans.* $4836.0675.

3. What is the difference between the areas of two parallelograms, the one 80 ft. long and having an altitude of 20 ft., the other having a length of 30 ft. and an altitude of 25 ft.?



534. TRIANGLES.—The Altitude of a Triangle is a perpendicular drawn from one of its angles to the base, or the base produced; as, C D.

RULE.—*To find the area of a triangle, multiply its base by half its altitude.*

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532. What is Mensuration?—533. What is a Parallelogram? What is its Base? What is its Altitude? Recite the rule for finding the area of a parallelogram.—534. What is the Altitude of a Triangle? Recite the rules for finding the area of a triangle.

Or, when the three sides are given, from half their sum subtract each side separately, multiply together the three remainders and the half sum, and extract the square root of their product.

4. What is the area of a triangle whose base is 12 feet and its altitude 3 yards? *Ans.* 54 sq. ft.

5. What is the area of a triangle whose sides are respectively 7, 11, and 12 feet? *Ans.* 37.94 sq. ft. +

6. In a triangular field whose sides are 18, 80, and 82 feet, how many square yards?

535. CIRCLES.—The Circumference, Diameter, and Radius of a Circle are defined on page 157.

RULES.—I. *To find the circumference of a circle, multiply the diameter by* 3.14159.

II. *To find the diameter, multiply the circumference by* .3183.

III. *To find the area, multiply* $\frac{1}{2}$ *the circumference by the diameter.*

Or, multiply the square of the circumference by .07958.

Or, multiply the square of the diameter by .7854.

7. The diameter of the earth being 7926 miles, what is its circumference? *Ans.* 24900.24234 mi.

8. Over what distance will a wheel 4 ft. 9 inches in diameter pass, in making four revolutions? *Ans.* 59.69021 ft.

9. If the tire of a wheel is 14.3235 ft. in circumference, what is its diameter?

10. What is the area of a circular plot requiring 40 rods of hedge to enclose it?

11. If I describe a circle with a rope 40 ft. long, fixed at one end, what will be its area? *Ans.* 5026.56 sq. ft.

12. A circle contains 415.4766 sq. inches, what is the square of its diameter? What is its diameter? *Last ans.* 23 in.

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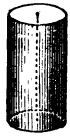
535. What is the Circumference of a circle? The Diameter? The Radius? Recite the rule for finding the circumference. The diameter. The area.

536. CYLINDERS.—A **Cylinder** is a body of uniform diameter, bounded by a curved surface, and two equal and parallel circles, either of which may be regarded as its base.



The Altitude of a cylinder is the perpendicular distance between its bases.

RULES.—I. *To find the surface of a cylinder, multiply the circumference of the base by the altitude, and to the product add twice the area of the base.*

II. *To find the solidity of a cylinder, multiply the area of the base by the altitude.*

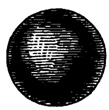
The base being a circle, its area may be found by Rule III., § 535.

13. How many square feet in the surface of a stove-pipe 20 feet long and 5 inches in diameter? *Ans.* 26.452 sq. ft. +

14. How many gallons (wine) will a cylindrical cistern hold, that is 15 ft. deep and 4 ft. across? *Ans.* 1410.048 gal.

15. A cylindrical piece of timber is 24 feet long and 18 inches across; what will it cost, at 20c. a cubic foot? *Ans.* $8.48.

537. SPHERES.—A **Sphere** is a body bounded by a curved surface, every point of which is equally distant from a point within, called the centre.



RULES.—I. *To find the surface of a sphere, multiply the square of the diameter by* 3.14159.

II. *To find the solidity of a sphere, multiply the cube of the diameter by* .5236.

15. Required the surface and solidity of a sphere 30 inches in diameter. *Ans.* 19 sq. ft., 91.431 sq. in.; 8 cu. ft., 313.2 cu. in.

16. The diameter of the earth is 7926 miles; if it were a perfect sphere, how many square miles would its surface contain?

17. Required the solidity of a sphere 2 yd. in diameter. How many square yards in its surface?

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536. What is a Cylinder? What is the Altitude of a cylinder? Recite the rule for finding the surface of a cylinder. For finding the solidity of a cylinder.—537. What is a Sphere? Give the rule for finding the surface of a sphere. For finding the solidity of a sphere.

538. Miscellaneous Examples.

1. If C and D retired at the same hour daily, but C rose at ¼ before 6 and D at half past 7, how much more working time had C than D in the years 1864 and 1865? *Ans.* 1279¼ hr.

2. How many acres, roods, &c., in a rectangular field 12 ch. 34 l. long and 10 ch. 85 l. wide? *Ans.* 13 A. 1 R. 22.224 sq. rd.

3. If 1 gal. 1 qt. 2 gi. of liquid passes through a filter in 1 hour, how much will pass through in 4 hr. 19 min. 24 sec.?

Ans. 5 gal. 2 qt. 1 pt. 1.58 gi.

4. How many square feet of glass in 12 windows, each having 12 panes, and each pane being 1 ft. 3′ by 11′? *Ans.* 165 sq. ft.

5. A grocer sold 10% of his stock of sugar, and then 10% of what was left. 60 cwt. 75 lb. remained; what was his original stock? *Ans.* 75 cwt.

6. If I divide ⅓ of a section of land into 13 equal parts, how many acres, &c., in each part? *Ans.* 16 A. 1 R. 25$\frac{11}{13}$ P.

7. Sold, Mobile, Feb. 1, 1866, 50 bales of cotton, averaging 426 lb. to the bale, at 45c. a pound. May 15, 1866, received the money, with legal interest from the date of sale. How much did I receive? *Ans.* $9806.52.

8. Find the amount of £1600 14s. 8d., at 4%, for 18 days.

9. When it is 10 minutes past 6 o'clock at Chicago, it is 22 min. 43 sec. past 6 at Cincinnati. What is the difference of longitude between these cities? *Ans.* 3° 10′ 45″.

10. A New York merchant, having £350 to pay in London, buys a draft for that amount with gold at 150, exchange standing at 109. He might in stead have remitted 5-20's, then selling at 104 in N. Y. and worth 64 in London. Would he have gained or lost by so doing, and how much? *Ans.* Gained $15.55⅙.

11. A rectangular piece of land containing half an acre is five times as long as it is broad. Required its length and breath.

Ans. Length, 20 rd.; breadth, 4 rd.

12. In a mixture of wine and cider, ½ of the whole + 25 gallons was wine, and ⅓ of the whole − 5 gallons cider. How many gallons were there of each? *Ans.* 85 gal. wine, 35 gal. cider.

13. Divide \$2000 into shares that shall be to each other as 8, 7, 6, and $\frac{1}{2}$. *Ans.* \$744.18$\frac{26}{43}$, \$651.16$\frac{12}{43}$, \$558.13$\frac{41}{43}$, \$46.51$\frac{7}{43}$.

14. How many rods of hedge will be required to enclose a circular plot containing 1 acre? To enclose a square plot containing an acre? To enclose an acre in the form of a right-angled triangle whose altitude is twice its base?

Ans. 44.83 rd. + ; 50.596 rd. + ; 66.231 rd. +

15. What will be the length of a diagonal from a lower corner to the opposite upper corner of a cubical vat 9 feet on each side?

Ans. 15.58 ft. +

16. There are two globes, one having a diameter of 10 in., the other a circumference of 37.69908 in. How many more square inches in the surface of one than in that of the other? How many more cubic inches in one than in the other? *First ans.* 138.22996 sq. in.

17. A person spent £100 for some geese, sheep, and cows, paying for each goose 1s., for each sheep £1, and for each cow £5. How many did he purchase of each kind, so as to have 100 in all? *Ans.* 80 geese, 1 sheep, 19 cows.

18. A hare is 50 leaps before a hound, and takes 4 leaps to the hound's 3, but 2 leaps of the hound are equal to 3 of the hare's. How many leaps must the hound take, before he catches the hare? *Ans.* 300 leaps.

19. A general, wishing to draw up his men in a square, found on the first trial that he had 39 men over. The second time, having placed one more man in rank, he needed 50 to complete the square. How many men had he? *Ans.* 1975 men.

20. A, B, and C, start from the same point, and travel in the same direction, round an island 20 miles in circumference. A goes 8 miles an hour, B 7, and C 11. In what time will they all be together? *Ans.* At the end of 5 hours.

21. From a cask containing 10 gallons of wine, a servant drew off 1 gallon each day, for five days, each time supplying the deficiency by adding a gallon of water. Afterwards, fearing detection, he again drew off a gallon a day for five days, adding each time a gallon of wine. How many gallons of water still remained in the cask? *Ans.* 2.418115599 gal.

22. If I purchase \$1200 worth of goods, $\frac{1}{3}$ on 3 months' credit, $\frac{1}{3}$ on 6 months, and $\frac{1}{3}$ on 9 months, what amount in cash would pay the bill, money being worth 7%? *Ans.* \$1159.64.

23. In the above example, what would be the equated time for paying the whole amount, \$1200, at once?

24. How many times will the second-hand of a watch go round its circle, in 12 wk. 2 hr. 15 min.? *Ans.* 121095 times.

25. Find the sum of the infinite series 1, $\frac{1}{4}$, $\frac{1}{16}$, &c. *Ans.* $1\frac{1}{3}$.

26. A and B had the same income. A saved $\frac{1}{5}$ of his; but B, spending \$120 a year more than A, at the end of 10 years was \$200 in debt. What was the income? *Ans.* \$500.

27. A father gave his five sons \$1000, to divide in such a way that each should have \$20 more than his next younger brother. What was the share of the youngest? *Ans.* \$160.

28. A and B, starting from opposite points of a fish-pond 536 feet in circumference, begin to walk around it at the same time, in the same direction. A goes 62 yards a minute, B 68 yards. In what time will B overtake A, and how far will A have walked?

Last ans. $923\frac{1}{9}$ yd.

29. A, B, and C, commence trade with \$3058.25, and gain \$610.65. The sum of A's and B's capital is to the sum of B's and C's as 5 to 7; and C's capital diminished by B's, is to C's increased by B's, as 1 to 7. What is each one's share of the gain?

Ans. A's, \$135.70; B's, \$203.55; C's, \$271.40.

30. A man left \$1000 to be divided between his two sons, one 14 years old, the other 18, in such a proportion that the share of each, being put at interest at 6%, might amount to the same sum when they reached the age of 21. How much did each receive?

Ans. Elder, \$546.15; younger, \$453.85.

31. If \$100 is divided between D, E, and F, so that E may have \$3 more than D, and F \$4 more than E, how much will each have? *Ans.* D, \$30; E, \$33; F, \$37.

32. P and Q have equal incomes. P contracts an annual debt amounting to $\frac{1}{7}$ of his; Q lives on $\frac{4}{5}$ of his, and at the end of 10 years lends P enough to pay off his debt, and has \$320 left. What is the income? *Ans.* \$560.

TABLES OF MONEYS, WEIGHTS, MEASURES, &c.

Federal Money.

10 mills,	1 cent.
10 cents,	1 dime.
10 dimes,	1 dollar.
10 dollars,	1 eagle.

Sterling Money.

4 farthings,	1 penny.
12 pence,	1 shilling.
20 shillings,	1 pound.
21 shillings,	1 guinea.

Troy Weight.

24 grains,	1 pennywt.
20 pennywts.,	1 ounce.
12 ounces,	1 pound.

Apothecaries' Weight.

20 grains,	1 scruple,	℈.
8 scruples,	1 dram,	ʒ.
8 drams,	1 ounce,	℥.
12 ounces,	1 pound,	℔.

Avoirdupois Weight.

16 drams,	1 ounce.
16 ounces,	1 pound.
25 pounds,	1 quarter.
4 quarters,	1 hundred-wt.
20 hundred-wt.,	1 ton.

Miscellaneous.

14 lb.,	1 stone (iron, lead).
100 lb.,	1 quintal.
100 lb.,	1 cask of raisins.
196 lb.,	1 barrel of flour.
200 lb.,	1 bar. beef, pork.

Linear Measure.

12 inches,	1 foot.
3 feet,	1 yard.
5½ yards,	1 rod.
40 rods,	1 furlong.
8 furlongs,	1 mile.

Cloth Measure.

2¼ inches,	1 nail.
4 nails,	1 quarter.
4 quarters,	1 yard.
3 quarters,	1 Ell Flemish.
5 quarters,	1 Ell English.
6 quarters,	1 Ell French.

Surveyors' Measure.

7.92 inches,	1 link.
100 links,	1 chain.
80 chains,	1 mile.

10 sq. chains,	1 acre.
640 acres, . . .	1 sq. mile.

Square Measure.

144 sq. inches,	1 sq. ft.
9 sq. feet,	1 sq. yd.
30¼ sq. yards,	1 sq. rd.
40 sq. rods,	1 rood.
4 roods,	1 acre.
640 acres,	1 sq. mi.

Cubic Measure.

1728 cubic inches,	1 cu. ft.
27 cubic feet,	1 cu. yd.
40 cu. ft. of round, or 50 cu. ft. hewn timber,	1 ton.
16 cubic feet,	1 cd. ft.
8 cord-feet,	1 cord.

Liquid Measure.

4 gills,	1 pint.
2 pints,	1 quart.
4 quarts,	1 gallon.
31½ gallons,	1 barrel.
2 barrels,	1 hogshd.
2 hhd.,	1 pipe.
2 pipes,	1 tun.

Beer Measure.

2 pints,	1 quart.
4 quarts,	1 gallon.
36 gallons,	1 barrel.
1½ barrels,	1 hogshead.

1 Beer Gallon = 282 cu. in.
1 Wine Gallon = 231 cu. in.

Dry Measure.

2 pints,	1 quart.
8 quarts,	1 peck.
4 pecks,	1 bushel.
36 bushels,	1 chaldron.

1 Small Measure = 2 quarts.
1 Bushel = 2150.42 cu. in.

Time.

60 seconds,	1 minute.
60 minutes,	1 hour.
24 hours,	1 day.
7 days,	1 week.
365 days,	1 year.
366 days,	1 leap year.
100 years,	1 century.

Circular Measure.

60 seconds (″),	1 minute,	′
60 minutes,	1 degree,	°
30 degrees,	1 sign,	S.
12 signs,	1 circle,	C.

Paper.

24 sheets,	1 quire.
20 quires,	1 ream.
2 reams,	1 bundle.
5 bundles,	1 bale.

Collections of Units.

12 units,	1 dozen.
12 dozen,	1 gross.
12 gross,	1 great gross.
20 units,	1 score.

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